ISSUED ON
31 JAN 2014

Part IB Paper 2: Structures

Examples Paper 5

Plastic theory

*Straightforward questions are marked by †; Tripos standard questions by *.*

Yield line analysis of plates and slabs

1. Figure 1 shows, in plan view, a ductile plastic plate, with its corners, A and D, resting on simple supports, and edge, BC, fully clamped. The rest of the perimeter is free. The fully plastic moment per unit length is m for all yield lines.

   Estimate the central point load, W, required to cause collapse: consider, at least, two distinct, simple mechanisms, each symmetrical about the perpendicular bisector of BC.

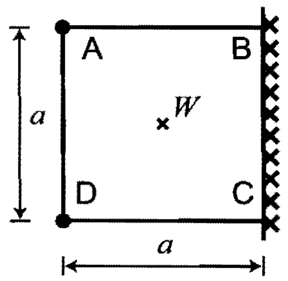


Figure 1

2. The square reinforced-concrete floor slab shown in Figure 2 is simply supported on edges, AB and BC; the other two edges are free. The slab reinforcement is uniform and isotropic, and the fully plastic moments of resistance per unit length are m and m' in sagging (hinge pointing down) and hogging (hinge pointing up), respectively.

   By considering two simple collapse mechanisms, each symmetrical about the diagonal, BD, estimate the uniform pressure, p, over the entire slab at which collapse will occur.

   What do you infer about the amount of reinforcement needed to resist bending in the two senses?

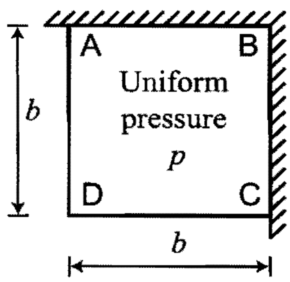


Figure 2

* 3. A plate in the form of a regular hexagon is shown in plan view in Figure 3. Edges BC, DE and FA are fully built-in whilst AB, CD and EF are free. The fully plastic moment of resistance per unit length is m for all yield lines.

Ignoring self-weight, estimate the magnitude of the central point load, W, to cause collapse.

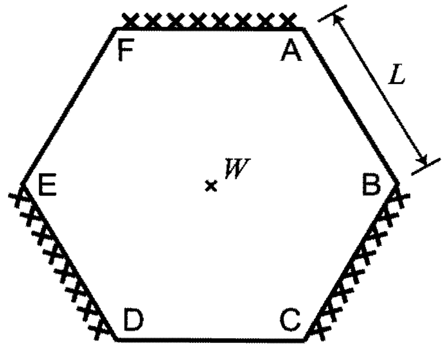


Figure 3

Slip line analysis of continua

4. Figure 4 shows, in cross-sectional view, a long rigid strip of breadth b, being pressed into the upper surface of a material, which may be considered to extend indefinitely into the page. The indentation force per unit length along the strip is F. The material yield stress in pure shear is k.

Determine the limiting value of F for the assumed circular slip surface, PQ, as a function of the semi-angle, α, subtended by the arc at the centre, C. Hence, find the lowest upper bound on F for the family of slip surfaces defined by α. (Note that the height of C above P varies with α.)

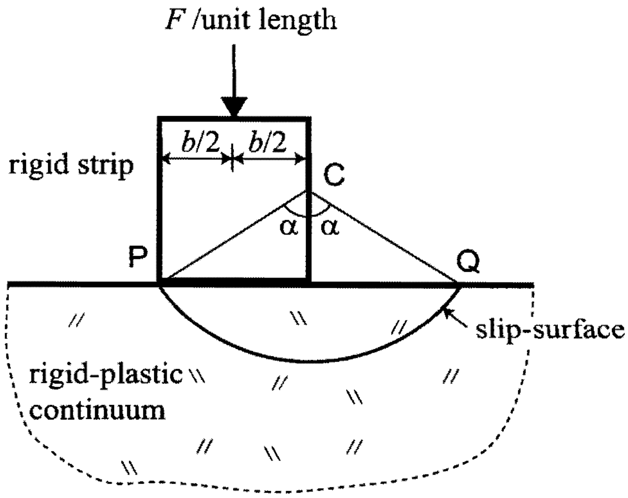


Figure 4

* 5. Figure 5 shows, for the strip described in question 4, another mode of indentation based on the relative displacements of five identical "rigid" triangular blocks; all other material is undisturbed.

(a) Estimate the upper bound on the distributed indentation force, F, in terms of the shear yield stress k, strip width b, and triangle height h. Hence, by varying h, find the lowest upper bound on F.

(b) Plot values of F over the range, $0.2 \leq (h/b) \leq 1.0$. Note the sensitivity, or otherwise, of F to variations in (h/b).

(c) Suggest another pattern of triangular blocks that would enable friction at the strip/material interface to feature in the upper bound calculation.

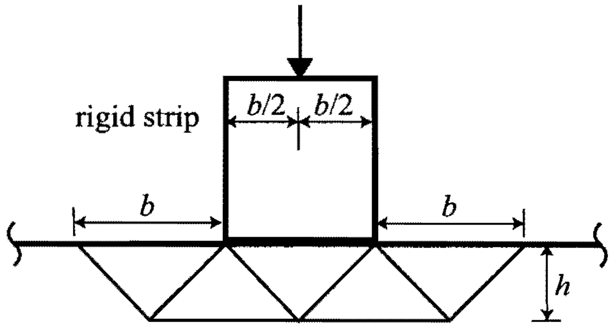


Figure 5

6. A trench is to be excavated and its vertical sides temporarily propped by horizontal jacks bearing on opposite sides. The jacks can exert a maximum force of 60 kN and it is proposed to spread their loading into the trench walls through rigid planks, 0.3 m wide and 1.5 m long. The soil is estimated to have an average shear strength of 10 kN/m^2.

Use the plane sliding block mechanism shown in Figure 6 to estimate the bearing load, F_p, at collapse for such a plank, neglecting end effects. Comment on the suitability of the size of this plank for the intended purpose.

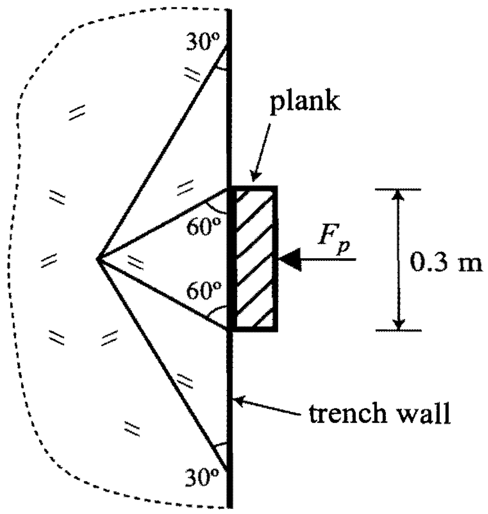


Figure 6

Lower bound analysis

† 7. Show the following equilibrium results (quoted but not derived in lectures) for a simply-supported beam of length, l. The moment sign convention is given in the Structures Data Book, and all forces are applied vertically downwards.

(a) When a central point load of magnitude, W, is applied, the largest bending moment is $M = -Wl/4$ at the centre of the beam, and the bending moment diagram is piecewise linear.

(b) When a point load of magnitude, W, is applied at a distance, a, from a support, the largest bending moment is $M = -Wa(l-a)/l$ at the point where the load is applied, and the bending moment diagram is piecewise linear.

(c) When a distributed load of magnitude, w per unit length, is applied to the entire beam, the bending moment at a distance, x, from a support is $M = -wx(l-x)/2$, and the largest bending moment is $M = -wl^2/8$.

(d) When a clockwise couple, C, is applied at the left hand support, the bending moment at a distance, x, from the *other* support is $M = -Cx/l$.

† 8. Find an optimal lower bound for the collapse load, W_c, of the propped cantilever shown in Figure 7. The beam has fully plastic moment, M_p.

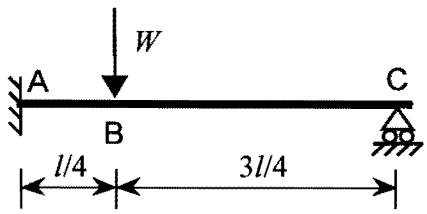


Figure 7

9. The two-span beam, ABC, shown in Figure 8 has been designed not to collapse when loaded with a distributed load, w = 10 kN/m. Show that a continuous beam with M_p = 15 kNm is a safe design.

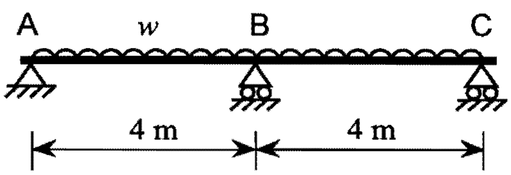


Figure 8

∗ 10. The two-span beam, ABC, shown in Figure 9 is to be designed so that it will not collapse under the indicated loads. It will be made from Grade 43 steel (σ_y = 245 N/mm^2).

(a) If the structure is to be made from a single continuous beam, choose a suitable Universal Beam (UB) from the Structures Data Book.

(b) If, instead, two spans are to be used, one for each span and welded together at B, choose a suitable UB for each span.

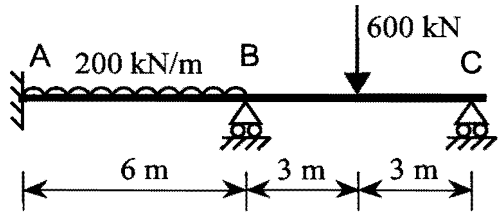
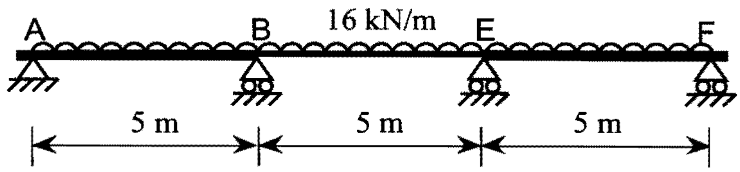


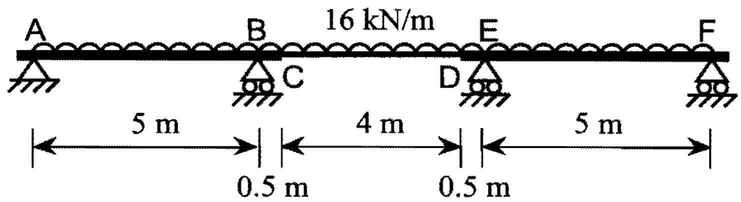
Figure 9

∗ 11. (a) The three-span beam shown in Figure 10(a) is to be made from three steel members, each of uniform section and connected together by full strength joints over the interior supports. It is to carry the given loads without failing. Show that a design with fully plastic moments of 38.3 kNm for AB, 25.0 kNm for BE, and 38.3 kNm for EF would be suitable.

(b) The same loads are to be carried by the revised design shown in Figure 10(b), where the joints between beams have been moved to C and D, 0.5 m from the interior supports. Show that an optimum design with fully plastic moments of 34.5 kNm for AC, 16.0 kNm for CD, and 34.5 kNm for DF would be suitable.



(a)



(b)

Figure 10

Suitable Tripos questions

Part IB Paper 2: 2013/6,5 2012/4,6 2011/5,6 2010/4,5,6; 2009/4.

ANSWERS

1. $W = 6m$ (other mechanisms give, e.g., $8m$ or $10m$).

2. $p_1 = 6m/b^2$, $p_2 = 12m'/b^2$.

3. $W = 5\sqrt{3}m$ (another symmetrical mechanism gives $W = 6\sqrt{3}m$).

4. $F = 4kb\alpha/\sin^2\alpha$, $F_{\min} = 5.52kb$ for $\alpha = 1.1655\ rad$ ($\approx$ 67 °).

5. $F = 2kb[2(h/b) + (b/h)]$, $F_{\min} = 5.66kb$ for $(h/b) = 1/\sqrt{2}$.

6. $F_p = 31.2$ kN.

8. $W_c = 28M_p/3l$.

10. (a) $Z_p \geq 2449$ cm^3, *e.g.* UB 533 × 210 × 101. (b) for AB, $Z_p \geq 1832$ cm^3, *e.g.* UB 533 × 210 × 82; for BC, $Z_p \geq 2755$ cm^3, *e.g.* UB 533 × 210 × 109.

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