Engineering Tripos Part IB, 2P7: Vector Calculus, 2020-21

Lecturer

Prof G Pullan [1]

Timing and Structure

Weeks 1-3 and 6-8 Michaelmas term, 2 lectures/week; weeks 4-5 Michaelmas term, 1 lecture/week. 14 lectures

Aims

The aims of the course are to:

- Provide the necessary background mathematics to ensure that students are confident in handling partial differential equations in vector form while maintaining a tangible physical appreciation of the manipulations involved.

Objectives

As specific objectives, by the end of the course students should be able to:

- Differentiate and integrate scalar functions of two or more variables including transformations to other co-ordinate systems.
- Manipulate vector differential equations including the gradient, divergence and curl operators while retaining a physical appreciation of the mathematical operations involved.
- Perform line, surface and volume integrals and understand their various physical interpretations.
- Set up conservation statements in both differential and integral form and be able to transform from one to the other using Gauss's theorem.
- Appreciate the physical significance of curl and its relationship to circulation via Stokes's theorem in simple examples.
- Solve common PDE's (particularly the Laplace, Poisson, heat conduction and wave equations) with simple boundary conditions by the method of separation of variables.
- Solve the diffusion (heat conduction) equation using the self-similar solutions method.

Content

The course provides an introduction to vector calculus and aims to familiarise the student with the ideas of the differential calculus (the vector gradient, divergence and curl) and the integral calculus (line, surface and volume integrals and the theorems of Gauss and Stokes). The physical interpretation of the mathematical ideas will be stressed throughout via applications which centre on the derivation and manipulation of the common partial differential equations of engineering. The analytical solution of simple partial differential equations by the method of separation of variables will also be discussed.

A knowledge of the following Part IA lecture material on functions of more than one variable will be assumed: representation of curves and surfaces (including parametric representation); partial differentiation; total and perfect differentials; Taylor series; maxima and minima.

The course will then consist of lectures on the following topics:

Vector functions and fields; field lines.

Vector differentiation; differentiation formulae.

The vector gradient and its physical interpretation;

Cylindrical and spherical polar co-ordinate systems.

The divergence and its physical interpretation; solenoidal fields; conservation statements;

Surface integrals; volume integrals; Gauss's divergence theorem; integral-differential transformations. Stokes's theorem.

The curl and its physical interpretation; irrotational fields; scalar potential; line integrals; conservative fields.

Types of PDE and boundary conditions; solution by separation of variables; examples of some common PDE's (Laplace, Poisson, heat conduction, wave equation). Solution of the diffusion (heat conduction) equation by the self-similar solution method.

Booklists

Please refer to the Booklist for Part IB Courses for references to this module, this can be found on the associated Moodle course.

Examination Guidelines

Please refer to Form & conduct of the examinations [2].

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