# Pärt 1A Paper 3: Electrical and Information Engineering, LINEAR CIRCUITS \& DEVICES <br> EXAMPLES PAPER 1 

This examples paper is based on material from lectures 1-4. Where possible, the lectures on which the questions are based are indicated, e.g. [L5]. As usual, questions containing material of tripos standard are marked *.

## Introductory question

Find the potential difference ( $V_{o u}$ ) across the $50 \Omega$ resistor in the circuit below, as well as the current, I.


## Voltage \& current generators

1. (a) A DC power supply consists of an EMF, $V_{0}$ in series with an internal resistance, $R_{S}$ as shown in Fig. 1. When two different test resistances are connected across the circuit as indicated, the output voltage and current are measured to be $600 \mathrm{~V}, 0.4 \mathrm{~A}$ and $650 \mathrm{~V}, 0.2 \mathrm{~A}$, respectively. Determine the quantities $V_{0} \& R_{S}$.
(b) Now consider the case where the power supply consists of a current source, $I_{0}$, in parallel with the internal resistance $R_{S}$. Assuming that the output voltages and currents are as above, now determine the quantities $I_{0} \& R_{\mathrm{S}}$. Do you notice anything about the relationship between these numbers?
[L1]

(a)

(b)

Fig. 1
2. Three batteries have open-circuit voltages of $6.5 \mathrm{~V}, 6.5 \mathrm{~V}$ and 6.0 V , and when each is individually connected to a $1 \Omega$ load, the corresponding terminal voltages drop by $0.5 \mathrm{~V}, 0.4 \mathrm{~V}$ and 0.5 V , respectively.
(a) Find the internal resistance of each battery.
(b) What is the terminal voltage ( $V_{T}$ ) when the three batteries are connected in parallel as shown in Fig. 2? Determine the current flowing through each battery.
(Hint - change the equivalent circuit for the batteries (which we have drawn as a voltage source in series with a resistance - a Thévenin circuit) to their Norton equivalents (i.e. a current source in parallel with a resistance). The currents can then simply be added together, and the resistances can be combined in parallel)


Fig. 2. Battery pack
(c) If this battery pack is now connected to a load of $0.1 \Omega$, what will be the new terminal voltage and the currents flowing through each battery?
(d) For the arrangement described in part (c), determine the electrical power dissipated in the load and the batteries. Where is the power dissipated in the batteries?
[L2]
3. Repeat Question 2(b) using nodal voltage analysis.
4. In the circuit of Fig. 3, find the voltage across and current through the $2 \Omega$ resistor using
(a) nodal voltage analysis
(b) mesh current analysis


Fig. 3
5. Determine the current through the $2 \Omega$ resistor in the circuit shown in Fig. 4 using nodal voltage analysis. (The relevant nodes, A-C are indicated in the figure).


Fig. 4.
6. For the circuit shown in Fig. 5, determine the number of independent loops required to find the currents flowing if the method of mesh analysis is used. By converting the voltage sources into their Norton equivalents, show that nodal analysis only requires two voltage unknowns. Hence calculate the potential of point $P$ with respect to ground, assuming that all resistors are $1 \Omega$.


Fig. 5.
7. Repeat Question 5 using a star-delta transformation of the centre three resistors ( $1 / 2 \Omega, 1 \Omega$ and 3 $\Omega$ ). Use the star-delta transformation equations from the electrical and information data book. Note that this reduces the number of nodes from 3 to 2 , so makes the problem easier to solve. A Norton to Thévenin transformation on each of the current sources reduces the maths still further.
[L4]

## Answers

Introductory question $\quad 250 \mathrm{mV}, 5 \mathrm{~mA}$

1. (a) $700 \mathrm{~V}, 250 \Omega$; (b) $2.8 \mathrm{~A}, 250 \Omega$
2. (a) $0.0833 \Omega, 0.0656 \Omega, 0.0909 \Omega$; (b) $6.36 \mathrm{~V}, 1.73 \mathrm{~A}, 2.19 \mathrm{~A},-3.92 \mathrm{~A}$; (c) $5.04 \mathrm{~V}, 17.5 \mathrm{~A}, 22.3 \mathrm{~A}$, 10.6 A ; (d) $254 \mathrm{~W}, 25.5 \mathrm{~W}, 32.6 \mathrm{~W}, 10.2 \mathrm{~W}$
3. 6.36 V
4. 2 A
5. 1 A
6. $4 ;-5.37 \mathrm{~V}$
7. 1A

Dr C. Durkan
Michaelmas 2013

