Engineering Tripos Part IIA, 3M1: Mathematical Methods, 2021-22

Module Leader
Prof M Girolami

Lecturers
Prof M Girolami, Dr A Kabla and Prof M Gales

Lab Leader
Prof M Girolami

Timing and Structure
Lent term. 16 lectures and coursework.

Aims
The aims of the course are to:

- Teach some mathematical techniques that have wide applicability to many areas of engineering.

Objectives
As specific objectives, by the end of the course students should be able to:

- Find the SVD of a matrix, and understand how this can be used to calculate the rank and pseudo-inverse of the matrix.
- Calculate the least squares solution of a set of linear equations.
- Understand how to apply Principal Component Analysis (PCA) to a problem.
- Apply PCA to reduce the dimensionality of an optimization problem and/or to improve the solution representation.
- Represent linear iterative schemes using linear algebra and understand what influences the rate of convergence.
- Understand the definitions and application areas of Stochastic Processes.
- Understand the principle of Markov Chains.
- Implement various sampling schemes to enable parameters of stochastic processes to be estimated.
- Understand the concepts of local and global minima and the conditions for which a global minimum can be obtained.
- Understand the algorithms of the different gradient search methods.
- Solve unconstrained problems using appropriate search methods.
- Solve constrained linear and non-linear optimization problems using appropriately selected techniques.
- Understand how Markov Chain-based algorithms can be used to give reasonable solutions to global optimisation problems.

Content
Linear Algebra provides important mathematical tools that are not only essential to solve many technical and
computational problems, but also help in obtaining a deeper understanding of many areas of engineering. Stochastic (random) processes are important in fields such as signal and image processing, data analysis etc. Optimization methods are routinely used in almost every branch of engineering, especially in the context of design.

**Linear Algebra (4L, Prof G Wells)**

- Revision of IB material
- Matrix norms, condition numbers, conditions for convergence of iterative schemes
- Positive definite matrices
- Singular Value Decomposition (SVD), pseudo-inverse of a matrix and least squares solutions of $Ax = b$
- Principal Component Analysis
- Markov matrices and applications

**Stochastic Processes (5L, Prof S Godsill)**

- Definition of a stochastic process, Markov assumption (with examples), the Chapman-Kolmogorov (CK) equation, conversion of a particular CK integral equation into a differential equation (for the case of Brownian motion)
- The general Fokker-Planck equation with particular examples (Brownian motion, Ornstein-Uhlenbeck process)
- Introduction to sampling Gibbs sampler, Metropolis Hastings, Importance sampling with applications.

**Optimization (7L, Prof M Girolami)**

- Introduction: Formulation of optimization problems; conditions for local and global minimum in one, two and multi-dimensional problems
- Unconstrained Optimization: gradient search methods (Steepest Descent, Newton’s Method, Conjugate Gradient Method)
- Linear programming (Simplex Method)
- Constrained Optimization: Lagrange and Karush-Kuhn-Tucker (KKT) multipliers; penalty and barrier functions
- Global optimisation: Simulated Annealing

**Coursework**

Exploring Principal Component Analysis for dimensional reduction and data representation.

There is no Full Technical Report (FTR) associated with this module.

**Booklists**

Please refer to the Booklist for Part IIA Courses for references to this module, this can be found on the associated Moodle course.

**Examination Guidelines**

Please refer to [Form & conduct of the examinations](http://www.teaching.eng.cam.ac.uk) [2].

**UK-SPEC**

The [UK Standard for Professional Engineering Competence (UK-SPEC)](http://www.teaching.eng.cam.ac.uk) [3] describes the requirements that have to be met in order to become a Chartered Engineer, and gives examples of ways of doing this.
UK-SPEC is published by the Engineering Council on behalf of the UK engineering profession. The standard has been developed, and is regularly updated, by panels representing professional engineering institutions, employers and engineering educators. Of particular relevance here is the 'Accreditation of Higher Education Programmes' (AHEP) document [4] which sets out the standard for degree accreditation.

The Output Standards Matrices [5] indicate where each of the Output Criteria as specified in the AHEP 3rd edition document is addressed within the Engineering and Manufacturing Engineering Triposes.