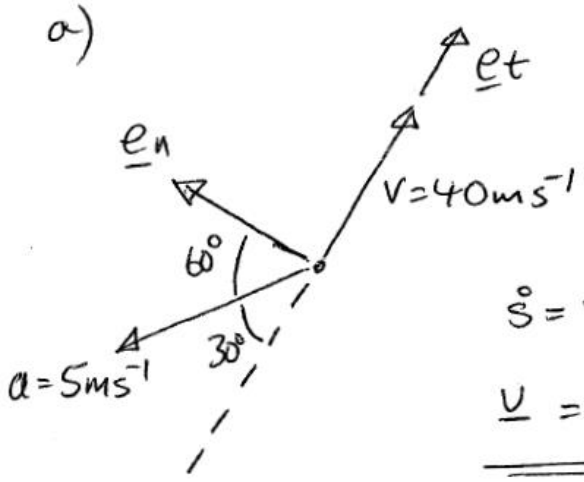
CUED IA Progress Test
17/Jan/2018:
Crib

Section A: Mechanical Engineering
Q1 (short): (2008: Q7)

7 a)



$$\dot{s} = V = 40\,ms^{-1}$$

$$\underline{V} = \dot{s}\,\underline{e}_t = 40\,\underline{e}_t \quad ms^{-1}$$

$$\underline{a} = \ddot{s}\,\underline{e}_t + \frac{\dot{s}^2}{\rho}\,\underline{e}_n$$

$$= -5\cos 30^\circ\,\underline{e}_t + 5\sin 30^\circ\,\underline{e}_n$$

$$\underline{a} = -\frac{5\sqrt{3}}{2}\,\underline{e}_t + \frac{5}{2}\,\underline{e}_n \quad ms^{-2}$$

b) $$\frac{\dot{s}^2}{\rho} = \frac{5}{2} \quad \therefore \rho = \frac{2\dot{s}^2}{5} = \frac{2 \cdot 40^2}{5} = 640\,m$$

Section A: Mechanical Engineering
Q2 (short): (2014: Q10)

10 (a) Radial acceleration $= 0$ after wire snaps

$$\therefore \ddot{r} - r\Omega^2 = 0$$

solving above for $\dot{r}(0) = L/2$, $\dot{r}(0) = 0$ we get

$$r(t) = A \cosh bt$$

where $A = \dfrac{L}{2}$

$$b = \Omega.$$

(b) when $r(t) = L$, $t = t_f$ say

$$L = \frac{L}{2} \cosh \Omega t_f$$

$$t_f = \frac{1}{\Omega} \cosh^{-1}(2).$$

Section A: Mechanical Engineering
Q3 (long): (2004: Q6)

6 a) If $r = 0.5$ m, extension $= 0.2$ m
so force in spring $= \underline{8\ N}$

$\therefore \frac{mV^2}{r} = \frac{0.25\ V^2}{0.5} = 8$

So $V^2 = 16 \rightarrow V = \underline{\underline{4\ m/s}}$

b) Momentum is conserved in the collision, but energy is not. Since the mass doubles, the velocity must halve.

Hence $V_{RADIAL} = \underline{\underline{0}}$, $V_{TANGENTIAL} = \underline{\underline{2\ m/s}}$

The combined mass still has 8 N acting on it, towards O

Hence $A_{RADIAL} = \frac{8}{0.5} = \underline{\underline{16\ m/s^2 \text{ inwards}}}$

and $A_{TANGENTIAL} = \underline{\underline{0}}$

c) The particle continues to travel round O, with varying r - it does not travel in an ellipse (this only happens if the central force is proportional to $1/r^2$, as in satellite orbits). Energy, and Moment of Momentum about O, will be conserved.

After the collision, energy $= \frac{1}{2}mV^2 + \frac{1}{2}ke^2$ Spring extension

$= \frac{1}{2} \times 0.5 \times 2^2 + \frac{1}{2} \times 40 \times 0.2^2$

$= 1 + 0.8 = \underline{1.8\ J}$

$M \text{ of } M = 0.5 \times 2 \times 0.5 = \underline{0.5\ kg\,m^2/s}$

6 c) (CONT)

Whenever velocity is perpendicular to OP, $mVr = 0.5 \rightarrow \underline{V = 1/r}$

So by energy, at max. and min. r,

$$\frac{1}{2} \times 0.5 \times \left(\frac{1}{r}\right)^2 + \frac{1}{2} \times 40 \times (r - 0.3)^2 = 1.8$$

$$0.25 + 20r^4 - 12r^3 + 1.8r^2 = 1.8r^2$$

$$\underline{\underline{20r^4 - 12r^3 + 0.25 = 0}}$$

This has solutions at $r \approx 0.391$ and $r = 0.5$, which are the minimum and maximum values for r after the collision.

Section B: Structural Mechanics
Q4 (short): (2006: Q3a)

3. a)

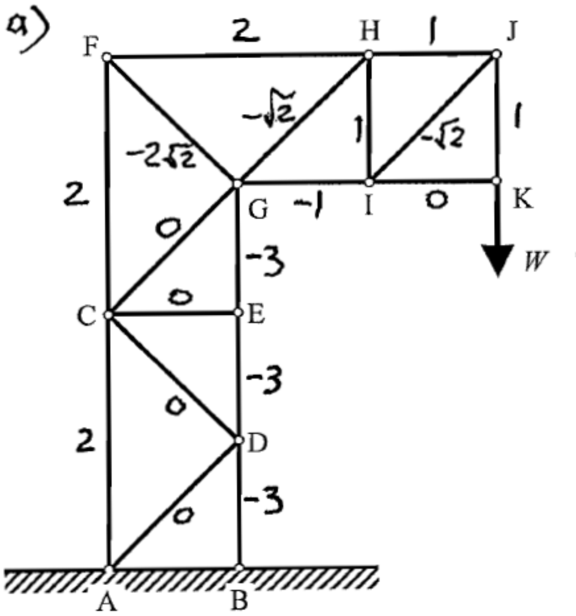


Fig. 3

T/W

A horizontal section anywhere through the tower ABGC shows zero force in the diagonals is required by horizontal equilibrium. Equilibrium at joints E and K requires $T_{CE} = T_{IK} = 0$. Considering whole structure, moments about A give $T_{BD} = -3W$, while vertical equilibrium gives $T_{AC} = 2W$.

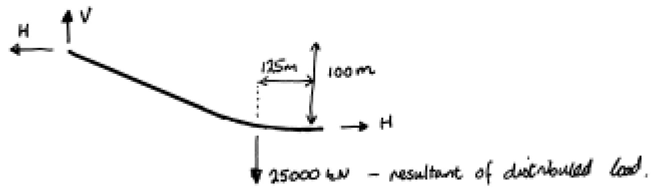
All other bar tensions found by sections or resolution of forces at joints. See Table.

Table f

bar	length	$\frac{T}{W}$
AC	$2L$	2
AD	$\sqrt{2}L$	0
BD	L	-3
CD	$\sqrt{2}L$	0
CE	L	0
CF	$2L$	2
CG	$\sqrt{2}L$	0
DE	L	-3
EG	L	-3
FG	$\sqrt{2}L$	$-2\sqrt{2}$
FH	$2L$	2
GH	$\sqrt{2}L$	$-\sqrt{2}$
GI	L	-1
HI	L	1
HJ	L	1
IJ	$\sqrt{2}L$	$-\sqrt{2}$
IK	L	0
JK	L	1

Section B: Structural Mechanics
Q5 (long): (1997: Q2abc + 2010: Q3)

2(a) Consider half cable



Equilibrium $\updownarrow$, $\underline{V = 25000 \text{ kN}}$

" $\circlearrowleft$ about left-hand support $H.100 = 25000\,(500-125)$

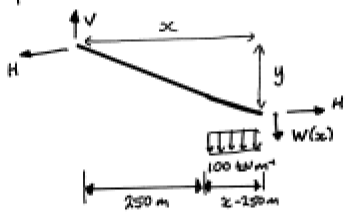
$\underline{H = 93750 \text{ kN}}$

b(i) for $0 \le x \le 250$ m, no external loads, constant slope

$$\frac{y}{x} = \frac{V}{H} = 0.267$$

$$\underline{y = 0.267.x}$$

b(ii) for $250 \le x \le 500$, draw free body of cable cut at x



Moments about cut

$$V.x = H.y + 100.(x-250).\left(\frac{x-250}{2}\right)$$

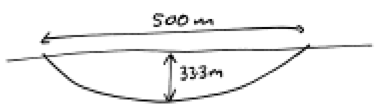
$$\therefore 25000x = 93750.y + 50\,(x^2 - 500x + 62500)$$

$$\therefore y = \frac{-50}{93750}\,(x^2 - 1000x + 62500)$$

Check, at $x = 500$m, $y = 100$m ✓, $\frac{dy}{dx} = 0$ ✓
at $x = 250$m, $y = 66.7$m from (i) and (ii) ✓

2(c) Length for $0 < x < 250$ is $\sqrt{250^2 + 66.7^2} = 258.7$ m

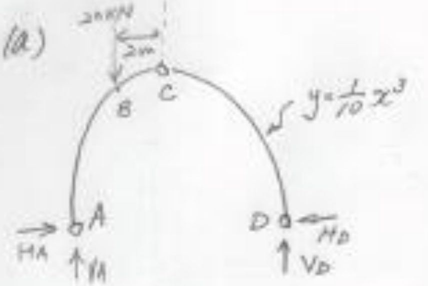
Length for $250 < x < 750$, use data book.



$$L = 500\left(1 + \frac{8 \times 33.3^2}{3 \times 500^2}\right) = 505.9 \text{ m}$$

$\therefore$ Total length $= 505.9 + 258.7 \times 2 \simeq \underline{1023\text{m}}$

3 (a)

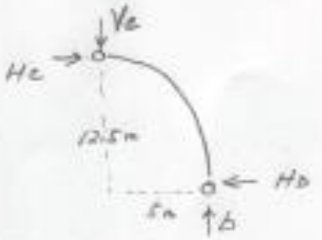


Moment at A

$V_D \times 10 - 20 \times 3 = 0$

$\underline{V_D = 6 \text{ kN}}$ //

$\Sigma V = 0 \quad \underline{V_A = 4 \text{ kN}}$ //



Using the right hand side only

Moment at C

$6 \times 5 - H_D \times 12.5 = 0$

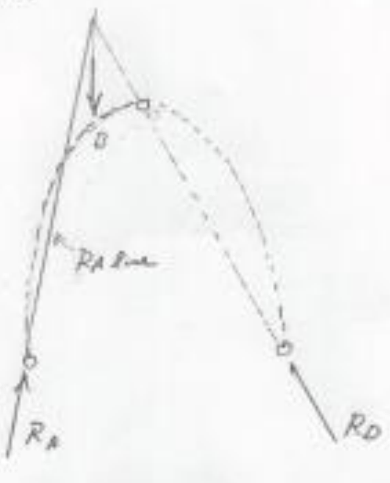
$\underline{H_D = 2.4 \text{ kN}}$ //

$\Sigma H = 0 \quad H_A = H_D \quad \underline{H_A = 2.4 \text{ NN}}$ //

Reaction at $A = \sqrt{(14)^2 + (2.4)^2} = \underline{14.20 \text{ kN}}$ //

Reaction at $D = \sqrt{(6)^2 + (2.4)^2} = \underline{6.46 \text{ kN}}$ //

(b)



Draw the force diagram like the one shown in the left figure using the scaled figure given in the question. By inspection, the largest separation between section AB and the RA-line is at B (2, 0.8). The maximum moment will be at this location.

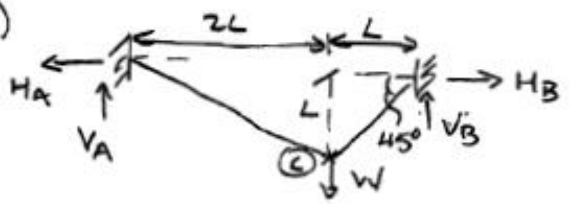
$M_B + 2.4(12.5 - 0.8) - 14 \times (5 - 2) = 0$

$\underline{M_B = 13.92 \text{ kN.m}}$ //

Note that the bending moment will change its sign along A to B. Deriving a general equation for M(x) and solving for dM(x) = 0 will not give the correct answer for the location of the maximum moment. It only gives the maximum at the negative side.

Section B: Structural Mechanics
Q6 (short): (2008: Q1)

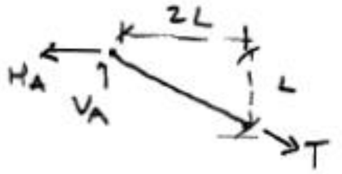
1 a)



$$\Sigma M_A = 0 \rightarrow V_B \cdot 3L - W \cdot 2L = 0 \qquad \therefore V_B = \tfrac{2}{3}W \uparrow$$

$$\Sigma V = 0 \rightarrow V_A + V_B = 0 \qquad \therefore V_A = \tfrac{1}{3}W \uparrow$$

FBD OF AC



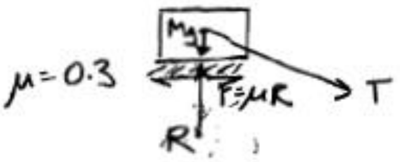
$$\Sigma M_C = 0 \rightarrow H_A \cdot L - V_A \cdot 2L = 0$$

$$H_A = 2V_A \qquad \therefore H_A = \frac{2}{3}W \leftarrow$$

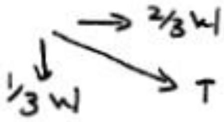
$$\Sigma H = 0 \rightarrow -H_A + H_B = 0$$

$$H_A = H_B \qquad \therefore H_B = \tfrac{2}{3}W \rightarrow$$

b) FBD at A



from part (a) - components of T



frictional force at point of sliding

$$R = Mg + W/3 \quad , \quad F = \mu R = H$$

for sliding not to occur when the applied horizontal component $H = 2/3\,W$

$$\mu(Mg + W/3) = \tfrac{2}{3}W \qquad \therefore M = 0.193W$$

$$M = \frac{17W}{9g}$$

Section C: Physical Principles of Electronics and Linear Circuits
Q7 (short): (2009: Q10)

10 (a) Gauss' Law: $\oint\!\!\oint_S \mathbf{D}.d\mathbf{S} = Q$

(b) Charges are confined to surface of both conductors, thus D = E = 0 for $r < 1$ mm and $r > 3$mm.

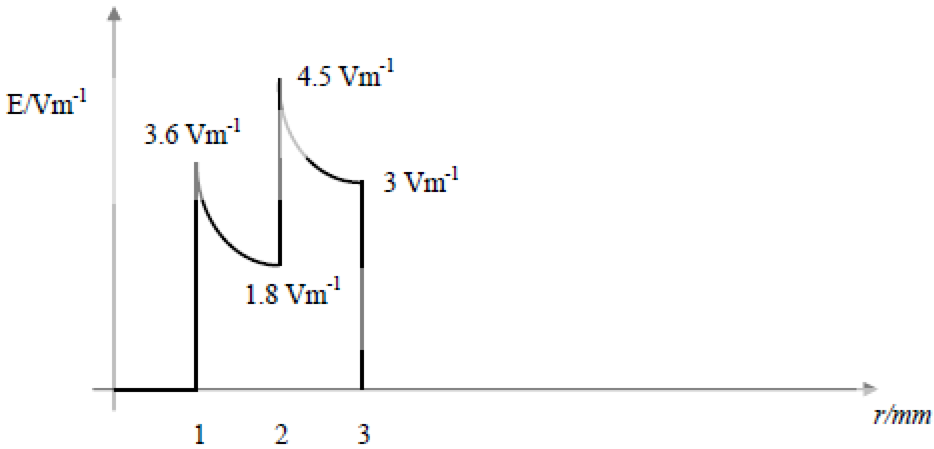
$D \times 2\pi r = \rho$ so $D = \rho/2\pi r$ and using $D = \varepsilon_0\varepsilon_r E$ gives $E = \rho/2\pi\varepsilon_0\varepsilon_r r$

Substituting in the numbers gives, in the material of relative permittivity 5:

E(1 mm) = 3.6 $\mathrm{Vm^{-1}}$ E(2 mm) = 1.8 $\mathrm{Vm^{-1}}$

In the material of relative permittivity 2:

E(2 mm) = 4.5 $\mathrm{Vm^{-1}}$ E(3 mm) = 3 $\mathrm{Vm^{-1}}$



(c) Find the voltage between inner and outer conductors by integrating the electric field from r = 1 mm to r = 3 mm, noting that the integration has to be broken down into two intervals:

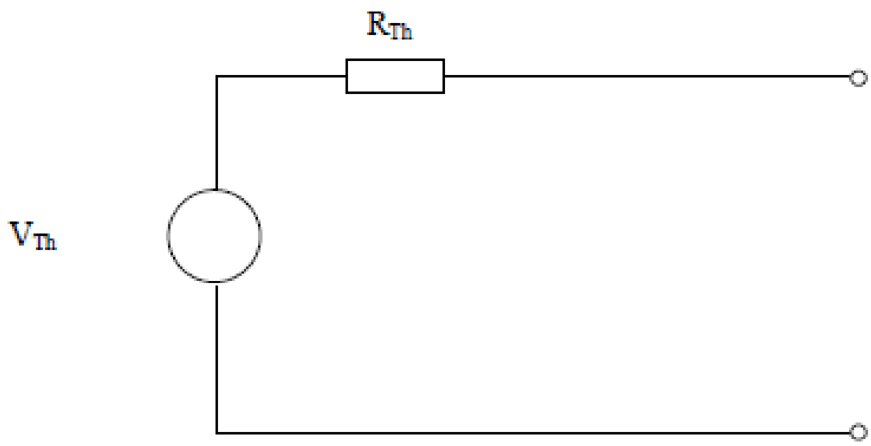
$$V = \int_{1mm}^{2mm} \frac{\rho}{2\pi\varepsilon_0\varepsilon_{r1} r} dr + \int_{2mm}^{3mm} \frac{\rho}{2\pi\varepsilon_0\varepsilon_{r2} r} dr = \frac{\rho}{2\pi\varepsilon_0}\left[\frac{1}{\varepsilon_{r1}}\ln 2 + \frac{1}{\varepsilon_{r2}}\ln 1.5\right] = 6.15mV$$

$$C_l = \frac{\rho}{V} = \frac{10^{-12}}{0.00615} = 0.163\mathrm{nFm^{-1}}$$

Section C: Physical Principles of Electronics and Linear Circuits
Q8 (short): (2009: Q2)

2 (short) (a) State Thevenin's and Norton's Theorems

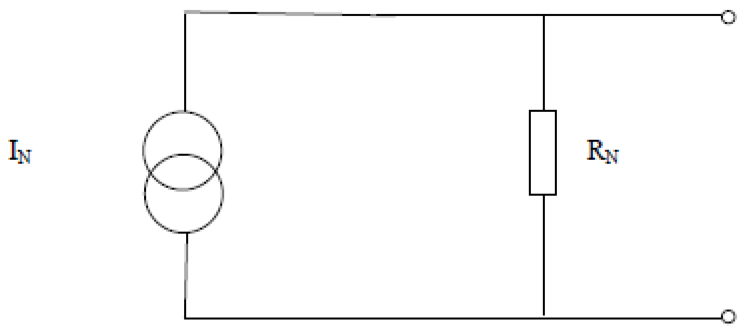
Thevenin: Any linear circuit may be represented as:



$V_{Th}=V_{Open\ Circuit}$

$R_{Th}=V_{Open\ Circuit}/I_{Short\ Circuit}$

Norton: Any linear circuit may be represented as:



$I_N = I_{\text{Short Circuit}}$
$R_N = V_{\text{Open Circuit}} / I_{\text{Short Circuit}}$

(b) Calculate the Thevenin and Norton equivalents of the circuit in Fig. 2

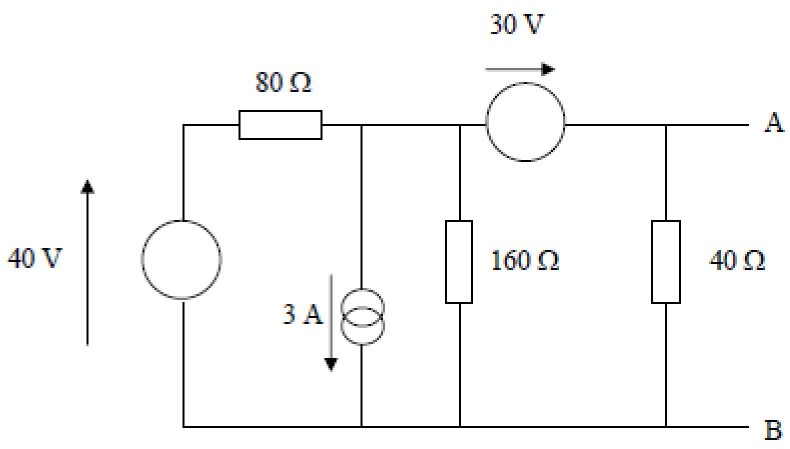
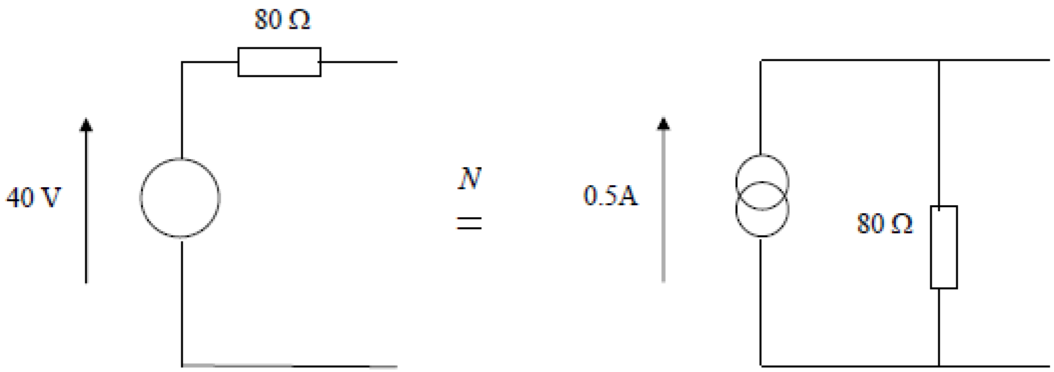
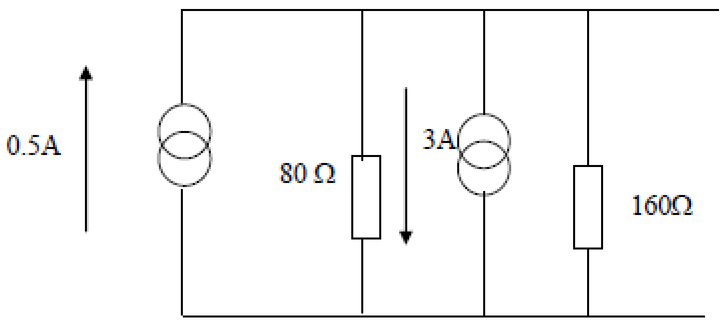


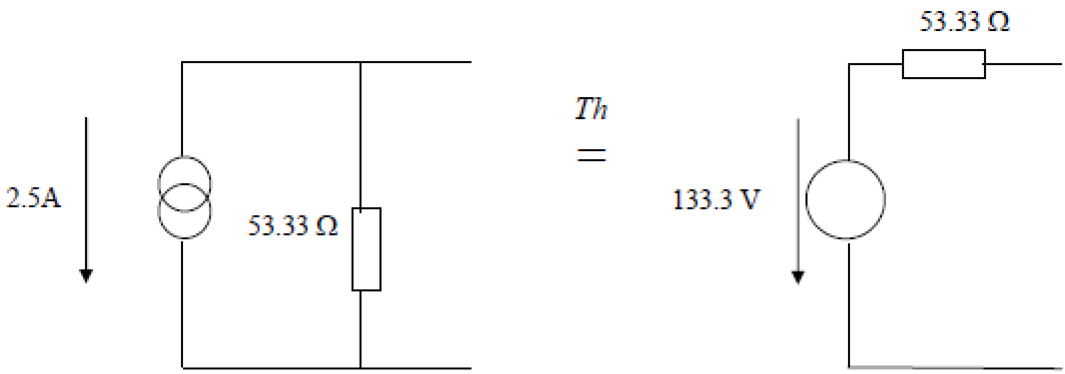
Fig.2



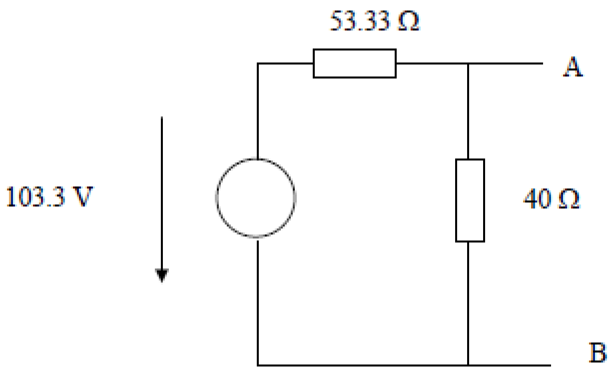
This gives



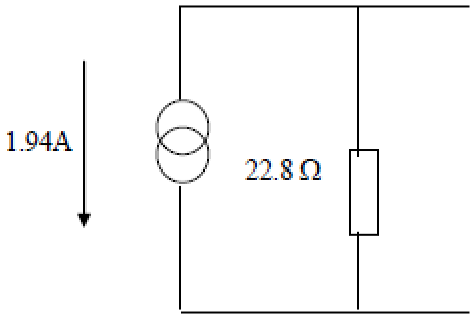
Which is equivalent to:



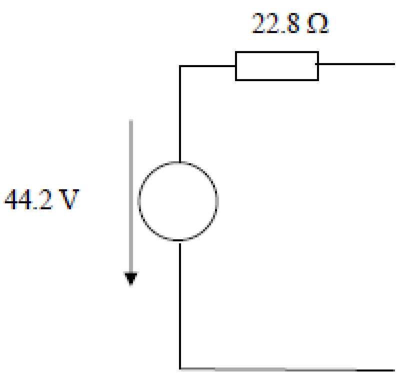
The circuit then becomes



Its Norton equivalent is then:



Its Thevenin equivalent is:

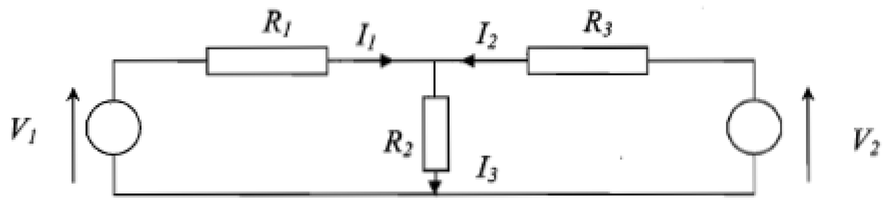


Section C: Physical Principles of Electronics and Linear Circuits
Q9 (long): (2007: Q1)

1 **(long)** (a) Explain briefly how the techniques of ***mesh current analysis*** and ***loop current analysis*** are used in d.c. and a.c electrical circuits.

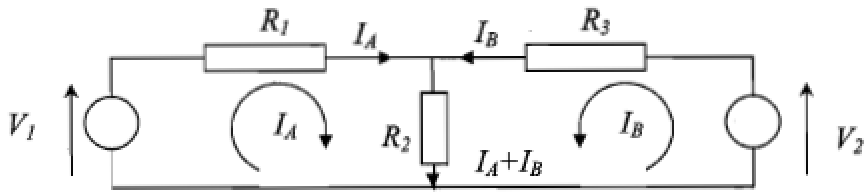
Mesh/loop current analysis refers to the application of Kirchhoff's voltage law to solve for unknown currents in a circuit. In mesh analysis the unknown currents flow through the individual circuit elements, and Kirchhoff's current law needs to be applied to enforce current conservation. In loop analysis, the unknown currents flow around each circuit loop, and the currents through the individual elements are appropriate sums of these loop currents. The number of equations and unknowns is equal to the number of independent circuit loops.

Examples:



Mesh analysis:

KCL $\Rightarrow I_3=I_1+I_2$
Then solve KVLx2



Loop Analysis
Solve KVLx2

(b) Figure 1(a) shows the circuit for an a.c. bridge. The voltage source supplies a sinusoidal waveform of frequency ω. At balance the current through the meter *M* is zero. Find the condition for balance in terms of R_1, R_2, R_3, R_4 and the ratio *L/C*.

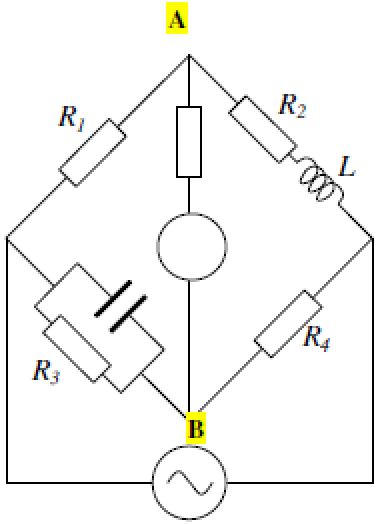


Fig. 1(a)

Different approaches are possible. Here we give one possible route.

At balance the current though meter M is zero. Thus the voltages at corners **A** and **B** must be the same. This gives:

$R_1R_4=(R_2+j\omega L)(R_3/j\omega C)/(R_3+1/j\omega C)$

$R_1R_4R_3j\omega C+R_1R_4=R_2R_3+j\omega LR_3$

$L/C=R_1R_4$

$R_1R_4=R_2R_3$

(c) Consider the bridge in Fig. 1(b). Find an expression for the frequency ω at which the bridge balances, in terms of R_1, R_2, R_3, R_4 and C.

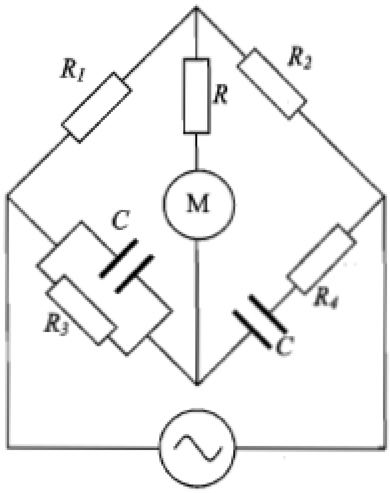


Fig. 1(b)

Again, different approaches are possible. Using the same as 1(b):

$R_1(R_4+/j\omega C)=R_2(R_3/j\omega C)/(R_3+1/j\omega C)$

$-R_1R_4R_3j\omega^2C^2+(R_4+R_3)R_1\omega C+jR_1=R_3R_2\omega C$

The real part does not give any indication on ω

Form the imaginary part we get

$\omega^2=1/C^2R_3R_4$

Section D: Mathematical methods
Q10 (short): (1996: Q2a)

2(i) evaluate z where $(\sin^{-1} z)^2 = 2\pi^2 i$

$$z = \sin\sqrt{2\pi^2 i}$$

or $z = \sin \pm(\pi + \pi i)$

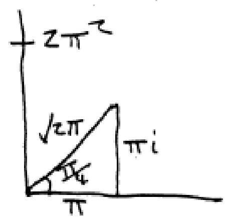
$$= \cancel{\sin(\pm\pi)}_{0}\cos \pm\pi i$$

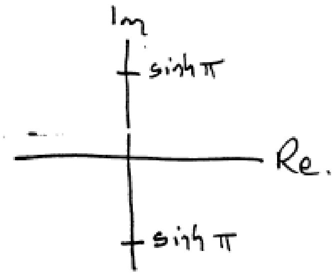
$$+ \cancel{\cos\pm\pi}_{-1}\sin\pm\pi i$$

$$= -1 . \sin \pm \pi i$$

$$= -i\sinh \pm\pi$$

$$- \pm i\sinh\pi$$





Section D: Mathematical methods
Q11 (short): (1998: Q3a)

3 a) $\sin x \dfrac{dy}{dx} + y\cos x = x\cos x$

$\dfrac{d}{dx}(y\sin x) = x\cos x$

$y\sin x + c = \int x\cos x = x\sin x - \int \sin x$

$= x\sin x + \cos x$

when $x = \pi/2$ $y = 1$ $\therefore 1 + c = \dfrac{\pi}{2}$

$c = \dfrac{\pi}{2} - 1$

$\therefore y = \dfrac{x\sin x + \cos x - \frac{\pi}{2} + 1}{\sin x}$

Section D: Mathematical methods
Q12 (long): (2011: Q4)

Q4. $\underset{\sim}{r} = \begin{pmatrix} 4 \\ -2 \\ 3 \end{pmatrix} + t \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$ Common normal

(a) $\underset{\sim}{a}$ $\underset{\sim}{r} = \begin{pmatrix} -7 \\ -2 \\ 1 \end{pmatrix} + s \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ $\underset{\sim}{n} = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 3 & 2 & 1 \end{vmatrix} = 3i - 5j + k$

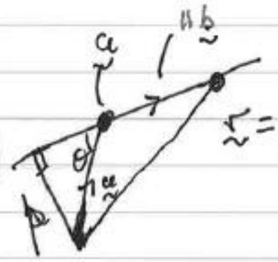
$\underset{\sim}{b}$

$\hat{n} = \frac{1}{\sqrt{35}} \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix}$

Shortest distance $= \hat{n} \cdot (\underset{\sim}{a} - \underset{\sim}{b})$

$= \frac{1}{\sqrt{35}} \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 11 \\ 0 \\ 2 \end{pmatrix} = \frac{35}{\sqrt{35}} = \sqrt{35}.$

(b)



$\underset{\sim}{r} = \underset{\sim}{a} + s\underset{\sim}{d}$

$|\underset{\sim}{a} \times \hat{\underset{\sim}{d}}| = |\underset{\sim}{a}| \sin\theta = h =$ shortest distance

$$p = \frac{|\underset{\sim}{a} \times \underset{\sim}{d}|}{|d|}.$$

Also: Component of $\underset{\sim}{a} \parallel \underset{\sim}{d} = \frac{|\underset{\sim}{a} \cdot \underset{\sim}{d}|}{|\underset{\sim}{d}|^2} \underset{\sim}{d}$

the rest is perpendicular, so $p = \left| \underset{\sim}{a} - \frac{\underset{\sim}{a} \cdot \underset{\sim}{d}}{|d|^2} d \right|$

(c) $\underset{\sim}{d} \times (\underset{\sim}{a} \times \underset{\sim}{d}) = \underset{\sim}{d} \cdot \underset{\sim}{d}\, \underset{\sim}{a} - (\underset{\sim}{d} \cdot a)\, \underset{\sim}{d} \Rightarrow \frac{|a \times d|}{|d|^2} = \left| \underset{\sim}{a} - \frac{a \cdot d}{|d|^2} \cdot \underset{\sim}{d} \right|$

↳ Direction: from origin to shortest distance to line: