Engineering Tripos Part IIA, 3M1: Mathematical Methods, 2019-20

Module Leader

Luca Magri [1]

Lecturers

Luca Magri, Prof G Wells and Prof S Godsill

Lab Leader

Luca Magri

Timing and Structure

Lent term. 16 lectures and coursework.

Aims

The aims of the course are to:

- Teach some mathematical techniques that have wide applicability to many areas of engineering.

Objectives

As specific objectives, by the end of the course students should be able to:

- Find the SVD of a matrix, and understand how this can be used to calculate the rank and pseudo-inverse of the matrix.
- Calculate the least squares solution of a set of linear equations.
- Understand how to apply Principal Component Analysis (PCA) to a problem.
- Apply PCA to reduce the dimensionality of an optimization problem and/or to improve the solution representation.
- Represent linear iterative schemes using linear algebra and understand what influences the rate of convergence.
- Understand the definitions and application areas of Stochastic Processes.
- Understand the principle of Markov Chains.
- Implement various sampling schemes to enable parameters of stochastic processes to be estimated.
- Understand the concepts of local and global minima and the conditions for which a global minimum can be obtained.
- Understand the algorithms of the different gradient search methods.
- Solve unconstrained problems using appropriate search methods.
- Solve constrained linear and non-linear optimization problems using appropriately selected techniques.
- Understand how Markov Chain-based algorithms can be used to give reasonable solutions to global optimisation problems.

Content

Linear Algebra provides important mathematical tools that are not only essential to solve many technical and computational problems, but also help in obtaining a deeper understanding of many areas of engineering. Stochastic (random) processes are important in fields such as signal and image processing, data analysis etc. Optimization methods are routinely used in almost of every branch of engineering, especially in the context of design.

Linear Algebra (4L, Prof G Wells)

- Revision of IB material
- Matrix norms, condition numbers, conditions for convergence of iterative schemes
- Positive definite matrices
- Singular Value Decomposition (SVD), pseudo-inverse of a matrix and least squares solutions of Ax = b
- Principal Component Analysis
- Markov matrices and applications

Stochastic Processes (5L, Prof S Godsill)

- Definition of a stochastic process, Markov assumption (with examples), the Chapman-Kolmogorov (CK) equation, conversion of a particular CK integral equation into a differential equation (for the case of Brownian motion)
- The general Fokker-Planck equation with particular examples (Brownian motion, Ornstein-Uhlenbeck process)
- Introduction to sampling Gibbs sampler, Metropolis Hastings, Importance sampling with applications.

Optimization (7L, Luca Magri)

- Introduction: Formulation of optimization problems; conditions for local and global minimum in one, two and multi-dimensional problems
- Unconstrained Optimization: gradient search methods (Steepest Descent, Newton's Method, Conjugate Gradient Method)
- Linear programming (Simplex Method)
- Constrained Optimization: Lagrange and Karush-Kuhn-Tucker (KKT) multipliers; penalty and barrier functions
- Global optimisation: Simulated Annealing

Coursework

Exploring Principal Component Analysis for dimensional reduction and data representation.

There is no Full Technical Report (FTR) associated with this module.

Booklists

Please see the Booklist for Part IIA Courses [2] for references for this module.

Examination Guidelines

Please refer to Form & conduct of the examinations [3].

UK-SPEC

This syllabus contributes to the following areas of the UK-SPEC [4] standard:

Toggle display of UK-SPEC areas.

GT1

Develop transferable skills that will be of value in a wide range of situations. These are exemplified by the Qualifications and Curriculum Authority Higher Level Key Skills and include problem solving, communication, and working with others, as well as the effective use of general IT facilities and information retrieval skills. They also include planning self-learning and improving performance, as the foundation for lifelong learning/CPD.

IA1

Apply appropriate quantitative science and engineering tools to the analysis of problems.

KU1

Demonstrate knowledge and understanding of essential facts, concepts, theories and principles of their engineering discipline, and its underpinning science and mathematics.

KU2

Have an appreciation of the wider multidisciplinary engineering context and its underlying principles.

E1

Ability to use fundamental knowledge to investigate new and emerging technologies.

E2

Ability to extract data pertinent to an unfamiliar problem, and apply its solution using computer based engineering tools when appropriate.

E3

Ability to apply mathematical and computer based models for solving problems in engineering, and the ability to assess the limitations of particular cases.

E4

Understanding of and ability to apply a systems approach to engineering problems.

P3

Understanding of contexts in which engineering knowledge can be applied (e.g. operations and management, technology, development, etc).

P8

Ability to apply engineering techniques taking account of a range of commercial and industrial constraints.

US1

A comprehensive understanding of the scientific principles of own specialisation and related disciplines.

US2

A comprehensive knowledge and understanding of mathematical and computer models relevant to the engineering discipline, and an appreciation of their limitations.

US3

An understanding of concepts from a range of areas including some outside engineering, and the ability to apply them effectively in engineering projects.

Last modified: 08/03/2020 17:41

Source URL (modified on 08-03-20): http://teaching.eng.cam.ac.uk/content/engineering-tripos-part-iia-3m1-mathematical-methods-2019-20

Links
[1] mailto:lm547@cam.ac.uk
[2] https://www.vle.cam.ac.uk/mod/book/view.php?id=364091&chapterid=49051
[3] http://teaching.eng.cam.ac.uk/content/form-conduct-examinations
[4] http://teaching.eng.cam.ac.uk/content/uk-spec