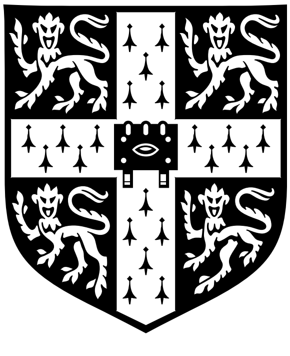
Electrical Data Book

2017 Edition



Cambridge University Engineering Department

Electrical Engineering Data Book

prepared by T.D. Wilkinson et al

2017 version

1 GENERAL PHYSICAL CONSTANTS

Electron rest mass	m_e	9.109×10^{-31} kg
Proton rest mass	m_p	1.673×10^{-27} kg
Neutron rest mass	m_n	1.675×10^{-27} kg
Proton / Electron mass ratio	m_p/m_e	1.836×10^{3}
Electronic charge	e	-1.602×10^{-19} C
Electronic charge / mass ratio	e/m_e	1.759×10^{11} C/kg
Velocity of light in vacuo	c	2.998×10^{8} m/s
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ H/m
Permittivity of free space	ε_0	8.854×10^{-12} F/m
Planck Constant	h	6.626×10^{-34} J s
Boltzmann Constant	k	1.381×10^{-23} J/K
Stefan-Boltzmann Constant	σ	5.670×10^{-8} J/K^4 m^2 s
Molar number (Avogadro's constant)	N_A	6.022×10^{26} kmol^{-1}
Faraday Constant	F	9.649×10^{7} C/kmol
Standard Volume of Perfect Gas	V_0	22.41 m^3/kmol
Molar (universal) gas constant	$\bar{R}$	8.314×10^{3} J/K kmol
Ice point	T_0	273.15 K
Standard Atmospheric Pressure	P_0	1.01325×10^{5} N/m^2 (=1 atm) (10^5 N/m^2 = 1 bar)
Standard Acceleration	g	9.80665 m/s^2

2 PROPERTIES OF MATERIALS AND SOLID STATE PHYSICS (Typical values)

2.1 Metals and Alloys etc.

	Resistivity at 20 °C Ω m	Temp. Coeff. of Resistance K^{-1} at 20°C	Temp. Coeff of Expansion K^{-1}	Specific Heat-Capacity J/kg K	Thermal Conductivity W/m K	Melting Point °C
Copper	1.72 x 10^{-8}	39 x 10^{-4}	25.5 x 10^{-6}	380	385	1083
Aluminium	2.8 x 10^{-8}	40 x 10^{-4}	16.7 x 10^{-6}	880	200	660
Tungsten	5.5 x 10^{-8}	45 x 10^{-4}	4.4 x 10^{-6}	140	160	3370
Manganin	44.5 x 10^{-8}	0.1 x 10^{-4}	18 x 10^{-6}		26	910
Nichrome	103 x 10^{-8}	1.5 x 10^{-4}	17 x 10^{-6}	450	13	1350
Carbon	4500 x 10^{-8}	−5 x 10^{-4}	5.4 x 10^{-6}	840	1.7	3500
Iron	100 x 10^{-8}	54 x 10^{-4}	11.6 x 10^{-6}	250	67	1537
Stainless steel	72 x 10^{-8}	-	9 x 10^{-6}	500	16	1427

2.2 Dielectrics

	Relative Permittivity	Dielectric Strength MV/m	tanδ at			Resistivity Ω m
			$50Hz$	$1MHz$	$1GHz$	
Mica	6	200	25 x 10^{-4}	3 x 10^{-4}	3 x 10^{-4}	10^{11} – 10^{15}
Glass	5	20	6 x 10^{-4}	8 x 10^{-4}	12 x 10^{-4}	10^{9} – 10^{12}
Porcelain	6	30	220 x 10^{-4}	75 x 10^{-4}	100 x 10^{-4}	-
Polystyrene	2.5	20	0.5 x 10^{-4}	0.7 x 10^{-4}	3.3 x 10^{-4}	-
P.T.F.E	2.1	20	5 x 10^{-4}	2 x 10^{-4}	2 x 10^{-4}	10^{15} – 10^{19}
Transfr. Oil	2.2	15	4 x 10^{-4}	5 x 10^{-4}	30 x 10^{-4}	-
Alumina	8.5	-	20 x 10^{-4}	-	-	-
Quartz	3.8	20	10 x 10^{-4}	-	-	10^{16}
Polythene	2.3	20	2 x 10^{-4}	-	-	10^{8} – 10^{14}
Polycarbonates	3.1	-	50 x 10^{-4}	-	-	10^{11} – 10^{14}

2.3 Semiconductors (properties at 300 K)

	Energy Gap eV	Mobilities Electron m^2/Vs	Hole m^2/Vs	Relative Permittivity
Germanium	0.67	0.39	0.19	16
Silicon	1.12	0.16	0.05	12
Gallium Arsenide	1.40	0.9	0.04	12.5
Indium Antimonide	0.16	7.0	0.07	17

2.4 Superconductors

	T_c K	B_c or B_{c2} at 0 K tesla (T)
Al	1.2	0.01
Pb	7.2	0.08
Nb	9.2	0.08
NbSn	18.4	24
YBaCuO	93	$\sim$ 100
T1BaCaCuO	125	$\sim$ 120

Critical current density is very variable:

NbSn will carry 10^9 A/m^2 in a field of 5 T at 4.2 K in wire form.

YBaCuO will carry 10^{10} A/m^2 in zero magnetic field at 77 K in thin film form.

Flux quantum $\frac{h}{2e}$ = 2.07 x 10^{-15} J s C^{-1}.

Energy gap $\sim$ 3500 T_c.

2.5 Solid state physics for crystalline materials

Density of states for nearly free electrons:

$$g(E) = \frac{4\pi(2m^*)^{\frac{3}{2}}}{h^3} E^{\frac{1}{2}}$$

where E is the energy measured from the bottom of the band.
For a semiconductor where $E_c - E_f > 3kT$, the electron density in the conduction band may be written as

$$n = N_c \exp\left\{ -\frac{(E_c - E_f)}{kT} \right\}$$

where E_c is the energy of the bottom of the band and the effective density of states, N_c, is given by

$$N_c = 2\frac{(2\pi m^* kT)^{\frac{3}{2}}}{h^3} m^{-3}$$

The equation of continuity for excess minority electrons is

$$\frac{\partial n}{\partial t} = -\frac{n}{\tau} + D\nabla^2 n + \mu\nabla \cdot (n\underline{\mathrm{E}})$$

where τ is the carrier life-time, D is the carrier diffusion coefficient, μ is the carrier mobility, and $\underline{\mathrm{E}}$ is the electric field.

Einstein's relation between mobility and diffusion coefficient:

$$D = \frac{kT}{e}\mu$$

2.6 Magnetic Materials

The number of magnetic materials in common use is very large and the data given here do not attempt to cover more than a few representative examples. For most purposes the materials may be grouped into four categories, as shown below, though there are some useful materials which lie outside these groups.

2.6.1 Group I: Materials used in the electrical power industry

These materials, notably cast steel and various silicon-iron alloys, are relatively inexpensive and have high values of maximum flux density. The silicon-iron alloys have moderately high permeabilities and fairly low hysteresis losses.

Magnetization curves are given in Fig.1.

2.6.2 Group II: The nickel-iron alloys

By comparison with Group I, these alloys are much more expensive, have much higher permeabilities, lower hysteresis loss and lower values of maximum flux density. They are widely used in the light-electrical industry. Several different alloys are available to provide different compromises between cost, permeability, maximum flux density and resistivity (to reduce eddy-current loss).

Small quantities of elements other than nickel and iron are sometimes added. Representative types of alloys are:

Percentage of nickel	Trade Names	Initial relative permeability	Maximum Flux Density Wb/m^2	Resistivity Ωm
70-90	Mumetal Permalloy C etc.	10000 to 30000	0.8	$6.0x10^{-7}$
45-50	Radiometal Permalloy B etc.	1800 to 2400	1.6	$5.5x10^{-7}$
35-45	Rhometal Permalloy D etc.	1500 to 2000	1.3	$9.0x10^{-7}$

Magnetization Curves are given in Fig.1.

2.6.3 Group III: Permanent magnet materials

Several alloys of nickel and aluminium, with or without cobalt, copper and titanium, have been developed for the manufacture of permanent magnets. These materials have high remanent magnetism and high coercive force.

Magnetism sintered from Barium Ferrite powder have been developed, and are characterised by low remanence and high coercive force.

Cobalt/rare earth materials which exhibit an extremely high maximum energy product are recent developments. Cobalt/samarium is available in two forms:

a. Sintered metal	Remanent flux density	0.87 T
	Coercive force	1280×10^3 A/m
b. Moulded powder in Epoxy	Remanent flux density	0.435 T
	Coercive force	640×10^3 A/m

This material may be machined.

Fig 1a

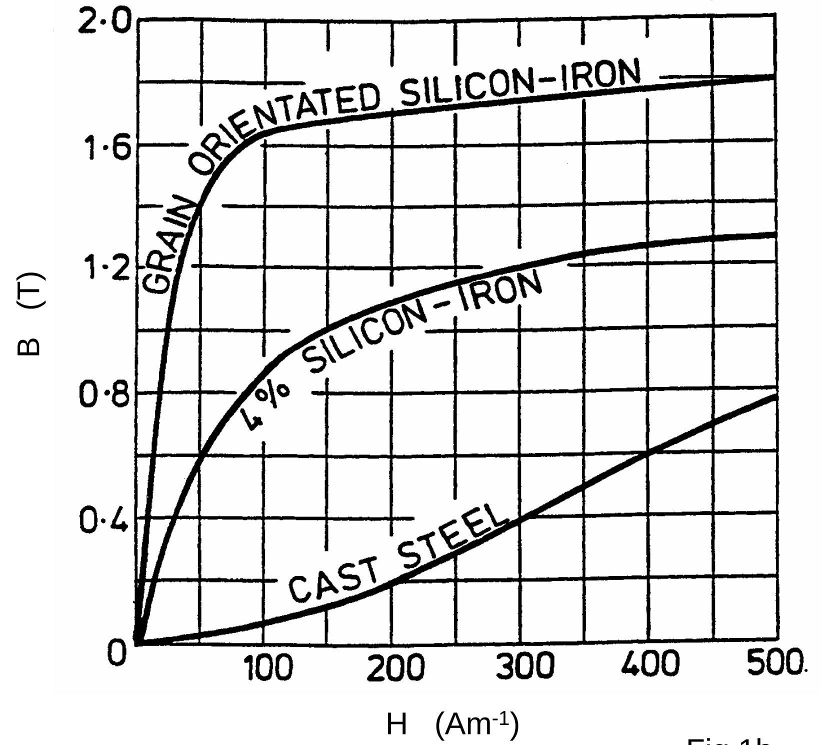


Fig 1b

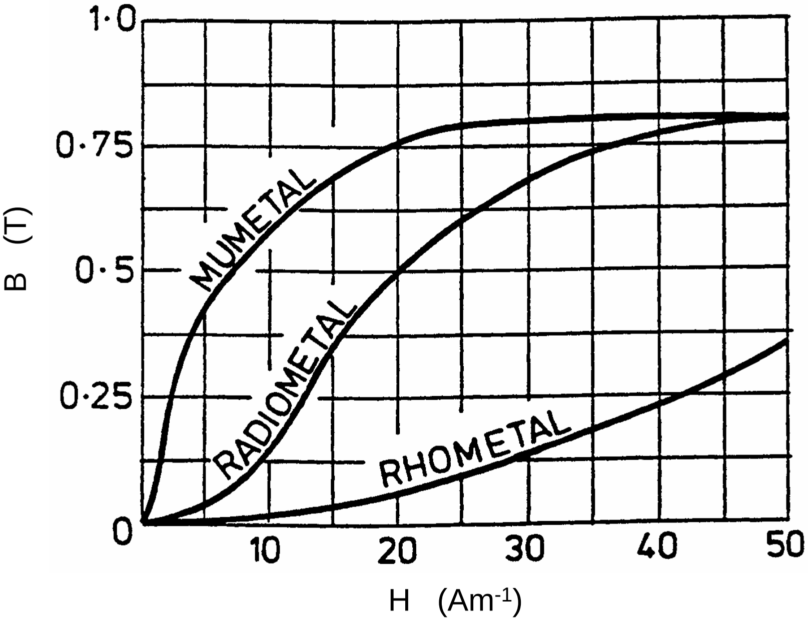


Figure 1: Magnetization curves: (a) Group I; (b) Group II.

The demagnetization characteristic is linear between the remanent flux density (H=0) and the coercive force (B=0).

Demagnetization curves are shown in Fig.2.

2.6.4 Group IV: Ferrites

In addition to the alloys mentioned above, a wide range of non-metallic ferrites has been developed for magnetic purposes. In general, the purely magnetic properties of the ferrites are inferior to those of the alloys, but the former have much higher resistivities and so can be used at much higher frequencies without serious eddy-current loss. A great many mixed ferrites of divalent metals have been prepared, but only three will be mentioned.

Manganese Zinc Ferrites

Different materials in this range, with different proportions of manganese and zinc, have useful magnetic properties in the frequency range from 1 kHz to 20 MHz. Other properties lie in the ranges:

Initial relative permeability	850 - 1500
Maximum flux density	0.34 - 0.40 Wb/m^2
Resistivity	0.5 - 1.0 Ωm

Nickel Zinc Ferrites

These materials cover a useful frequency range from 1 kHz to 200 MHz and have the following properties:

Initial relative permeability	20 - 650
Maximum flux density	0.19 - 0.32 Wb/m^2
Resistivity	10^3 Ωm

Barium Ferrite

This material is quite different from those mentioned above and is used in the manufacture of permanent magnets. Representative properties are:

Remanent flux density	0.36 Wb/m^2
Coercive force	1.1 x 10^5 A/m

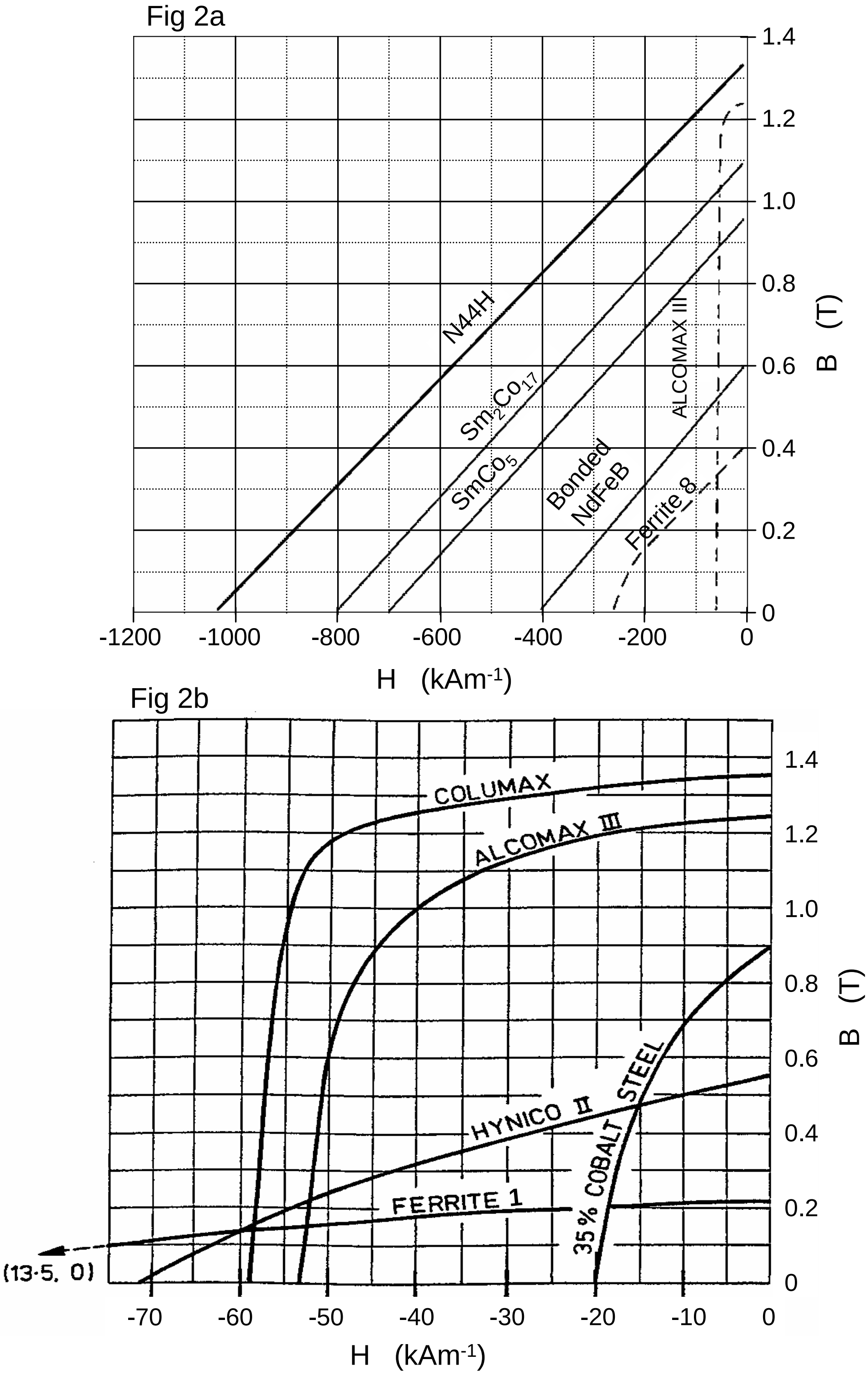


Figure 2: Demagnetization curves.

3 ELECTROMAGNETISM

3.1 Electromagnetic Fields. Fundamental Variables and Equations

I : Current, ampere (A)
V : Potential, volt (V)
Q : Charge, coulomb (C)
ρ : Charge density (C m^{-3})
ϕ : Magnetic flux, weber (Wb)
ϕ' : Magnetic flux linkage, weber (Wb)
$\mathbf{J}$: Current density (A m^{-2})
$\mathbf{B}$: Magnetic flux density, tesla (T)
$\mathbf{H}$: Magnetic field intensity, (A m^{-1})
$\mathbf{M}$: Magnetisation (A m^{-1})
$\mathbf{D}$: Electric flux density, (C m^{-2})

$\mathbf{E}$: Electric field intensity, (V m^{-1})
$\mathbf{P}$: Electrical polarisation, dipole density per unit volume (C m^{-2})
ε : Dielectric permittivity, (F m^{-1}) — Free space $\varepsilon_0 = 8.854 \times 10^{-12}$ F m^{-1}
ε_r : Relative permittivity (dielectric constant)
μ : Magnetic permeability (H m^{-1}) — Free space $\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}
μ_r : Relative permeability
c : Speed of light in a vacuum, 3×10^8 m s^{-1}
σ : Electrical conductivity, (S m^{-1})

Constitutive relations

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}), \qquad \mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P},$$

For linear materials:

$$\mathbf{B} = \mu \mathbf{H}, \quad \mu = \mu_0 \mu_r, \quad \mathbf{D} = \varepsilon \mathbf{E}, \quad \varepsilon = \varepsilon_0 \varepsilon_r, \quad \mathbf{J} = \sigma \mathbf{E}$$

3.2 Electrostatic equations

Electrostatic potential between points 1 and 2,

$$V_2 - V_1 = -\int_1^2 \mathbf{E} \cdot \mathbf{dl} \qquad \text{(vector form)}$$

$$V_2 - V_1 = -\int_1^2 E dl \qquad \text{(scalar form)}$$

Maxwell - Faraday equation

$$\oint_C \mathbf{E} \cdot \mathbf{dl} = -\int_S \dot{\mathbf{B}} \cdot \mathbf{dS}$$

Capacitance

$$Q = CV, \qquad I = C\frac{dV}{dt}$$

$$C = \frac{\varepsilon A}{d} \qquad \text{(parallel plate capacitor)}$$

Energy stored in a capacitance

$$W = \frac{1}{2} C V^2$$

Virtual work

$$F = \frac{1}{2} V^2 \frac{\partial C}{\partial x}$$

3.3 Magnetostatic equations

Biot-Savart law

$$dH = \frac{I}{4\pi r^2} dl sin\theta \qquad \text{(scalar form)}$$

$$d\mathbf{H} = \frac{I}{4\pi r^3} \mathbf{dl} \times \mathbf{r} \qquad \text{(vector form)}$$

Magnetic Flux

$$\phi = \int_S \mathbf{B} \cdot \mathbf{dS} \qquad \text{(vector form)}$$

$$\phi = BA \qquad \text{(scalar form, where A is the area of the surface S)}$$

Magnetic flux linkage

$$\phi' = N\phi \qquad N = \text{number of turns in a coil}$$

3.4 Gauss's Law

Scalar form for Gaussian surface S perpendicular to the lines of flux:

$$DA = Q \qquad (Q \text{ is the charge enclosed by the surface } S\text{, of area } A)$$

Associated with Maxwell's equations:

$$\oint_S \mathbf{D} \cdot \mathbf{dS} = Q, \qquad \oint_S \mathbf{B} \cdot \mathbf{dS} = 0 \quad \text{always.}$$

3.5 Ampère's Law

For a closed path C (360 degree circulation) of length l with current of I through N turns:

$$Hl = NI \quad \text{(scalar form),} \qquad \oint_C \mathbf{H} \cdot \mathbf{dl} = NI \quad \text{(vector form).}$$

Maxwell – Ampère equation (generalised circuit law):

$$\oint_C \mathbf{H} \cdot \mathbf{dl} = \int_S \left(\mathbf{J} + \dot{\mathbf{D}}\right) \cdot \mathbf{dS}$$

where S is any open surface bounded by a closed curve C.

3.6 Maxwell's Equations (point form)

$$\nabla \times \mathbf{E} = -\dot{\mathbf{B}}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \dot{\mathbf{D}}$$

$$\nabla \cdot \mathbf{D} = \rho, \quad \nabla \cdot \mathbf{B} = 0$$

3.7 Gradient law

Gradient of potential:

$$\begin{aligned} \mathbf{E} &= -\nabla V \quad \text{(electrostatic only)} \\ &= -\left(\underline{i}\frac{\partial V}{\partial x} + \underline{j}\frac{\partial V}{\partial y} + \underline{k}\frac{\partial V}{\partial z}\right) \text{ in Cartesian coordinates.} \end{aligned}$$

3.8 Poisson's Equation

$$\nabla^2 V = -\frac{\rho}{\varepsilon},$$

(becomes Laplace's equation for $\rho = 0$).

3.9 Laplace's Equation

In rectangular coordinates

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

In cylindrical coordinates

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} + \frac{\partial^2 V}{\partial z^2} = 0$$

In spherical coordinates

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi^2} = 0$$

3.10 Transmission Lines

3.10.1 Lossless Transmission Lines

L = loop inductance / unit length
C = shunt capacitance / unit length
Wave velocity = $v = \frac{1}{\sqrt{LC}}$
Wavelength $\lambda = \frac{2\pi v}{\omega}$
Characteristic impedance $Z_0 = \sqrt{\frac{L}{C}}$
Phase constant $\beta = \frac{2\pi}{\lambda} = \frac{\omega}{v}$
Propagation constant = $j\beta$
For sinusoidal time variation, with forward and reverse waves superposed,

$$\begin{aligned} V(x,t) &= \left(V_F e^{-j\beta x} + V_B e^{j\beta x}\right) e^{j\omega t} \\ I(x,t) &= \left(I_F e^{-j\beta x} + I_B e^{j\beta x}\right) e^{j\omega t} \end{aligned}$$

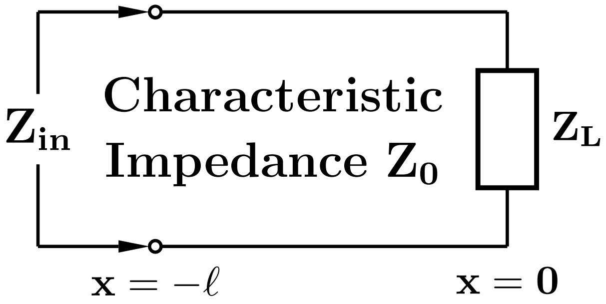
with

$$I_F = \frac{V_F}{Z_0} \qquad I_B = -\frac{V_B}{Z_0}$$

At $x = 0$, with load Z_L, voltage reflection coefficient:

$$\begin{aligned} \rho_L &= \frac{V_B}{V_F} = \frac{Z_L - Z_0}{Z_L + Z_0} \\ \frac{Z_L}{Z_0} &= \frac{1 + \rho_L}{1 - \rho_L} \end{aligned}$$

For any value of x, $\rho(x) = \rho_L e^{j2\beta x}$



Input impedance $\quad Z_{in} = Z_0 \frac{Z_L + Z_0 j\tan(\beta l)}{Z_0 + Z_L j\tan(\beta l)}$

3.10.2 Lossy Transmission Lines

R = series/loop resistance / unit length
G = shunt conductance / unit length
Characteristic impedance $Z_0 = \sqrt{\dfrac{R+j\omega L}{G+j\omega C}}$
Propagation constant = $\gamma = (\alpha + j\beta) = \sqrt{(R+j\omega L)(G+j\omega C)}$

$$\begin{aligned} V(x,t) &= (V_F e^{-\gamma x} + V_B e^{\gamma x})e^{j\omega t} \\ I(x,t) &= (I_F e^{-\gamma x} + I_B e^{\gamma x})e^{j\omega t} \\ Z_{in} &= Z_0 \frac{Z_L + Z_0 j \tanh(\gamma l)}{Z_0 + Z_L j \tanh(\gamma l)} \\ \rho(x) &= \rho_L e^{2\gamma x} \end{aligned}$$

3.10.3 Propagation of waves in media

μ = permeability
ε = permittivity
n = refractive index=$\sqrt{\varepsilon_r \mu_r}$
Intrinsic impedance $\eta = \sqrt{\dfrac{\mu}{\varepsilon}}$
Wave velocity $v = \dfrac{1}{\sqrt{\mu\varepsilon}}$
Phase constant $\beta = \omega\sqrt{\mu\varepsilon}$
Wave propagation

$$\begin{aligned} E_x &= \mathrm{Re}\left\{\mathbf{E}_{xF} e^{j(\omega t - \beta z)} + \mathbf{E}_{xB} e^{j(\omega t + \beta z)}\right\} \\ H_y &= \mathrm{Re}\left\{\mathbf{H}_{yF} e^{j(\omega t - \beta z)} + \mathbf{H}_{yB} e^{j(\omega t + \beta z)}\right\} \end{aligned}$$

Reflection coefficient $\rho_L = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$
Snell's Law: $\dfrac{\sin\theta_i}{\sin\theta_t} = \dfrac{n_t}{n_i} = \dfrac{\eta_i}{\eta_t}$
The intensity of an electromagnetic wave (complex Poynting vector) $= \dfrac{1}{2}\mathrm{Re}\,(\mathbf{E} \times \mathbf{H}^*)$

3.10.4 Reflection of EM waves perpendicular to a boundary

An incident electric field E_i perpendiclar to a boundary will create reflected (E_r) and transmitted (E_t) wave such that:

$$E_i + E_r = E_t$$

3.10.5 Propagation of waves in conducting media

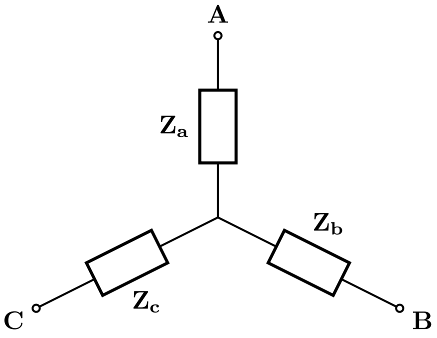
σ= conductivity $\neq 0$
Propagation constant = $\gamma = (\alpha + j\beta) = \sqrt{j\omega\mu(\sigma + j\omega\varepsilon)}$.
Intrinsic impedance $\eta = \sqrt{\dfrac{j\omega\mu}{(\sigma + j\omega\varepsilon)}}$
Wave propagation

$$\begin{aligned} E_x &= \mathrm{Re}\left\{\mathbf{E}_{xF} e^{-\gamma z} + \mathbf{E}_{xB} e^{\gamma z}\right\} e^{j\omega t} \\ H_y &= \mathrm{Re}\left\{\mathbf{H}_{yF} e^{-\gamma z} + \mathbf{H}_{yB} e^{\gamma z}\right\} e^{j\omega t} \end{aligned}$$

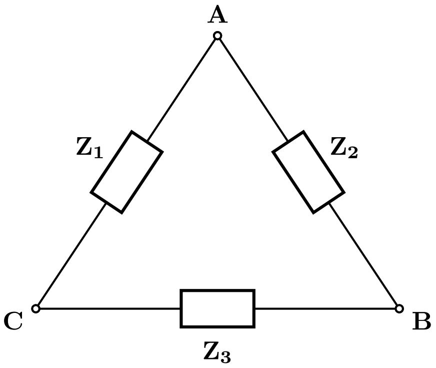
Reflection coefficient $\rho_L = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$

4 CIRCUITS INCLUDING LOGIC

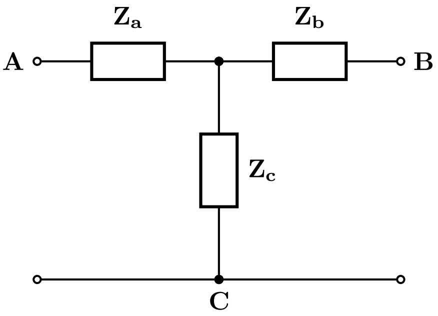
4.1 Star-Delta Transformation (Y - mesh or T - π)



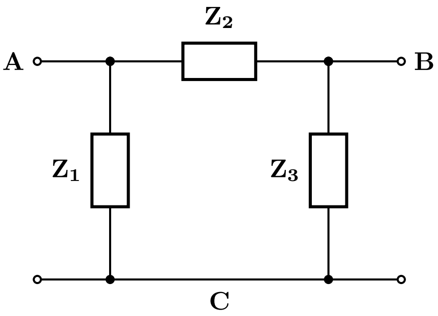
STAR (Y)



DELTA (MESH)



T



π

$$Z_a = \frac{Z_1 Z_2}{Z_1 + Z_2 + Z_3}$$

$$Z_b = \frac{Z_2 Z_3}{Z_1 + Z_2 + Z_3}$$

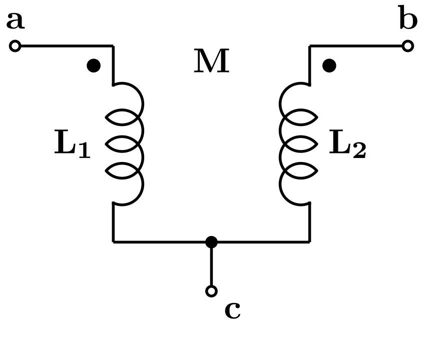
$$Z_c = \frac{Z_3 Z_1}{Z_1 + Z_2 + Z_3}$$

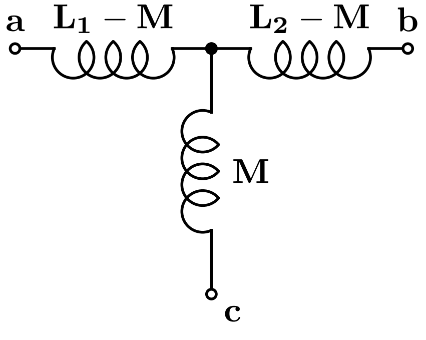
$$Z_1 = Z_c + Z_a + \frac{Z_c Z_a}{Z_b}$$

$$Z_2 = Z_a + Z_b + \frac{Z_a Z_b}{Z_c}$$

$$Z_3 = Z_b + Z_c + \frac{Z_b Z_c}{Z_a}$$

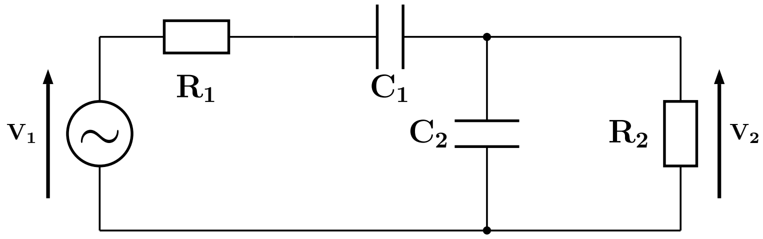
4.2 Tee-equivalent of Coupled Coils





4.3 Coupling Circuits

C_1 is usually relatively large and C_2 small.



At midband, when the effects of C_1 and C_2 can be ignored,

$$v_2 = v_1 \frac{R_2}{R_1 + R_2}$$

At high frequencies, v_2 drops to 70% of the midband value when $1/\omega_2 C_2 = R_1 R_2/(R_1 + R_2)$.
At low frequencies, v_2 drops to 70% of the midband value when $1/\omega_1 C_1 = R_1 + R_2$.

ω_1 and ω_2 are known as the lower and upper half power angular frequencies. They are also the -3dB, turnover, or 45° phase shift angular frequencies, which are other names to describe the same condition.

4.4 Resonant Circuits

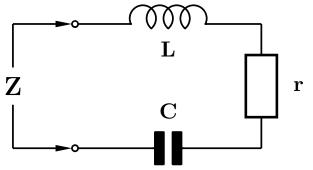
Undamped resonant angular frequency, ω_0, is given when $\omega_0{}^2 LC = 1$.

Quality factor $Q = \omega U/P$ where U is the total stored energy in the system, and P is the mean power dissipation.

Series Resonant Circuit:

$$Q_0 = \frac{\omega_0 L}{r} = \frac{1}{r\omega_0 C}, \qquad Z \approx r\left(1 + 2jQ_0 \frac{\delta\omega}{\omega_0}\right)$$

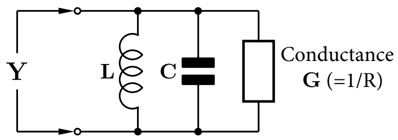
at frequencies close to resonance



Parallel Resonant Circuit:

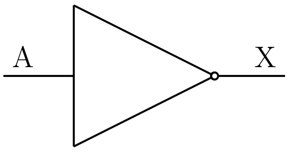
$$Q_0 = \frac{1}{\omega_0 LG} = \frac{\omega_0 C}{G}, \qquad Y \approx G\left(1 + 2jQ_0 \frac{\delta\omega}{\omega_0}\right)$$

at frequencies close to resonance

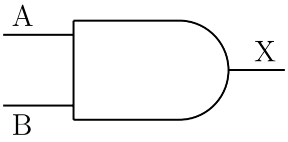


For series resonant circuit, $Z = r(1 \pm j)$ when $\delta\omega/\omega_0 = \pm 1/(2Q_0)$.
Half power bandwith $= (1/Q)\times$ resonant frequency.

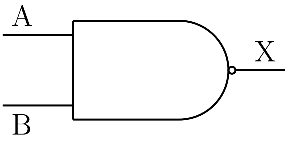
4.5 Logic



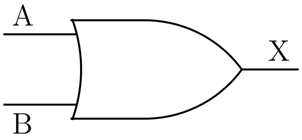
NOT: $X = \overline{A}$



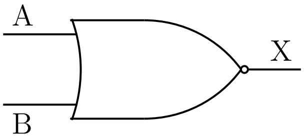
AND: $X = A \cdot B$



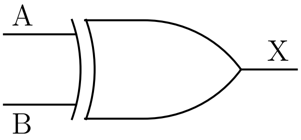
NAND: $X = \overline{A \cdot B}$



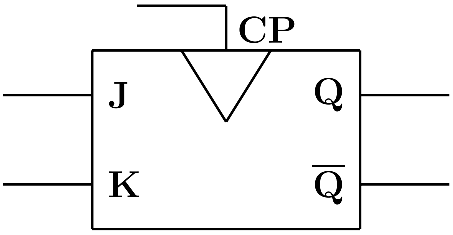
OR: $X = A + B$



NOR: $X = \overline{A + B}$



EXCLUSIVE OR: $X = A\overline{B} + \overline{A}B$



J-K Flip-Flop

Excitation table			
Q_n	Q_{n+1}	J	K
0	0	0	X
0	1	1	X
1	1	X	0
1	0	X	1

Truth table		
J_n	K_n	Q_{n+1}
0	0	Q_n
0	1	0
1	0	1
1	1	$\overline{Q}_n$

In the excitation table X represents a ‘don’t care’ state.

4.6 Boolean Algebra

Commutation

$$A + B = B + A$$

$$A \cdot B = B \cdot A$$

Association

$$A + (B + C) = (A + B) + C$$

$$A \cdot (B \cdot C) = (A \cdot B) \cdot C$$

Absorption

$$A + (A \cdot B) = A$$

$$A \cdot (A + B) = A$$

Distribution

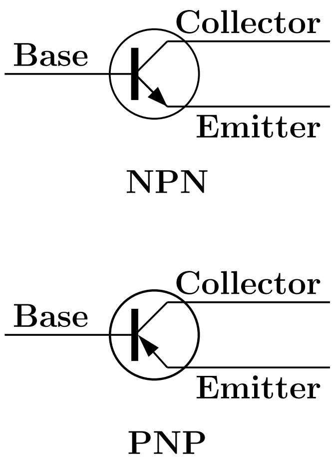
$$A \cdot (B + C) = A \cdot B + A \cdot C$$

$$A + (B \cdot C) = (A + B) \cdot (A + C)$$

De Morgan’s Theorems: $\overline{A + B} = \overline{A} \cdot \overline{B}$, $\overline{A \cdot B} = \overline{A} + \overline{B}$
where the bar indicates the inversion operation, $\overline{A}$ meaning ‘not A’.

5 SMALL SIGNAL EQUIVALENT CIRCUIT OF TRANSISTORS OPERATING AT LOW FREQUENCIES, INCLUDING THE OPERATIONAL AMPLIFIER

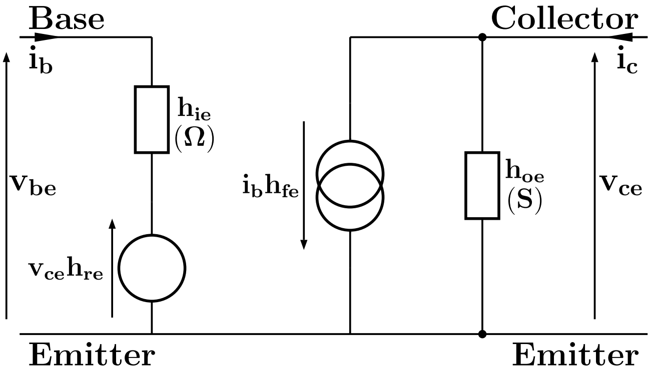
5.1 Bipolar Transistors



NPN

PNP

DEVICE SYMBOLS



AC SMALL SIGNAL MODEL

The currents and voltages shown on the diagram are small a.c. signals superimposed upon a specified d.c. operating condition. The hybrid parameters (h) depend on the d.c. operating condition and relate the signals by these equations:

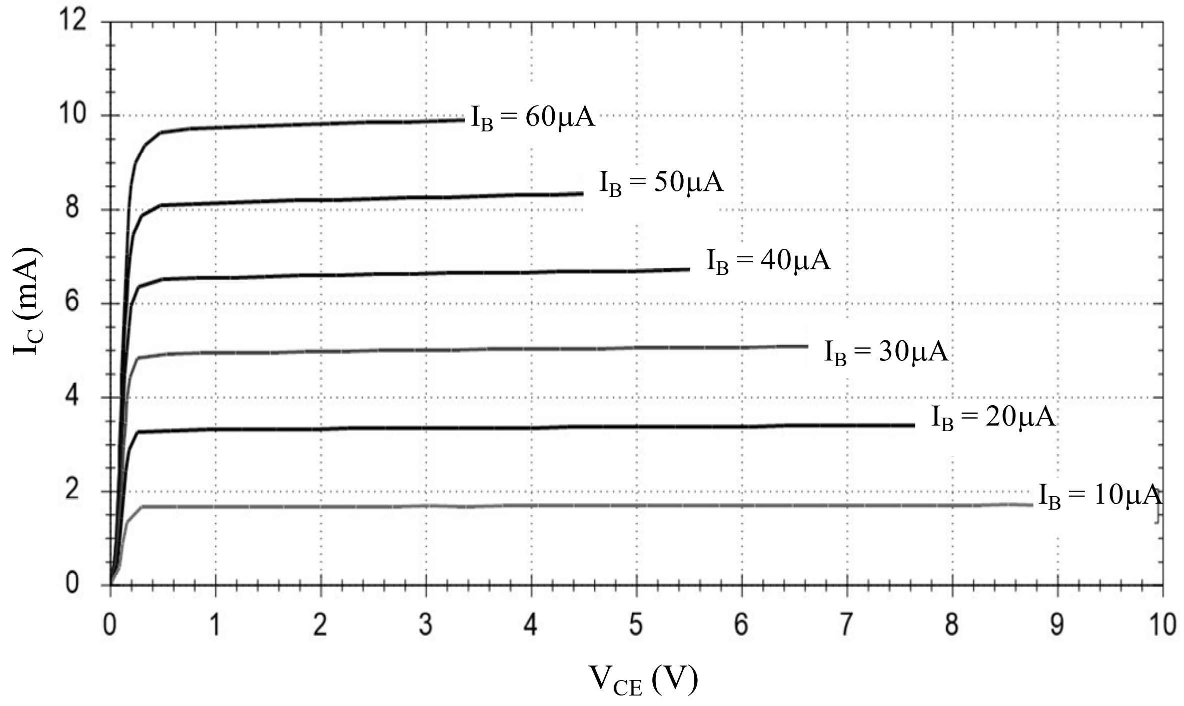
$$\begin{aligned} v_{be} &= h_{ie}i_b + h_{re}v_{ce} \\ i_c &= h_{fe}i_b + h_{oe}v_{ce} \end{aligned}$$

5.2 NPN Bipolar junction transistor BC182L - basic data

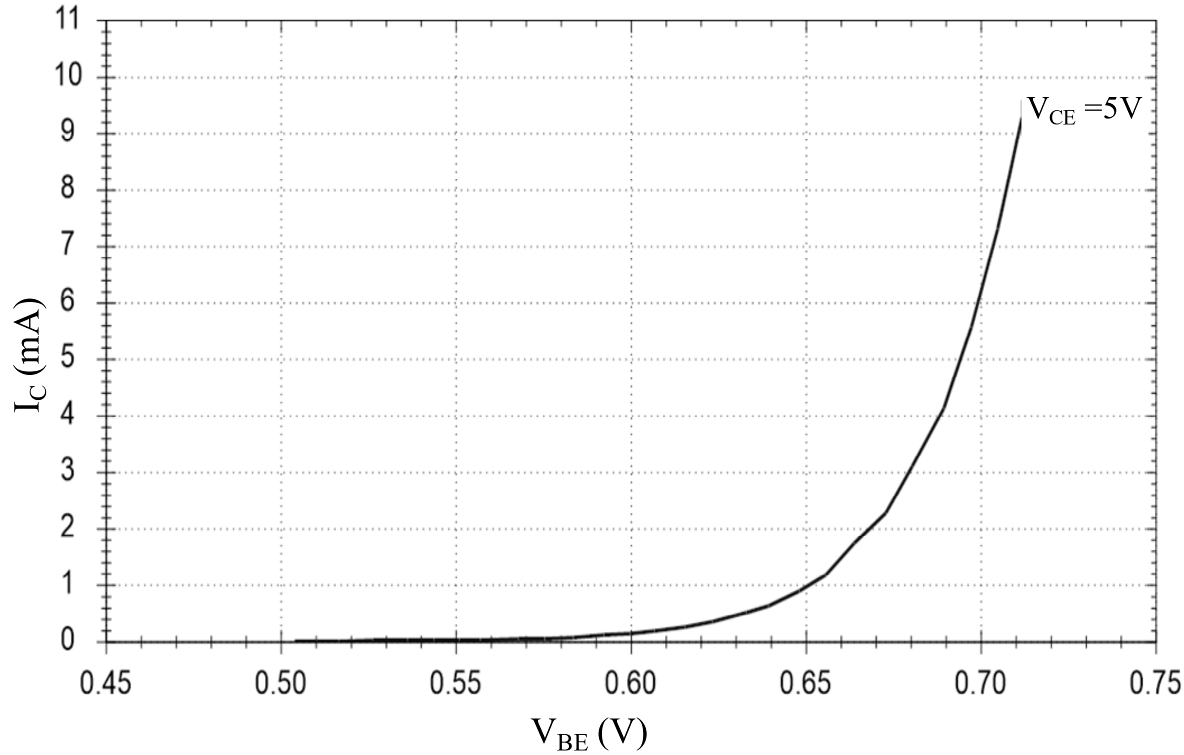
Typical values at 1 kHz, junction temperature of 25°C and at d.c. operating conditions of $I_c = 2$ mA, $V_{CE} = 5$ V are:

h_{ie}: 3.2 to 8.5 kΩ, h_{re}: 2×10^{-4}
h_{fe}: 240 to 500, h_{oe}: 30 to 100 μS

h_{re} and h_{oe} may often be neglected.

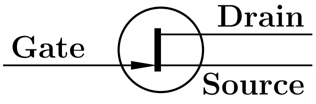


BC182L INPUT CHARACTERISTIC

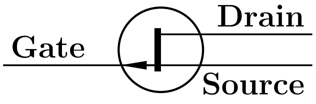


BC182L OUTPUT CHARACTERISTIC

5.3 Junction Field-Effect Transistors

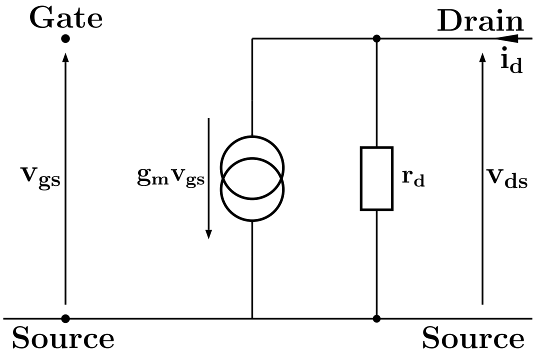


N-Channel FET



P-Channel FET

DEVICE SYMBOLS



AC SMALL SIGNAL MODEL

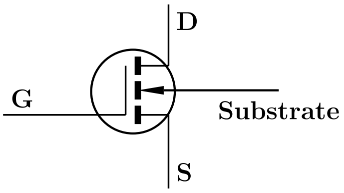
The currents and voltages shown on the diagram are small a.c. signals superimposed upon a specified d.c. operating condition. The device parameters depend on the d.c. operating condition and relate the signals by this equation:

$$i_d = g_m v_{gs} + v_{ds}/r_d$$

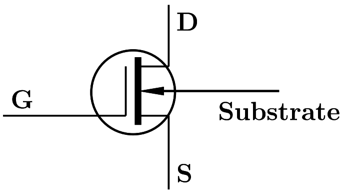
Typical values for the low-power silicon transistor $2N3819$ are:

$$g_m : 2 - 6.5\,\mathrm{mS}, \qquad r_d :> 20\,\mathrm{k\Omega}.$$

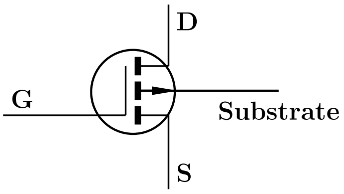
5.4 Insulated Gate Field-Effect Transistors (including MOSFETs)



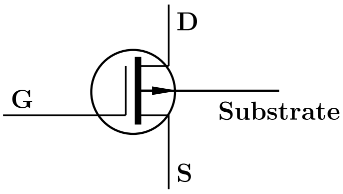
Enhancement
N-Channel



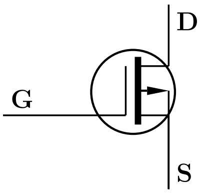
Depletion
N-Channel



Enhancement
P-Channel



(a)

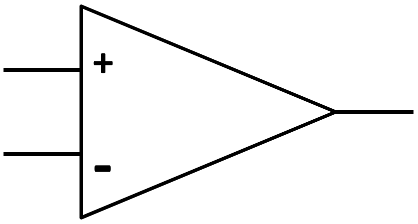


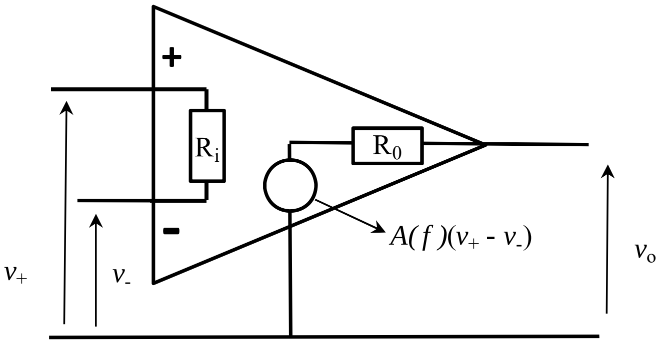
(b)

Depletion
P-Channel

In many cases the substrate is internally connected to the source electrode and the symbol for the p-channel depletion FET is as shown in Fig.(b).
The circuit model is the same as for the junction FET.

5.5 The Operational Amplifier

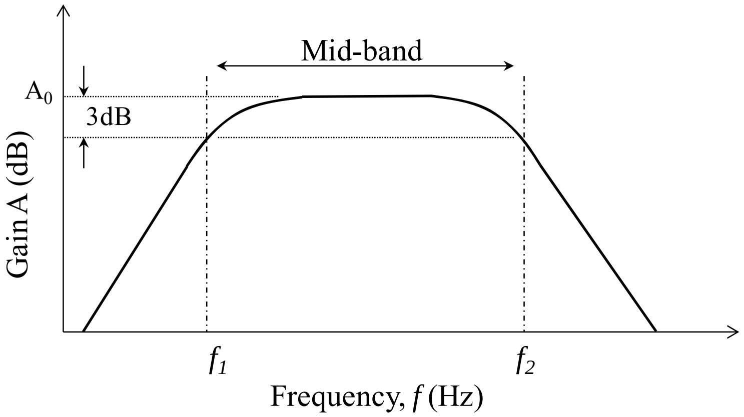




CIRCUIT SYMBOL EQUIVALENT AC MODEL

Where *A(f)* is the open loop gain of the operational amplifier, R_i is the input resistance and R_0 is the output resistance. For an Ideal operational amplifier $A(f) = \infty$, $R_i = \infty$, $R_0 = 0$.

5.6 Frequency response of first order circuits



PLOT OF GAIN VS FREQUENCY

The frequency dependent gain of an amplifier, *A(f),* can be expressed in the form:

$$A(f) = (\text{low freq. cut off, } f_1) \times A_o \times (\text{high freq. cut off, } f_2)$$

$$A(f) = \left(\frac{1}{1 + \frac{f_1}{jf}} \right) A_0 \left(\frac{1}{1 + \frac{jf}{f_2}} \right)$$

where A_o is the gain at mid-band frequencies.

5.7 3dB points of first order systems

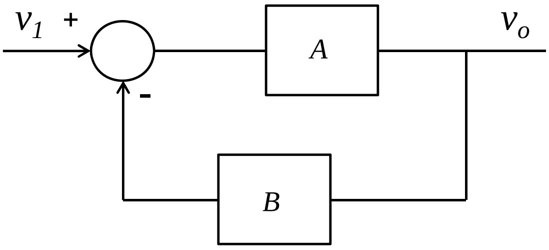
If the first order characteristic (such as voltage gain) of the frequency response is in the complex ratio form:

$$\frac{A+jB}{C+jD},$$

then the 3dB frequency (where the power is reduced by a half, or if in terms of voltage, where the voltage is reduced by $\frac{1}{\sqrt{2}}$, 0.7, 70%) can be found from the denominator condition:

$$C = D.$$

5.8 Basic Principles of Negative Feedback



$$Gain = \frac{v_o}{v_1} = \frac{A}{(1 + AB)}$$

VOLTAGE FEEDBACK SYSTEM

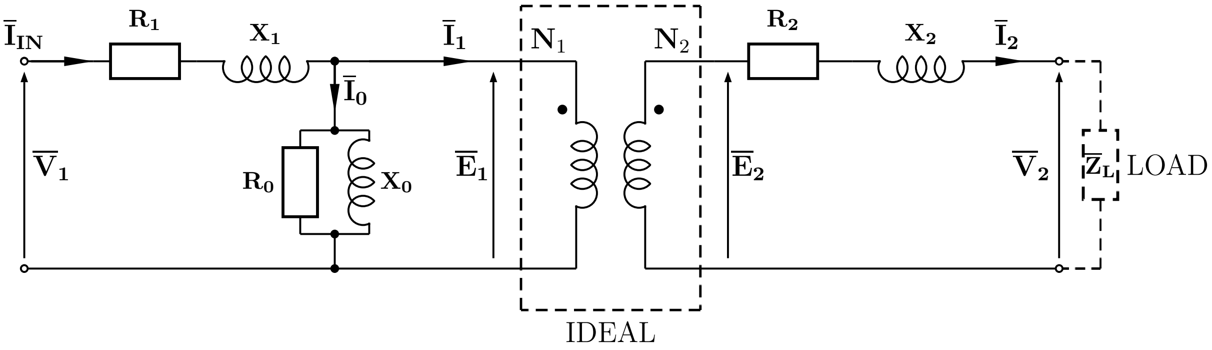
For **voltage amplifiers**, reducing the gain by (1 + *AB*)

- (i) Increases the input resistance by (1 + *AB*)
- (ii) Decreases the output resistance by (1 + *AB*)
- (iii) Increases the upper 3 dB frequency
- (iv) Reduces the lower 3 dB frequency
- (v) Reduces the variation in open loop gain by (1 + *AB*).

6 ELECTRICAL POWER AND MACHINES

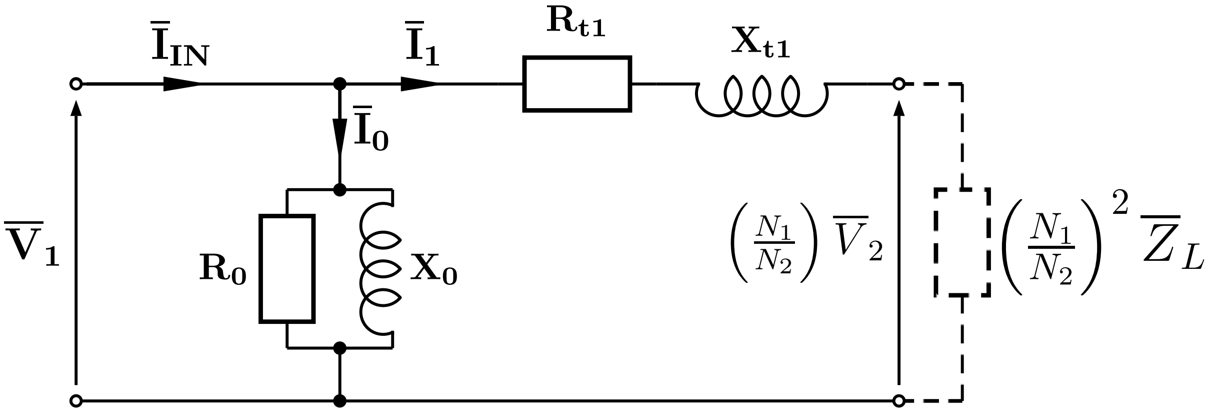
6.1 Transformer

6.1.1 Complete equivalent circuit



$$\begin{aligned}\bar{I}_1 N_1 &= \bar{I}_2 N_2 \\ \frac{\bar{E}_1}{N_1} &= \frac{\bar{E}_2}{N_2}\end{aligned}$$

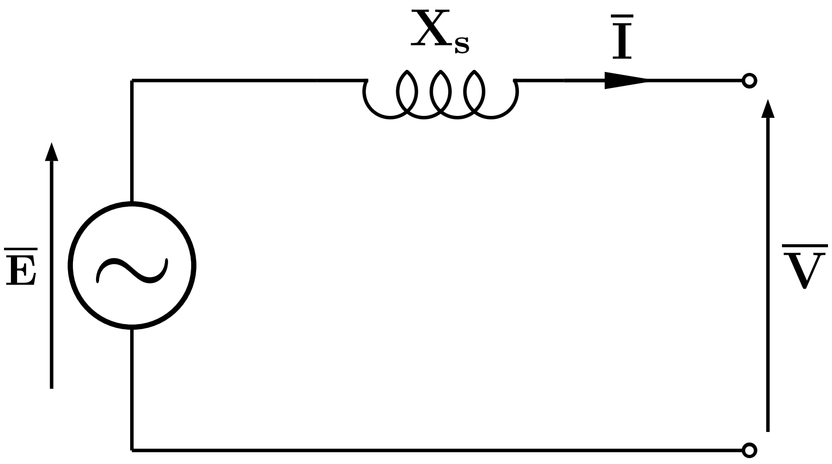
6.1.2 Simplified Equivalent Circuit



$$\begin{aligned}R_{t1} &= R_1 + \left(\frac{N_1}{N_2}\right)^2 R_2 \\ X_{t1} &= X_1 + \left(\frac{N_1}{N_2}\right)^2 X_2\end{aligned}$$

6.2 Three-phase synchronous machine

6.2.1 Equivalent circuit for cylindrical rotor machine (motor)



V = Terminal phase voltage
I = Input phase current
X_s = Synchronous reactance (per phase)
E = Generated phase emf

6.2.2 Basic relationships

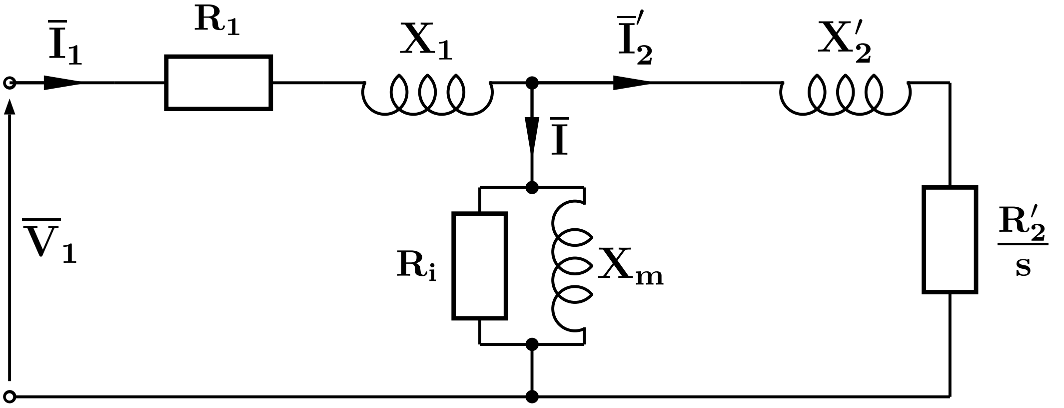
ω_s = Synchronous speed (rad/s)
ω = Supply frequency (rad/s)
p = Number of pole-pairs
δ = Electrical load angle

$$\omega_s = \frac{\omega}{p}$$

$$\text{Electromagnetic torque, } T = \frac{3VE}{\omega_s X_s}\sin\delta$$

6.3 Three-phase induction motor

6.3.1 Equivalent circuit



6.3.2 Basic relationships

$$\text{Slip, } s = \frac{\omega_s - \omega_r}{\omega_s}$$

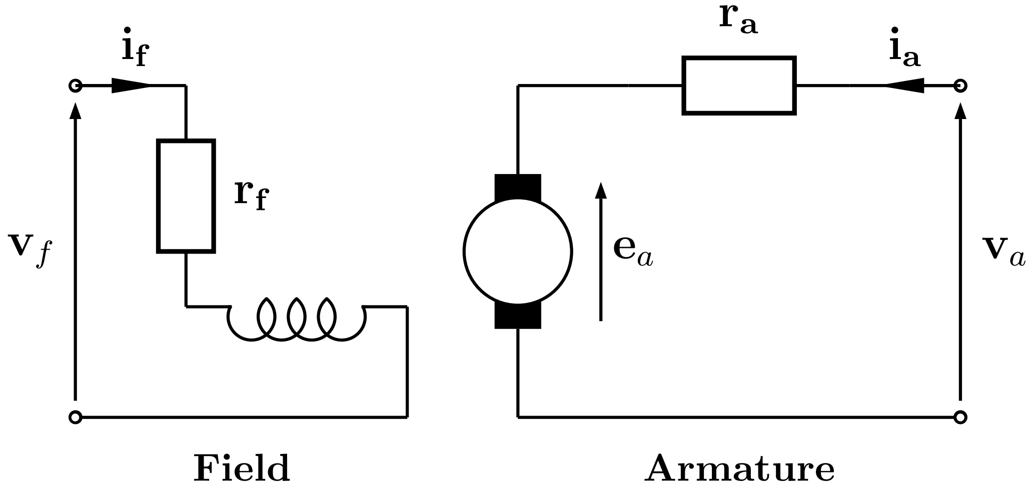
$$\text{Total torque, } T = \frac{3\bar{I}_2'^2}{\omega_s}\frac{R_2'}{s}$$

ω_s = Synchronous speed as defined in 6.2.2.

ω_r = Rotor speed (rad/s)

6.4 Separately-excited d.c. motor

6.4.1 Equivalent circuit for separately-excited motor



6.4.2 Basic relationships

$$e_a = K\phi\omega$$

$$T = K\phi i_a$$

K = emf constant

ϕ = flux per pole $\qquad \phi = \phi(i_f)$

ω = rotor speed

i_a = armature current

T = torque

6.5 Per-Unit Calculations

VA_b = Base VA (three-phase)

V_b = Base line voltage

$$Z_b = \frac{V_b^2}{VA_b}$$

$$I_b = \frac{VA_b}{\sqrt{3}V_b}$$

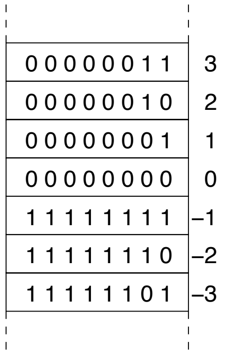
Change of base VA: $Z_{pu(2)} = Z_{pu(1)}\dfrac{VA_{b(2)}}{VA_{b(1)}}$

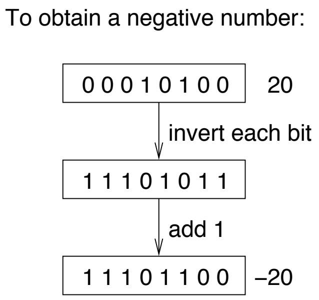
7 MICROPROCESSORS

7.1 Decimal - Hex - ASCII conversion

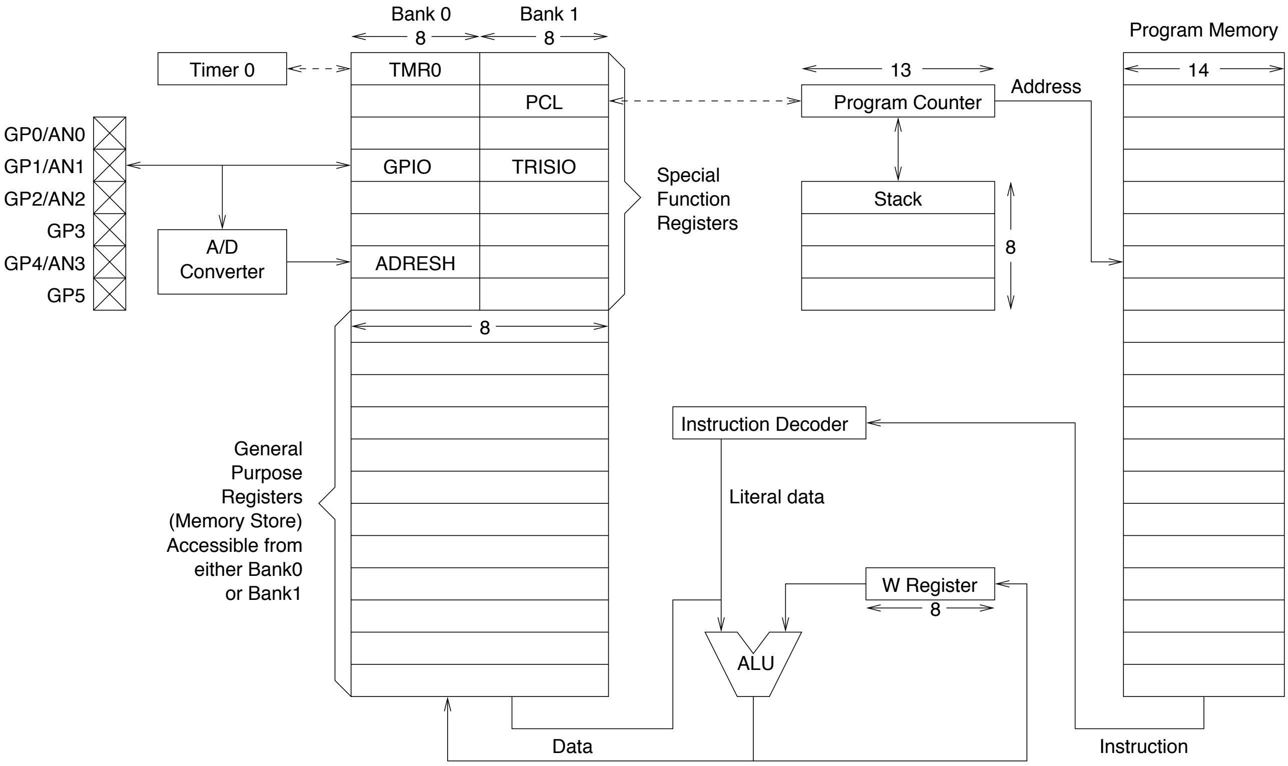
DEC	HEX	ASCII	DEC	HEX	ASCII	DEC	HEX	ASCII	DEC	HEX	ASCII
0	00	`^@`	32	20	`SPACE`	64	40	`@`	96	60	`` ` ``
1	01	`^A`	33	21	`!`	65	41	`A`	97	61	`a`
2	02	`^B`	34	22	`"`	66	42	`B`	98	62	`b`
3	03	`^C`	35	23	`#`	67	43	`C`	99	63	`c`
4	04	`^D`	36	24	`$`	68	44	`D`	100	64	`d`
5	05	`^E`	37	25	`%`	69	45	`E`	101	65	`e`
6	06	`^F`	38	26	`&`	70	46	`F`	102	66	`f`
7	07	`^G`	39	27	`'`	71	47	`G`	103	67	`g`
8	08	`^H`	40	28	`(`	72	48	`H`	104	68	`h`
9	09	`^I`	41	29	`)`	73	49	`I`	105	69	`i`
10	0A	`^J`	42	2A	`*`	74	4A	`J`	106	6A	`j`
11	0B	`^K`	43	2B	`+`	75	4B	`K`	107	6B	`k`
12	0C	`^L`	44	2C	`,`	76	4C	`L`	108	6C	`l`
13	0D	`^M`	45	2D	`-`	77	4D	`M`	109	6D	`m`
14	0E	`^N`	46	2E	`.`	78	4E	`N`	110	6E	`n`
15	0F	`^O`	47	2F	`/`	79	4F	`O`	111	6F	`o`
16	10	`^P`	48	30	`0`	80	50	`P`	112	70	`p`
17	11	`^Q`	49	31	`1`	81	51	`Q`	113	71	`q`
18	12	`^R`	50	32	`2`	82	52	`R`	114	72	`r`
19	13	`^S`	51	33	`3`	83	53	`S`	115	73	`s`
20	14	`^T`	52	34	`4`	84	54	`T`	116	74	`t`
21	15	`^U`	53	35	`5`	85	55	`U`	117	75	`u`
22	16	`^V`	54	36	`6`	86	56	`V`	118	76	`v`
23	17	`^W`	55	37	`7`	87	57	`W`	119	77	`w`
24	18	`^X`	56	38	`8`	88	58	`X`	120	78	`x`
25	19	`^Y`	57	39	`9`	89	59	`Y`	121	79	`y`
26	1A	`^Z`	58	3A	`:`	90	5A	`Z`	122	7A	`z`
27	1B	`^[`	59	3B	`;`	91	5B	`[`	123	7B	`{`
28	1C	`^\`	60	3C	`<`	92	5C	`\`	124	7C	`|`
29	1D	`^]`	61	3D	`=`	93	5D	`]`	125	7D	`}`
30	1E	`^^`	62	3E	`>`	94	5E	`^`	126	7E	`~`
31	1F	`^_`	63	3F	`?`	95	5F	`_`	127	7F	`DEL`

7.2 Binary representation of negative numbers (twos complement)





7.3 Schematic of PIC Microprocessor



GPIO Register (register location 0x05, bank 0)

U-0	U-0	R/W-x	R/W-x	R/W-x	R/W-x	R/W-x	R/W-x
—	—	GPIO5	GPIO4	GPIO3	GPIO2	GPIO1	GPIO0
bit 7							bit 0

Bits 0-5	General purpose input or outputs (NB GPIO3 can only be an input)
Bit 6-7	Unimplemented (read as 0)

TRISIO Register (register location 0x85, bank 1)

U-0	U-0	R/W-x	R/W-x	R-1	R/W-x	R/W-x	R/W-x
—	—	TRISIO5	TRISIO4	TRISIO3	TRISIO2	TRISIO1	TRISIO0
bit 7							bit 0

Bits 0-5	Set (=1) means equivalent GPIO pin is input Clear (=0) means equivalent GPIO pin is output NB TRISIO3 is always 1
Bit 6-7	Unimplemented (read as 0)

STATUS Register (register location 0x03, bank 0)

IRP	RP1	RP0	$\overline{\text{TO}}$	$\overline{\text{PD}}$	Z	DC	C
bit 7							bit 0

C	Carry Flag
DC	Digital Carry Flag
Z	Zero Flag
PD	Power Down
TO	Time Out
RP0	Register Bank (0 sets bank 0, 1 sets bank 1)
RP1	Not Used
IRP	Not Used

Other Registers

W	Working register
PC	Programme Counter (13 bits, using PCL, 0x02, for lower 8 bits and PCLATH, 0x0A, for upper 5 bits)
FSR	File select register (0x04, bank 0)
INDF	Indirect file register (used for indirect addressing) - doesn't have a physical address

7.4 PIC Instruction Set

File Register Instructions that operate with whole bytes

These instructions take a memory location F and most take a destination *d* as arguments. The instructions operate on the contents of the memory location specified by F. The destination *d* can be the source memory location specified by F (e.g. addwf 0x20,F where the value of the bit d in the instruction is 1) or in the working register specified by W (e.g. addwf 0x20,W where the value of d is 0). The default is d = 1.

mnemonic	args	Description	Cycles	Opcode			
addwf	F,*d*	Add W and F and store the result in *d*	1	00	0111	dfff	ffff
andwf	F,*d*	AND W and F and store the result in *d*	1	00	0101	dfff	ffff
clrf	F	Clear F	1	00	0001	1fff	ffff
comf	F,*d*	Complement F and store the result in *d*	1	00	1001	dfff	ffff
decf	F,*d*	Decrement F and store the result in *d*	1	00	0011	dfff	ffff
decfsz	F,*d*	Decrement F and store the result in *d*, if the result is zero then skip the next instruction	1(2)	00	1011	dfff	ffff
incf	F,*d*	Increment F and store the result in *d*	1	00	1010	dfff	ffff
incfsz	F,*d*	Increment F and store the result in *d*, if the result is zero then skip the next instruction	1(2)	00	1111	dfff	ffff
iorwf	F,*d*	Inclusive OR W with F and store the result in *d*	1	00	0100	dfff	ffff
movf	F,*d*	Copy F to *d* (*d* = F)	1	00	1000	dfff	ffff
movwf	F	Copy W to F (F = W)	1	00	0000	1fff	ffff
rlf	F,*d*	Rotate F left through Carry and store the result in *d*	1	00	1101	dfff	ffff
rrf	F,*d*	Rotate F right through Carry and store the result in *d*	1	00	1100	dfff	ffff
subwf	F,*d*	Subtract W from F and store the result in *d*	1	00	0010	dfff	ffff
swapf	F,*d*	Swap low and high 4 bits of F and store the result in *d*	1	00	1110	dfff	ffff
xorwf	F,*d*	Exclusive OR W with F and store the result in *d*	1	00	0110	dfff	ffff

File Register Instructions that operate with bits

These instructions take a memory location F and a bit b(=0..7) in that memory location as arguments.

bcf	F,b	Clear bit b in F	1	01	00bb	bfff	ffff
bsf	F,b	Set bit b in F	1	01	01bb	bfff	ffff
btfsc	F,b	Test bit b in F and skip the next instruction if that bit is clear (zero)	1(2)	01	10bb	bfff	ffff
btfss	F,b	Test bit b in F and skip the next instruction if that bit is set (one)	1(2)	01	11bb	bfff	ffff

Literal Instructions

These instructions take an 8 bit integer argument Q which is the number used in the instruction (by contrast to File instructions where the argument is the memory location of the number used).

addlw	Q	Add Q to W and store the result in W	1	11	111x	qqqq	qqqq
andlw	Q	AND Q with W and store the result in W	1	11	1001	qqqq	qqqq
iorlw	Q	Inclusive OR Q with W and store the result in W	1	11	1000	qqqq	qqqq
movlw	Q	Copy Q to W (W=Q)	1	11	00xx	qqqq	qqqq
retlw	Q	Return from Subroutine (Pull program counter from stack) and Copy Q to W (W=Q)	2	11	01xx	qqqq	qqqq
sublw	Q	Subtract W from Q and store the result in W	1	11	110x	qqqq	qqqq
xorlw	Q	Exclusive OR Q with W and store the result in W	1	11	1010	qqqq	qqqq

Call and Goto

These instructions take an 11 bit program destination P.

call	P	Push program counter onto the stack and Jump to program location P	2	10	0ppp	pppp	pppp
goto	P	Jump to program location P	2	10	1ppp	pppp	pppp

Instructions with no arguments

clrw		Clear W (W=0)	1	00	0001	0xxx	xxxx
clrwdt		Clear Watchdog timer (if the watchdog timer overflows, the PIC is reset)	1	00	0000	0110	0100
nop		No operation	1	00	0000	0xx0	0000
retfie		Return from interrupt (Pull program counter from stack and enable interrupts)	2	00	0000	0000	1001
return		Return from subroutine (Pull program counter from stack)	2	00	0000	0000	1000
sleep		Go into standby mode	1	00	0000	0110	0011