Part IA Paper 1: Mechanical Engineering

MECHANICS

EXAMPLES PAPER 2

Questions marked with a † are of a straightforward nature: those marked * of Tripos standard.

URLs for some web pages related to examples paper questions may be found at
www.eng.cam.ac.uk/~hemh/IAexamples.htm

Kinematics of a rigid body

1.† A 3-D rigid body is rotating about an axis at an angular rate of 10 rads^{-1}. The positive direction of the axis is defined by $3\underline{i} + 2\underline{j}$. If a fixed point O in the rigid body lies on the axis, determine the velocity of a point P in the body when the instantaneous vector $\underline{OP}$ is $(20\underline{i} + 10\underline{j})$ mm.

2. † A playground roundabout of radius 2 m shown in Fig. 1 rotates in an anticlockwise direction about O at 1.5 rads^{-1} . A child holds on to the roundabout at A , a distance of 2 m from O . A second child holds on to the roundabout at B , a distance of 1.5 m from O .

(a) Express the angular velocity of the roundabout as a vector.
(b) What is the angular velocity of each child?
(c) What is the angular velocity of the vector $\underline{AB}$?
(d) Determine the velocities of A and B.
(e) Determine the velocity of A relative to B , and the velocity of B relative to A.
(f) Show that the velocity of B is equal to the velocity of A plus the velocity of B relative to A.

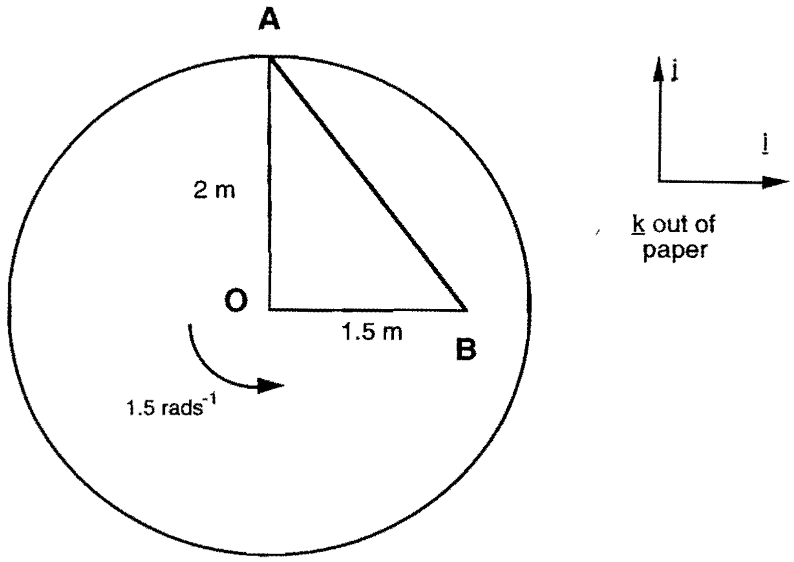


Figure 1

3. † A person throws a uniform rigid stick AB , which is 0.8 m long, and it goes spinning across the horizontal surface of a frozen lake. At a particular instant the velocity of end A is as shown in Fig. 2. The direction of the velocity of end B is a shown in Fig. 2 and is known to be correct. The magnitude of the velocity at B is *thought* to be 8 ms^{-1}.

(a) Is the magnitude of the velocity of the end B correct? If not, what should it be?
(b) Determine the angular velocity of the stick as a vector.
(c) Find the velocity of the centre of gravity of the stick:
 (i) using $\underline{v}_G = \underline{v}_A + \underline{\omega} \times \underline{r}_{G/A}$;
and (ii) by taking the average of $\underline{v}_A$ and $\underline{v}_B$.

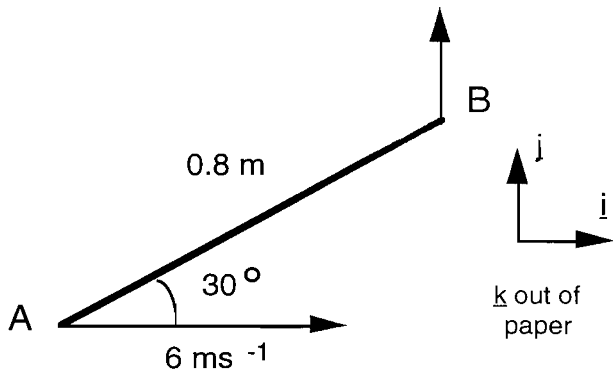


Figure 2

4. The cylinder shown in Fig. 3 rolls freely at an absolute angular speed of 15 $rads^{-1}$ on the surface of a conveyor belt moving at a constant speed of 2 ms^{-1}. Assuming *no slipping* occurs between the cylinder and the belt at A , determine for the instant shown:

(a) the velocity of B relative to A
(b) the absolute velocity of B
(c) the velocity of C relative to B
(d) the absolute velocity of C, using answers (b) and (c)
(e) the velocity of C relative to A
(f) the absolute velocity of C, using answer (e).

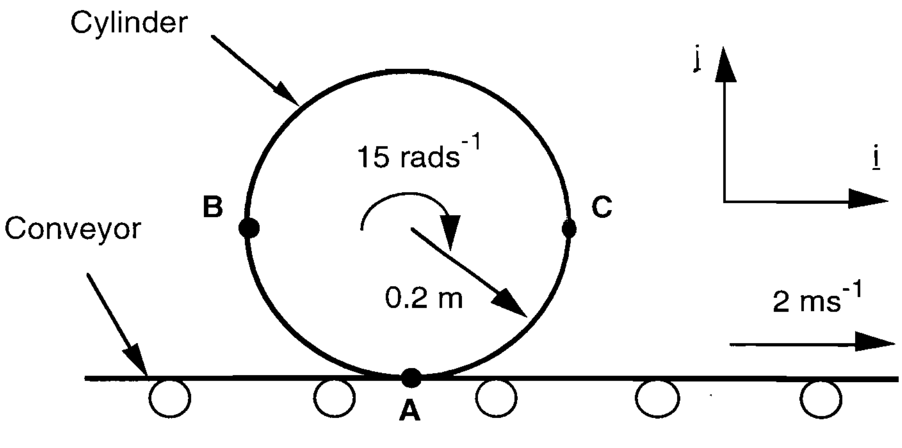


Figure 3

5. Point A, coordinates (1,1,0) m, and point B, coordinates (2,0,1) m, are fixed within a 3-D rigid body and have absolute velocities $(4\underline{i} + 2\underline{k})$ ms^{-1} and $(9\underline{i} - \underline{j} - 4\underline{k})$ ms^{-1} respectively. Check that the given velocities are consistent with the body being rigid.

Find the angular velocity $\underline{\omega} = \omega_X \underline{i} + \omega_Y \underline{j} + \omega_Z \underline{k}$ of the body when the component ω_Z in the $\underline{k}$ direction is

(i) $\omega_Z = 0$ rads^{-1} and (ii) $\omega_Z = 1$ rads^{-1}.

This leads to an apparent contradiction, that is we have found two *different* angular velocities giving the same relative velocity $\underline{v}_{B/A}$ for two points fixed in a rigid body. Subtract the two angular velocity vectors. Show that this difference gives an angular velocity vector that is parallel to $\underline{AB}$. Use this to explain the apparent contradiction.

Note 1: This question involves a 3-D body and is therefore not strictly part of the IA Mechanics course. However, it demonstrates a very important concept about angular velocities. It should be treated as an interesting challenge question.

Instantaneous centres

6. (a)† Locate the instantaneous centre for the stick in Fig. 2 and use it to confirm that the stick's angular velocity is $15\,\underline{k}$ rads^{-1}.

(b)† The same stick is shown in Fig. 4 but the motion is different. Locate the instantaneous centre at the instant shown.

(c) Locate the instantaneous centre for the cylinder shown in Fig. 3.

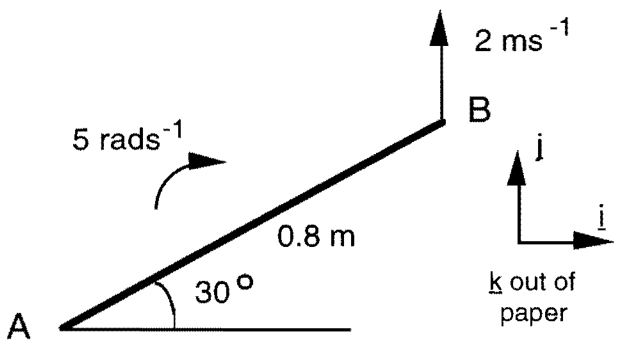


Figure 4

7. A plank 4 m long has one end on horizontal ground and rests against the top corner of a vertical wall 2·5 m high. The bottom end is sliding away from the wall towards the right at a rate of 1·5 ms^{-1}. Locate the instantaneous centre for the plank at the instant when the bottom end is 2 m from the wall and determine:

(a) the angular velocity of the plank,

(b) the velocity of the top end of the plank

(c) the point on the plank which has the smallest speed.

8. For the slider-crank mechanism shown in Fig. 5 (dimensions in mm), locate the instantaneous centres of ABC and BD. Note that EC is rigidly fixed to AB such that AEC is a right angle. Using the instantaneous centres determine, for the instant shown, the angular velocity of ABC, the velocity of point C and the angular velocity of the crank BD.

Figure 5 is drawn full scale. It is suggested that you either trace (or prick through) the figure on to your own paper, or draw directly on the figure below. Carefully measured dimensions from the figure will be accurate enough. Do not measure angles and/or use trigonometry.

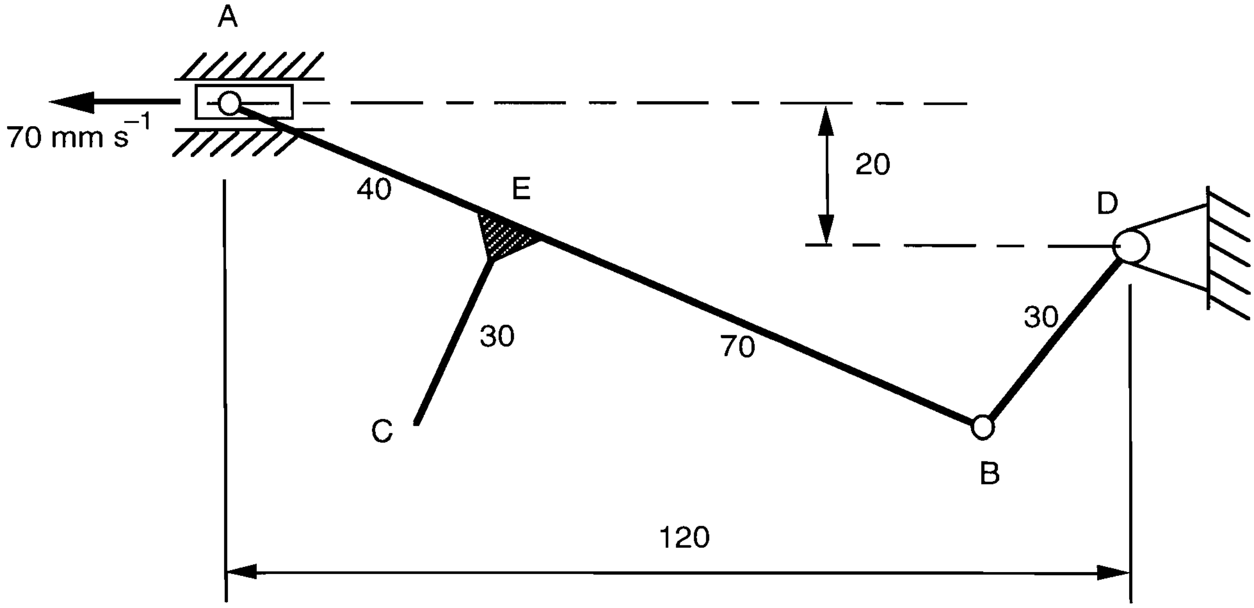


Figure 5

Velocity diagrams I

9. Draw velocity diagrams for the following:

(a)† points A and B in Fig. 1. Identify the velocity of A relative to B and the velocity of B relative to A on the diagram. A scale of 20 mm to represent 1 ms^{-1} is recommended.

(b)† points A and B in Fig. 2. Confirm from the diagram that the angular velocity of vector AB is 15 $\underline{k}$ rads $^{-1}$. A scale of 10 mm to represent 1 ms^{-1} is recommended.

(c)† points A , B and C in Fig. 3. Identify the velocity of B relative to A and the absolute velocity of B on the diagram. A scale of 20 mm to represent 1 ms^{-1} is recommended.

(d) points A , B , C , D and E in Fig. 5. Confirm the values for the angular velocity of ABC , the velocity of point C and the angular velocity of the crank BD that were calculated in question 8. A scale of 10 mm to represent 10 mms^{-1} is recommended.

10.* Figure 6 shows a section through a thick cylindrical axle which is rigidly fixed to a cylindrical roller. Bar A touches the axle at X and bar B touches the roller at Y. Both bars are constrained to move in the horizontal direction only and *no slipping* takes place at X or Y.

If bar A has a velocity of 0.8 $\underline{i}$ ms^{-1} and bar B a velocity of - 0.6 $\underline{i}$ ms^{-1} , draw a velocity diagram showing clearly the velocities of G , X , Y and P . Determine the values for the angular velocity of the cylinder, the velocity of G , the velocity of P , and the velocity of Y relative to P .

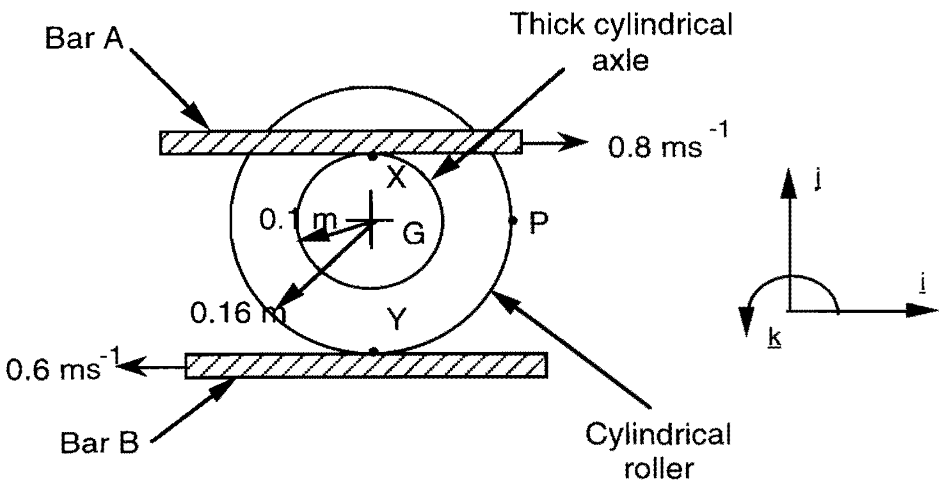


Figure 6

ANSWERS

1. $\dfrac{-100}{\sqrt{13}}\underline{k}$ mms^{-1}

2. (a) $\underline{\omega} = 1.5\,\underline{k}$ rads^{-1} (b) $1.5\,\underline{k}$ rads^{-1} (c) $\underline{\omega}_{AB} = 1.5\,\underline{k}$ rads^{-1}

   (d) $\underline{v}_A = -3.0\,\underline{i}$ ms^{-1} ; $\underline{v}_B = 2.25\,\underline{j}$ ms^{-1}

   (e) $\underline{v}_{A/B} = -3.0\,\underline{i} - 2.25\,\underline{j}$ ms^{-1} ; $\underline{v}_{B/A} = 3.0\,\underline{i} + 2.25\,\underline{j}$ ms^{-1}

3. (a) No - it should be 10.39 ms^{-1} (b) $\underline{\omega} = 15.0\,\underline{k}$ rads^{-1}

   (c) $\underline{v}_G = 3.0\,\underline{i} + 5.18\,\underline{j}$ ms^{-1}

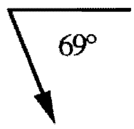
4. (a) $\underline{v}_{B/A} = 3\,\underline{i} + 3\,\underline{j}$ ms^{-1} (b) $\underline{v}_B = 5\,\underline{i} + 3\,\underline{j}$ ms^{-1}

   (c) $\underline{v}_{C/B} = -6\,\underline{j}$ ms^{-1} (d) $\underline{v}_C = 5\,\underline{i} - 3\,\underline{j}$ ms^{-1}

   (e) $\underline{v}_{C/A} = 3\,\underline{i} - 3\,\underline{j}$ ms^{-1} (f) $\underline{v}_C = 5\,\underline{i} - 3\,\underline{j}$ ms^{-1}

5. $(\underline{i} + 5\underline{j})$ rads^{-1} ; $(2\underline{i} + 4\underline{j} + \underline{k})$ rads^{-1}

6. (a) $0.4\,\underline{j}$ m rel to A (b) $0.4\,\underline{i}$ m rel to B (c) $-0.13\,\underline{j}$ m rel to A

7. (a) 0·37 rads^{-1} anticlockwise (b) 1·0 ms^{-1} 

   (c) The point instantaneously in contact with the corner of the wall.

8. 0.4 rads^{-1} anti-clockwise; 53 mms^{-1} 

; 2·2 rads^{-1} clockwise

10. $\omega = -5{\cdot}38\,\underline{k}$ rads^{-1} ; $v_G = 0{\cdot}26\,\underline{i}$ ms^{-1} ; $\underline{v}_P = 0{\cdot}26\,\underline{i} - 0{\cdot}86\,\underline{j}$ ms^{-1}

    $v_{Y/P} = -0.86\,\underline{i} + 0.86\,\underline{j}$ ms^{-1}

Suitable practice Tripos questions can be found at
<http://www2.eng.cam.ac.uk/~hemh/IAexamples.htm>.

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