## Sart 1A Paper 3: Electrical and Information Engineering, EXAMPLES PAPER 4 Linear Circuits $\boldsymbol{\&}$ Devices

This examples paper is based on material from lectures 13-18. Where possible, the lectures on which the questions are based are indicated, e.g. [L15]. Questions containing material of tripos standard are marked *.

## Small-Siqnal Models

1. (i) For the FET with the characteristics shown in Fig. 1 below, graphically estimate the values of the small-signal parameters $g_{m}$ and $r_{d}$ at the operating point, $P$.


Fig. 1
(ii) The small-signal model of the circuit in Fig. 2(a) is shown in Fig. 2(b). What do $R_{1}, R_{2} \& R_{3}$ represent?
[L13]


Fig. 2(a)
2. An FET has small-signal parameters $g m=4 \mathrm{mS}$ and $r_{d}=10 \mathrm{k} \Omega$ at the following working point:

$$
V_{D S}=8 \mathrm{~V}, I_{D}=2.5 \mathrm{~mA}, V_{G S}=2 \mathrm{~V}
$$

Design a common-mode amplifier circuit using this FET, a +30 V power supply ( $V_{D D}$ ) and a potential divider to generate the gate bias voltage. Then
(a) Determine the value of the circuit drain resistor and the ratio of potential divider resistances (in the gate bias circuit) that will enable the above operating point to be achieved.
(b) Draw the small-signal equivalent circuit of your design and use it to calculate the smallsignal voltage gain of the circuit.
(c) Now assume a further constraint - the input resistance of the circuit must be greater than $100 \mathrm{k} \Omega$. Now determine values of the potential divider resistors that satisfy this requirement.
3. The circuit shown below in Fig. 3 is known as a source-follower (i.e. the output "follows" the input, so the gain $\sim 1) . \mathrm{R}_{1}=2 \mathrm{M} \Omega$. The FET has small-signal parameters $g m=5 \mathrm{mS}, r_{d}=50 \mathrm{k} \Omega$. The operating point for the transistor is $V_{D S}=7 \mathrm{~V}, I_{D}=2.6 \mathrm{~mA}, V_{G S}=-3 \mathrm{~V}$.
(a) Calculate values for R2 \& R3 that will enable the desired operating point to be achieved.
(b) Draw the small-signal equivalent circuit.
(c) Using the small-signal equivalent circuit, calculate the circuit's input resistance, output resistance and voltage gain.


Fig. 3

## Coupling capacitors, generic amplifiers

4. A common-source amplifier is shown below in Fig. 4. The small signal-parameters for the FET are

$$
g_{m}=4 \mathrm{mS}, r_{d}=10 \mathrm{k} \Omega
$$

What value of output coupling capacitor, $C$, is needed between the drain and the load, assuming that the load resistance is $500 \Omega$, and the voltage across the load can drop to $70 \%$ of its maximum value at 20 Hz ?


Fig. 4

* 5. In the amplifier circuit of Fig. 5, the field-effect transistor has mutual conductance $g_{m}=4 \mathrm{mS}$ and an effectively infinite drain resistance, $r_{d}$.


Fig. 5

Derive a general expression for the signal gain $v_{0} / v_{1}$ as a function of $g_{m}, R_{1}, R_{2}, C \&$ angular frequency, $\omega$.

Calculate the gain at frequencies of $1.59 \mathrm{~Hz}, 159 \mathrm{~Hz}$ and 15.9 kHz .
6.

The circuit of an amplifier and its load is shown in
Fig. 6


Fig. 6
(a) With the switch between the load and the amplifier in the DIRECT position, as shown, and at a frequency when the effect of $C_{4}$ can be ignored, what is the value of the signal voltage gain. $v_{4} / v_{2}$ ? This is commonly called the mid-band gain.
(b) When the effect of $C_{4}$ cannot be ignored, show that the gain $v_{4} / v_{2}$ drops to $70 \%$ (or $1 / \sqrt{2}$ ) of its mid-band value at frequency $f 4 \mathrm{~Hz}$ given by:

$$
\frac{1}{2 \pi f_{4} C_{4}}=\frac{R_{3} \cdot R_{4}}{\left(R_{3}+R_{4}\right)}
$$

For the circuit values given, find $f 4$.
(c) Consider the switch between the load and the amplifier to be in the A.C. position and the signal frequency now being very much lower than $f_{4}$ so that the effect of $\mathrm{C}_{4}$ can be ignored. Show that the gain $v_{4} / v_{2}$ drops to $70 \%$ (or $1 / \sqrt{2}$ ) of its mid-band value at a frequency $f 3$ given by:

$$
\frac{1}{2 \pi f_{3} C_{3}}=R_{3}+R_{4}
$$

For the circuit values given, find $f_{3}$.
(d) Explain why $f_{3}$ and $f_{4}$ are also known as the "turnover", half-power or -3 dB frequencies.
If $R_{2}=1 \mathrm{M} \Omega$ and $R_{1}$ is the only output load, find the mid-band power gain of the amplifier numerically and in decibels.
If the amplifier is preceded by another stage whose gain is +40 dB and a "volume" control between the two amplifiers is set to -4 dB (i.e. a loss), what is the overall gain in decibels?
7. (a) In the circuit of Fig 7(a), if the input voltage, $v_{i n}$ is 3 mV rms from a microphone, $\mathrm{R}_{1}=3$ $\mathrm{k} \Omega, \mathrm{R}_{2}=500 \mathrm{k} \Omega$, and the Op -Amp is ideal, calculate the output voltage, $v_{\text {out }}$.
(b) In the circuit of Fig 7(b), if the input voltage, $v_{i n}$ is 5 mV rms from a microphone, $\mathrm{R}_{3}=99$ $\mathrm{k} \Omega, \mathrm{R}_{4}=1 \mathrm{k} \Omega$, and the Op-Amp is ideal, calculate the output voltage, $v_{\text {ou }}$.
(c) Now assume that the Op-Amps have finite values of gain, $A$ and input resistance, $\mathrm{R}_{\mathrm{i}}$, but have negligible output resistance. Derive algebraic expressions for the gain $v_{o u t} / v_{i n}$ of each circuit. Show that these expressions reduce to those given in the lectures for ideal op-amps.


Fig 7(a)


7(b)
8. Figure 8 shows the circuit of a summing amplifier. Assuming that the op-amp is ideal, show that $v_{\text {out }}$ is given by:

$$
v_{\text {out }}=-\left(v_{1} \frac{R_{f}}{R_{1}}+v_{2} \frac{R_{f}}{R_{2}}\right)
$$



Fig. 8
(a) The summing circuit is required to produce an output $-\left(200 v_{l}+40 v_{2}\right)$. If $R_{f}=100 \mathrm{k} \Omega$, what values are required for $R_{1} \& R_{2}$, given that the signal sources providing $v_{1} \& v_{2}$ each have an internal output impedance of $200 \Omega$ ?
(b) The op-amp is powered from a voltage supply that delivers $+V$ and $-V$, relative to earth. If $v_{I}$ and $v_{2}$ have amplitudes 10 mV and 50 mV , respectively, determine the minimum power supply voltages required for the circuit to work correctly.
9. The amplifier circuit shown in Fig. 9 is called a transimpedance amplifier. It is commonly used to amplify very small ac currents and convert them to a voltage. The op-amp used has a gain, A of $10^{4}$, but is otherwise ideal.
(a) Derive an expression for the output voltage per unit input current (this is known as the transimpedance).
(b) Estimate the half-power bandwidth of the circuit for $\mathrm{C}=60 \mathrm{pF}$ and $\mathrm{R}=10^{10} \Omega$.
(c) Unfortunately, such a high value of R will have some stray capacitance, $\mathrm{C}_{\mathrm{S}}$, associated with it, which is in parallel with R . If $C_{S}=2 \mathrm{pF}$, estimate the upper -3 dB point for the circuit.


Fig. 9

## Answers:

1. (i) $4.6 \mathrm{mS}, 40 \mathrm{k} \Omega$, (ii) $R_{1}=R_{G}, R_{2} \& R_{3}=R_{D} \& r_{d}$.
2. (a) $8.8 \mathrm{k} \Omega, 14$, (b) -18.7 , (c) $1.5 \mathrm{M} \Omega$ from the gate to the power supply line, $107 \mathrm{k} \Omega$ from the gate to ground.
3. (a) $R_{3}=5 \mathrm{k} \Omega, \mathrm{R}_{1}=\mathrm{R}_{2}=2 \mathrm{M} \Omega$, (c) gain $=0.958, \mathrm{R}_{\text {in }}=1 \mathrm{M} \Omega, \mathrm{R}_{\text {out }}=192 \Omega$.
4. $2.08 \mu \mathrm{~F}$.
5. $-g_{m} \mathrm{R}_{1}\left(1+\mathrm{j} \omega \mathrm{C}_{2} \mathrm{R}_{2}\right) /\left(1+g_{m} \mathrm{R}_{2}+\mathrm{j} \omega \mathrm{C}_{2} \mathrm{R}_{2}\right),-9,-53.5,-80$
6. (a) 90 , (b) 1 MHz , (c) 1 Hz , (d) $59.5 \mathrm{~dB}, 95.5 \mathrm{~dB}$
7. (a) 500 mV rms , (b) 500 mV rms , (c) $-A R_{2} R_{i} /\left(A R_{1} R_{i}+R_{2} R_{i}+R_{1} R_{i}+R_{1} R_{2}\right),->-R_{2} / R_{1}$ when $A$ is large; $A\left(R_{i} R_{3}+R_{i} R_{4}\right) /\left(A R_{4} R_{i}+R_{3} R_{i}+R_{4} R_{i}+R_{4} R_{3}\right)->A R_{i} R_{4} /\left(R_{3}+R_{4}\right)$ when $A$ is large.
8. (a) $R_{1}=300 \Omega, R_{2}=2300 \Omega$. (b) Power supplies should be at least $+4 V$ and $-4 V$.
9. (a) $-\mathrm{AR} /((\mathrm{A}+1)+\mathrm{j} \omega \mathrm{CR})$, (b) 2.7 kHz , (c) 8 Hz

Dr C. Durkan Lent 2014

