

P2: Structural Mechanics, Examples paper 2

Straightforward questions are marked †.

Tripos standard questions are marked *.

Cables

1. A flexible cable weighing 10 N/m is stretched between two support points at the same level and 90 m apart.
 - (a) If the dip at midspan is 1.7 m, find the reactions at the supports and the tension in the cable at midspan by considering equilibrium of one half of the cable. (Assume that the weight of the cable applies a load of 10 N per metre of *horizontal* span). Also calculate the length of the suspended cable.
 - (b) A downward point load of 500 N is now applied at midspan. What is the vertical component of the reaction at the supports? If the dip at midspan is found to be 2.1 m, what is now the horizontal component of the reaction at the supports?
 - * (c) The load is moved to a point at a horizontal distance of 30 m from one end, and the dip at that point is found to be 1.9 m. Find the vertical and horizontal components of the reaction at the supports, and the dip at midspan.
- * 2. Figure 1 shows a flexible cable OA, loaded vertically by a distributed load (including self-weight) which varies linearly in intensity from w to $3w$ across the horizontal span L . Point A is at height h above O. The tension in the cable is such that the tangent at O is horizontal. Find the tension at A and derive an equation for the shape of the cable.

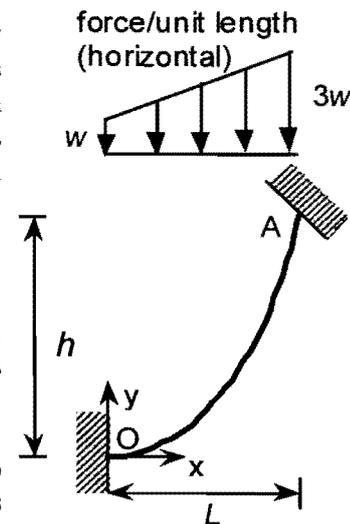


Figure 1

- Pressure Vessels
3. Figure 2 shows a thin-walled drinks can of thickness t , standing upright on a table. The weight of the can is negligibly small.
 - † (a) The can is pressure tested at uniform gauge pressure p_0 (i.e. p_0 above the atmospheric pressure). Find expressions for the circumferential stresses σ_c and longitudinal stresses σ_ℓ in the cylindrical part of the can, and calculate their values for $h = 200$ mm, $r = 25$ mm, $t = 0.1$ mm, $p_0 = 2$ bar (1 bar = 0.1 N/mm²).
 - (b) The can is filled up to point A with a still drink of density ρ . The pressure above the liquid is atmospheric. The pressure beneath the surface is hydrostatic. Find an expression for σ_c between points A and B: point B is a distance h below A, and well in the cylindrical region. What is the value of σ_ℓ in this region?

(c) The can is filled up to point A with a fizzy drink of density $\rho = 1000 \text{ kg/m}^3$. The gauge pressure p_0 above A is 2 bar. Find σ_c and σ_ℓ at point B, for $h = 150 \text{ mm}$. What is the percentage error if one assumes $\rho \cong 0$ in this calculation? Comment on this result.

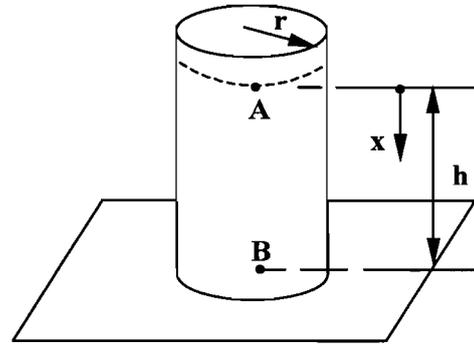


Figure 2

Arches

4. Figure 3 shows a symmetrical three-pin arch of given span $6L$ and rise $2L$. In the system of axes shown, points A, B, and C have coordinates $(L, 2\alpha L)$, $(2L, 2\beta L)$, and $(3L, 2L)$ respectively, where α and β are unknown "shape factors". The arch supports a deck which transfers five equal loads W to the arch, as shown. The self-weight of the arch itself can be neglected.

(a) By considering equilibrium of the whole arch, find the vertical component of the reactions at the abutments C and F. By drawing a free-body diagram for portion OC of the arch, calculate the horizontal components of the reaction at the abutments, and hence the forces acting on OC at point O.

* (b) Obtain an expression for the bending moment at A in terms of α , and for the bending moment at B in terms of β . If $\alpha = 0.2$ and $\beta = 0.5$, determine the bending moment at A.

* (c) Now assume α and β can be adjusted. Find the values of α and β which cause the bending moments at both A and B to be zero. For these values of α and β , explain why the arch is everywhere free from bending moment for the loading shown. Show that points O, A, B, C, D, E and F lie on a parabola.

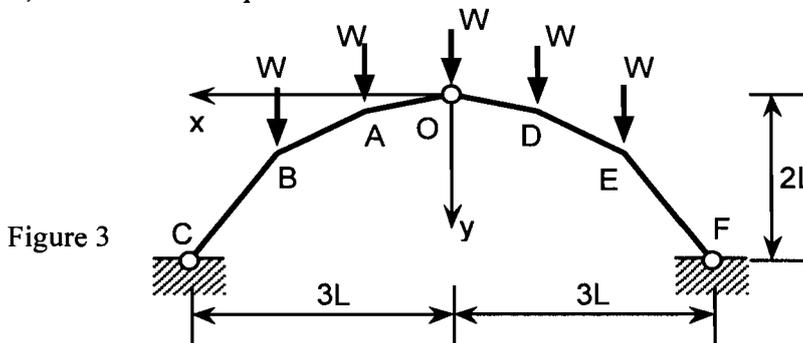


Figure 3

* 5. A symmetrical three-pin arch AOB is shown in Fig. 4. The shape of one half is defined by the equation

$$y = h(x/L)^3, \quad 0 \leq x \leq L$$

relative to the axes shown. A vertical force W may act anywhere on the right-hand half of the arch. Find the location and magnitude of the maximum bending moment that could be induced by W in the left-hand half OA.

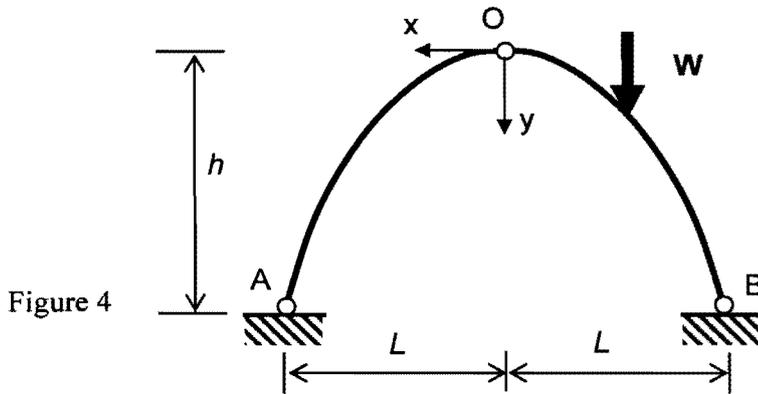


Figure 4

Bar Forces in Pin-Jointed Trusses

All figures in this section are drawn to scale: lines may be drawn on them and measurements made. Find solutions only for the loads specified: neglect self-weight unless instructed otherwise.

- † 6. Find the support reactions at A and E acting on the truss shown in Fig. 5. By resolving forces at the joints or by using the method of section as appropriate, find the axial forces in the members DE, GF, and GC.

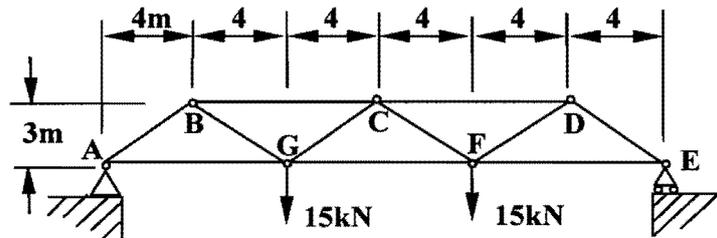


Figure 5

7. The self-weight of the regular truss shown in Fig. 6 may be represented by forces of 5 kN at each of the lower joints A-F. The truss is just lifted off its support at F by the rope GH. Determine the direction of the resultant reaction provided by the support at A, the axial force in the rope, the magnitude of the reaction at A, and the axial force in the member CD.

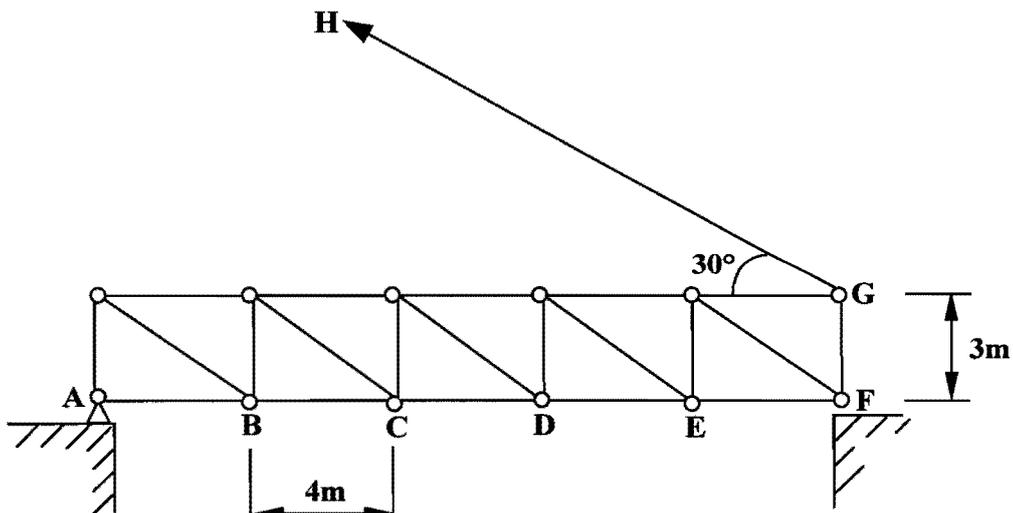


Figure 6

8. Figure 7 shows a tower. A smooth pulley turns on the pin at B. The rope ABCD runs over the pulley at B and round a second pulley at C. The rope is used to lift a weight of 10 kN at A by means of a winch at D. The straight lines in the figure are all horizontal, vertical or inclined at 45° .

(a) Find the magnitudes and directions of the reactions on the tower from the pinned foundations at E and F, both graphically and by using equations of equilibrium.

(b) Find the axial forces in each of the diagonal members of the tower.

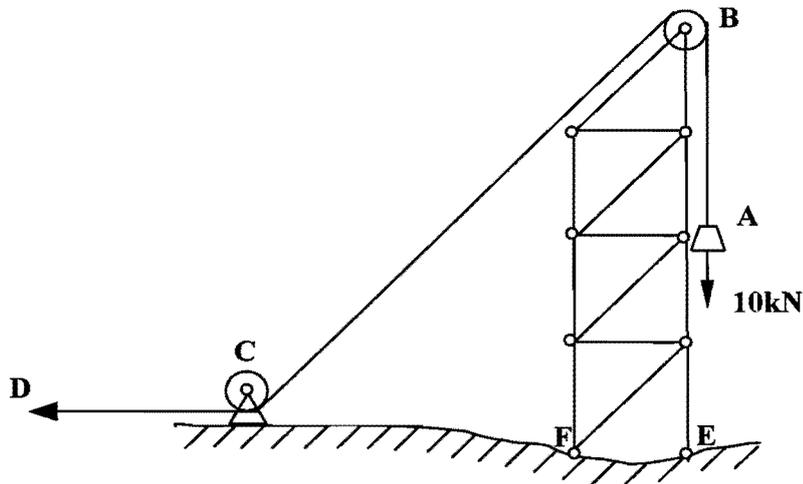


Figure 7

9. Figure 8 shows the upper part of a crane. The rolling load W can be moved along the lower chord of the boom. Find the axial force in member CD for the following load cases:

(a) if the load is at A (consider equilibrium of joints), and

(b) if the load is at B (use the method of section).

Could the method of section be used to find result (a)?

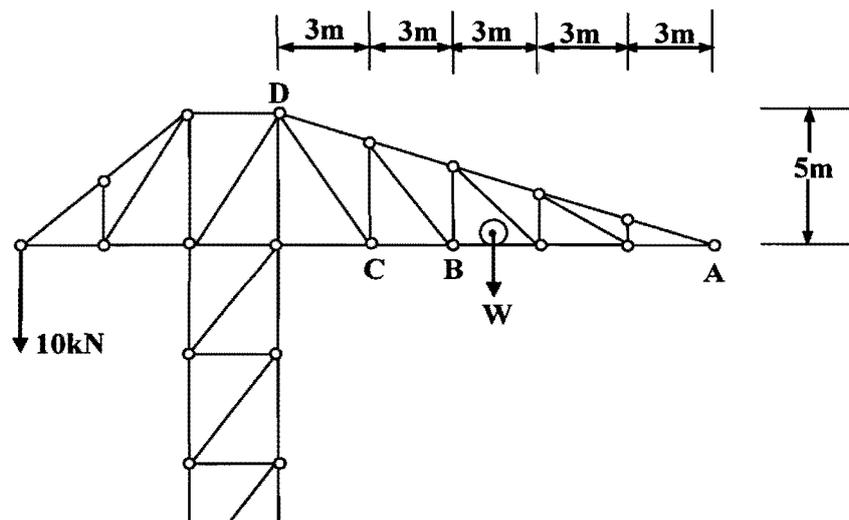
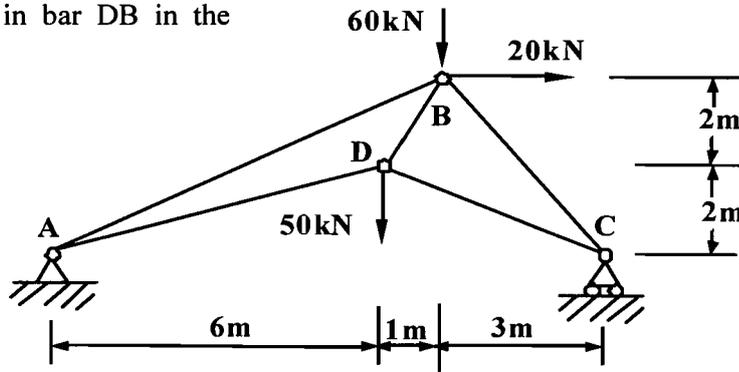


Figure 8

10. Find the axial force in bar DB in the truss shown in Fig. 9.

Figure 9



11. Find the axial forces in all the bars of the truss shown in Fig. 10, for each of the following loading systems:

(a) $P_1 = P_2 = W$

(b) $P_1 = -P_2 = W$

Enter your results in the table below.

Then, using the principle of superposition, find the tensions in all the bars due to

(c) $P_1 = 8 \text{ kN}, P_2 = 0$

and find the tensions in bars AB and DE due to

(d) $P_1 = 10 \text{ kN}, P_2 = 4 \text{ kN}.$

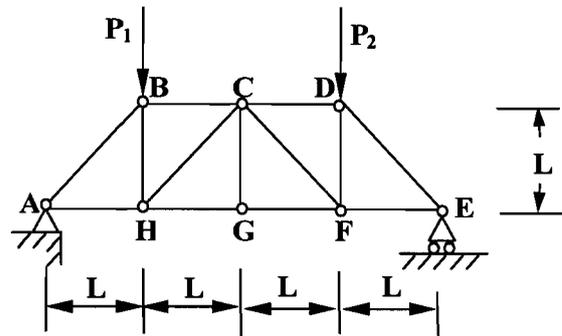


Figure 10

Member	(a)	(b)			(c)			(d)
AB								
DE								
BC								
CD								
AH								
FE								
HG								
GF								
BH								
DF								
CH								
CF								
CG								

Suitable Past Tripos Questions

- 2005 Q5a,b,d
 2006 Q2, Q3a
 2007 Q1a, Q2a
 2008 Q1, Q2b, Q3
 2009 Q2a
 2010 Q1a, Q3
 2011 Q3a, Q5a
 2012 Q2a, Q3
 2013 Q4a

Answers

- (a) $R_y = 450 \text{ N}$; $R_x = T = 5960 \text{ N}$; 90.086 m (b) 700 N ; 10180 N ;
 (c) 617 N ; 783 N ; 10000 N ; 1.76 m .
- $wL\sqrt{4 + \left(\frac{5L}{6H}\right)^2}$; $y = \frac{h}{5}\left(\frac{x}{L}\right)^2\left(3 + 2\frac{x}{L}\right)$
- (a) $\sigma_c = 50 \text{ N/mm}^2$; $\sigma_t = 25 \text{ N/mm}^2$
 (b) $\sigma_c(x) = \frac{\rho g x r}{t}$; $\sigma_t = 0$;
 (c) $\sigma_c = 50.4 \text{ N/mm}^2$; $\sigma_t = 25 \text{ N/mm}^2$; 0.7% error in σ_c .
- (a) $\frac{5W}{2}$; $\frac{9W}{4}$; $\frac{W}{2}$ downwards and $\frac{9W}{4}$ from right to left; (b) $\frac{2WL}{5}$; $\frac{WL}{4}$ (clockwise = positive)
 (c) $\alpha = \frac{1}{9}$; $\beta = \frac{4}{9}$
- $\frac{WL}{3\sqrt{3}}$ at $x = \frac{L}{\sqrt{3}}$
- $R_A = 15 \text{ kN}$; $T_{DE} = -25 \text{ kN}$; $T_{GF} = 40 \text{ kN}$; $T_{GC} = 0 \text{ kN}$
- 49° to the vertical; $T_{GH} = 24 \text{ kN}$; $R_A = 27.5 \text{ kN}$; $T_{CD} = 8 \text{ kN}$
- (a) $R_E = 11.2 \text{ kN}$ downwards; $R_F = \frac{40}{\sqrt{2}}$ upwards and $\frac{10}{\sqrt{2}}$ from left to right
 (b) All -10 kN
- (a) Zero; (b) $0.87W$; Yes
- 174 kN
-

Member	(a)	(b)	(c)	(d)
AB	$-\sqrt{2}W$	$-W/\sqrt{2}$	$-6\sqrt{2}$	$-17/\sqrt{2}$
DE	$-\sqrt{2}W$	$+W/\sqrt{2}$	$-2\sqrt{2}$	$-11/\sqrt{2}$
BC	$-W$	$-W/2$	-6	
CD	$-W$	$+W/2$	-2	
AH	W	$+W/2$	$+6$	
FE	W	$-W/2$	$+2$	
HG	W	0	$+4$	
GF	W	0	$+4$	
BH	0	$-W/2$	-2	
DF	0	$+W/2$	$+2$	
CH	0	$+W/\sqrt{2}$	$2\sqrt{2}$	
CF	0	$-W/\sqrt{2}$	$-2\sqrt{2}$	
CG	0	0	0	

MJD/JML
 Michaelmas 2013