CUED IA Progress Test
16/Jan/2019:
Crib
Section A: Mathematical methods: multiple choice
Q1 (short): (bespoke)

(i) 

\[ \cosh^{-1}(-2) = x + iy \quad (x, y \text{ real}) \]

\[-2 = \cosh(x + iy) \]
\[= \cos(iz - y) \]
\[= \cos(ix) \cos y + \sin(ix) \sin y \]
\[-2 = \cosh x \cos y + i \sinh x \sin y \]

\[\Rightarrow \cosh x \cos y = -2 \quad 1 \]
\[\& \sinh x \sin y = 0 \quad 2 \]
\[\Rightarrow x = 0 \text{ or } y = n\pi \]

1 & \text{x = 0 } \Rightarrow \cosh(0) \cos y = -2 
\[\text{NO REAL SOLUTIONS!} \]

2 & y = n\pi \quad \cos(n\pi) = (-1)^n 
\[\Rightarrow (-1)^n \cosh x = -2 \]
\[\Rightarrow n \text{ odd } \& x = \pm \cosh^{-1}(2) \]

\[\cosh^{-1}(-2) = \pm \cosh^{-1}(2) + i(2k+1)\pi \]

\[\cosh^{-1}(-2) = \pm 1.32 + i 3.14 (2k+1) \text{ to 2 d.p.} \]

\[C \]
\[
\frac{\ln(1+x)}{x - \frac{1}{2} x^2} = \frac{x - \frac{1}{2} x^2 + \frac{1}{3} x^3 - \frac{1}{4} x^4 + O(x^5)}{x (1 - \frac{1}{2} x)}
\]

\[
= (1 - \frac{1}{2} x + \frac{1}{3} x^2 - \frac{1}{4} x^3 + O(x^4))
\times (1 + \frac{1}{2} x + \frac{1}{4} x^2 + \frac{1}{8} x^3 + O(x^4))
\]

\[
= 1 + \left(\frac{1}{2} - \frac{1}{3}\right)x
+ \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{3}\right)x^2
+ \left(\frac{1}{8} - \frac{1}{8} + \frac{1}{6} - \frac{1}{4}\right)x^3
+ O(x^4)
\]

\[
= 1 + \frac{1}{3} x^2 - \frac{1}{12} x^3 + O(x^4)
\]

\[B\]
Section A: Mathematical methods: multiple choice
Q2 (short): (bespoke)

(i)

\[ a_{n+1} + 2a_n - a_{n-1} = 0 \]
\[ \lambda^2 + 2\lambda - 1 = 0 \]
\[ \lambda = \frac{-2 \pm \sqrt{2^2 + 4}}{2} = -1 \pm \sqrt{2} \]

\[ a_n = A(-1 + \sqrt{2})^n + B(-1 - \sqrt{2})^n \]

\[-1 + \sqrt{2} \approx 0.4 \quad \iff \quad \left| -1 + \sqrt{2} \right| < 1 \Rightarrow \text{Decay} \]

\[-1 - \sqrt{2} \approx -2.4 \quad \iff \quad \left| -1 - \sqrt{2} \right| > 1 \Rightarrow \text{Grows} \]

\[ \Rightarrow a_n \sim B(-2.4)^n \quad n \to +\infty \]

\[ \Rightarrow \frac{a_{n+1}}{a_n} \sim (-2.4) \]

\[ \boxed{E} \]
\[ \begin{align*}
X + aY &= 5 \\
-2X + 6Y + aZ &= 2 \\
3Y + Z &= C \\
\end{align*} \]

\[ \text{Need } \det \begin{bmatrix} 1 & a & 0 \\
-2 & 6 & a \\
0 & 3 & 1 \end{bmatrix} = 0 \]

\[ 1(6 - 3a) - a(-2) = 0 \]

\[ 6 - 3a + 2a = 0 \]

\[ a = 6 \]

**Eons:**

\[ \begin{align*}
X + 6Y &= 5 \\
-2X + 6Y + 6Z &= 2 \\
3Y + Z &= C \\
\end{align*} \]

**Eliminate** \( X \Rightarrow \)

\[ 18Y + 6Z = 12 \]

\[ 3Y + Z = 2 \]

\[ C \]

\[ \Rightarrow C = 2 \]
4. (a) The auxiliary equation is
\[ \lambda^2 + (3 + a)\lambda + 3a = 0 \iff (\lambda + 3)(\lambda + a) = 0 \]
So, assuming \( a \neq 3 \) (repeated root), the complementary function is
\[ y = Ae^{-3x} + Be^{-ax} \]
Assuming \( a \neq 2 \) (complementary function the same as the right hand side), the particular integral is \( y = Ce^{-2x} \). Substitute into the differential equation to find \( C \):
\[
4Ce^{-2x} + -2(3 + a)Ce^{-2x} + 3aCe^{-2x} = e^{-2x} \iff -2C + +aC = 1 \\
\iff C = \frac{1}{a - 2}
\]
The general solution is therefore
\[ y = Ae^{-3x} + Be^{-ax} + \frac{e^{-2x}}{a - 2} \]
The boundary conditions tell us that
\[
0 = A + B + \frac{1}{a - 2} \\
0 = -3A - aB - \frac{2}{a - 2}
\]
Multiplying the top equation by 3 and adding the bottom one:
\[ 0 = (3 - a)B + \frac{3}{a - 2} \iff B = \frac{1}{(a - 2)(a - 3)} \]
Substituting into the top equation:
\[ A = \frac{1}{(a - 2)(a - 3)} - \frac{1}{a - 2} = \frac{-1 - a + 3}{(a - 2)(a - 3)} = \frac{-a - 2}{(a - 2)(a - 3)} = \frac{1}{3 - a} \]
The solution satisfying the boundary conditions is therefore
\[ y = \frac{e^{-3x}}{3 - a} + \frac{e^{-ax}}{(a - 2)(a - 3)} + \frac{e^{-2x}}{a - 2} \] [12]
(b) When $a = 2$, the first term in the above expression is simply $e^{-3x}$. For the other terms, we substitute $\epsilon = a - 2$ to get

\[
y = e^{-3x} + \frac{e^{-(\epsilon+2)x}}{\epsilon(\epsilon - 1)} + \frac{e^{-2x}}{\epsilon} = e^{-3x} - \frac{e^{-(\epsilon+2)x}(1 - \epsilon)^{-1}}{\epsilon} + \frac{e^{-2x}}{\epsilon}
\]

\[
= e^{-3x} - \frac{e^{-2x}e^{-\epsilon x}(1 + \epsilon + O(\epsilon^2))}{\epsilon} + \frac{e^{-2x}}{\epsilon} = e^{-3x} - \frac{e^{-2x}}{\epsilon} \left[ e^{-\epsilon x}(1 + \epsilon + O(\epsilon^2)) - 1 \right]
\]

\[
= e^{-3x} - \frac{e^{-2x}}{\epsilon} \left[ (1 - \epsilon x + O(\epsilon^2))(1 + \epsilon + O(\epsilon^2)) - 1 \right] = e^{-3x} - e^{-2x}(1 - x + O(\epsilon))
\]

\[
= e^{-3x} + (x - 1)e^{-2x} \text{ in the limit as } \epsilon \to 0 \quad [12]
\]

(c) Instead of taking the limit, we could have returned to the particular integral in (a) and tried a function of the form $Cxe^{-2x}$. For the case $a = 3$, we could take a limit as in (b), or return to (a) and change the complementary function to

\[
y = (A + Bx)e^{-3x} \quad [6]
\]

Examiner’s remarks: This question asked the candidates to solve the differential equation $\frac{d^4y}{dx^4} + (3 + a)\frac{dy}{dx} + 3ay = e^{-2x}$. The question initially asked the candidates to assume $a \neq 2$ (so the complementary function has nothing in common with the right hand side) and $a \neq 3$ (so the auxiliary equation does not have repeated roots). Almost everyone derived the auxiliary equation $\lambda^2 + (3 + a)\lambda + 3a = 0$ but then distressingly few spotted the obvious factorisation, launching instead into the standard formula for solving a quadratic equation. Nevertheless, since the candidates knew where they were going in part (a), which was a “show that” question, algebraic slips were corrected and almost everyone scored full marks. In part (b), candidates were asked to find the solution when $a = 2$ by writing $\epsilon = a - 2$ and taking the limit as $\epsilon \to 0$. Responses were very poor, with only a handful of students finding the right answer using power series. Most instead tried to use L'Hôpital’s rule, which can produce the right answer (a few succeeded) but of course you first have to rewrite the indeterminate part of the expression as a single numerator over a single denominator, with both numerator and denominator tending to zero in the limit: most candidates seemed unaware of this prerequisite. In part (c), candidates were asked how else they might find the solution when $a = 2$ or $a = 3$. Only a few suggested trying a different particular integral or complementary function, with most instead suggesting power series in place of L'Hôpital’s rule, or vice versa.
Potential energy $V(x) = -2mg \left( \frac{L^2 - (L - x)^2}{2} \right) + \frac{1}{2} k x^2$

$V(x) = -mg \sqrt{\frac{x^2 + xL}{4}} + \frac{1}{2} k x^2$

(b) $V'(x) = kx - mg \frac{L - x/2}{2 \left( \frac{1}{4} xL - \frac{x^2}{4} \right)}$

at $x = 45^\circ$, $L - x = \frac{L}{\sqrt{2}}$ or $x = (\sqrt{2} - 1)L$

$k \cdot (2 - \sqrt{2}) L = \frac{mg L/\sqrt{2}}{2 \cdot \frac{L}{\sqrt{2}}}$

$k = \frac{mg}{2(2 - \sqrt{2}) L}$
Q10.

(a) \[ \ddot{v} = -200 \hat{e}_\theta \]

\[ \ddot{a} = \frac{10}{\sqrt{2}} \hat{e}_r - \frac{10}{\sqrt{2}} \hat{e}_\theta \]

The radial component of the acceleration can be calculated as below:

\[ r \dot{\theta} = -200 \]

\[ \therefore \dot{\theta} = -0.2 \text{ rad s}^{-1} \]

\[ \ddot{r} - r \dot{\theta}^2 = 7.07 \text{ ms}^{-1} \]

\[ \therefore \ddot{r} = 47.07 \text{ ms}^{-1} \]

(b) If \( R \) is the instantaneous radius of curvature:

\[ a_\perp = \frac{s^2}{R} \]

\[ \therefore R = \frac{\sqrt{2} \times (200)^2}{10} \]

\[ \therefore R = 5.66 \text{ km} \]
Q9 chain

Let us define the linear density of the chain as $\rho = M/L$. Denoting the height of the chain above the table by $z$, the mass of the chain in the air is $\rho z$.

a) The chain is being pulled at a constant speed $v$, so we have

$$z = vt$$

The gravitational pull on the chain is $\rho zg = \rho vgt$, so the total force is $F - \rho vgt$ that has to equal the rate of change of momentum,

$$F - \rho vgt = \frac{d}{dt} (mv)$$

where $m = \rho z = \rho vt$ is the mass in the air. While the chain is being lifted, the mass in the air is changing,

$$\frac{d}{dt} (\rho vt v) = \rho v^2$$

After the chain is fully lifted, its momentum does not change any more, so the force required is just that to balance the weight. Therefore

$$F = \begin{cases} 
\rho g vt + \rho v^2 & 0 < t < L/v \\
Mg \quad (= \rho Lg) & L/v < t 
\end{cases}$$

b) For the case of constant pulling force $F$, again the total force must equal the rate of change of momentum. Momentum is still mass $\times$ velocity, but the velocity is not a constant any more, so $mv = \rho z \times \dot{z}$, and therefore

$$F - \rho zg = \frac{d}{dt} (\rho z \dot{z})$$

$$F = \rho gz + \rho z \ddot{z} + \rho \dot{z}^2$$
The proposed solution is the quadratic polynomial \( z = a + bt + ct^2 \). Using the initial condition \( z = 0 \) at \( t = 0 \) yields \( a = 0 \). Substituting solution into the differential equation, we have

\[
(a + bt + ct^2)2c + (b + 2ct)^2 + g(a + bt + ct^2) = F/\rho.
\]

We equate coefficients of powers of \( t \),

\[
2ac + b^2 + ga = F/\rho \\
2bc + 4bc + gb = 0 \\
x^2 + 4c^2 + gc = 0
\]

Notice that the second and third equations are redundant. Together with the first equation, they give \( b \) and \( c \),

\[
c = -g/6 \\
b = \sqrt{F/\rho}
\]

So the complete solution is

\[
z = \sqrt{F/\rho} t - gt^2/6
\]

c) The chain leaves the table when \( z = L \), so the corresponding time \( t \) satisfies

\[
-\frac{g}{6}t^2 + \sqrt{F/\rho} t - L = 0
\]

Using the the quadratic formula gives

\[
t = \frac{\sqrt{F/\rho} - \sqrt{F/\rho - 2Lg/3}}{g/3}
\]

d) A real solution exists when \( F > 2L\rho g/3 \). This is less than the force required to balance the weight of the chain, \( L\rho g \). Using this minimum force, the chain would momentarily leave the table and then fall back down again.
9. (a) \[ F = \rho g v t + \rho v^2 \] for \( 0 < t \leq L/v \) and \[ F = \rho L g \] for \( L/v < t \)

(b) \[ z = t \sqrt[3]{\frac{F}{\rho}} - \frac{t^2 g}{6} \]

(c) \[ t = \frac{3}{g} \left( \sqrt[3]{\frac{F}{\rho}} - \sqrt[3]{\frac{F}{\rho} - \frac{2Lg}{3}} \right) \]

(d) \[ F > \frac{2L\rho g}{3} \]
In the limit, the two lower cylinders roll away from each other, losing contact as the top cylinder falls, slipping at $B = D$.

Fig. 1

If three forces act on a rigid body, their lines of action must pass through a point if the body is to be in moment equilibrium. Consider each of the lower cylinders. The line of action of the contact force at $B$ must pass through $A$, and that of the contact force at $D$ must pass through $C$.

The contact normals at $B$ and $D$ are at $60^\circ$ to the horizontal (equilateral triangle), so $30^\circ$ to the vertical. A trigonometric construction for the line of action $AB$ shows it inclined at $15^\circ$ to the vertical, and therefore $15^\circ$ to the contact normal. We therefore require

$$\mu \geq \tan 15^\circ \approx 0.268$$

for equilibrium.
Performance average mark 4.9/10
34 perfect answers 12.84

Alternative perfect solutions were obtained by putting \( F = \mu N \) at contact point B, for example, and taking moments of force for equilibrium about point A so that only \( FE \)N come into the equation. The geometry gives
\[ \mu = \frac{1}{2 + \sqrt{3}} = 2 - \sqrt{3} = 0.268 \] as before.
But \( F \) may accidentally be reversed in direction, causing 4 marks to be lost. Friction must always oppose relative motion, and by symmetry the top cylinder tends to fall vertically while the bottom cylinders move away from each other by rolling. The friction \( F \) must therefore oppose outward movement of the lower cylinders.

Unfortunately many candidates got stuck looking at the vertical equilibrium of the top cylinder which does not even need friction at B and D to support it, as many noticed. By missing the clue that the "lower cylinders may roll but not slip" on the rough floor, these candidates never studied moment equilibrium, and missed the point of the question.
Q8 (a)

Note for completeness all joints have been calculated in the crib but only a subset of these calculations are required to find the bar forces.

* A common mistake was to forget $H_A$ at Joint A and as a result calculate a non-zero value for $T_{AD}$. But this mistake should have been easily identified since by inspection at Joint D there is no horizontal reaction so $T_{AD}$ must be zero.
2 (continued)

b) - Virtual work solution

For virtual force system apply unit horizontal force at C

\[ \Sigma H = 0 \]
\[ -T_{AC} \cos 45^\circ + 1 = 0 \quad \therefore T_{AC} = \sqrt{2} \]
\[ \Sigma V = 0 \]
\[ -T_{CD} - T_{AC} \sin 45^\circ = 0 \quad \therefore T_{CD} = \sqrt{2} \]

<table>
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<th>Bar</th>
<th>force</th>
<th>length</th>
<th>( e )</th>
<th>( T^* )</th>
<th>( T^* e )</th>
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<tr>
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<td>(-W)</td>
<td>(L)</td>
<td>(-1)</td>
<td>(0)</td>
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<tr>
<td>BC</td>
<td>(-W)</td>
<td>(L)</td>
<td>(-1)</td>
<td>(0)</td>
<td>(0)</td>
</tr>
<tr>
<td>CD</td>
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<td>(L)</td>
<td>(-1)</td>
<td>(-1)</td>
<td>(1)</td>
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<tr>
<td>AC</td>
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<td>(\sqrt{2})</td>
<td>(2\sqrt{2})</td>
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<tr>
<td>AD</td>
<td>0</td>
<td>(L)</td>
<td>(0)</td>
<td>(0)</td>
<td>(0)</td>
</tr>
</tbody>
</table>

\[ P^* \delta_{ch} = \Sigma T^* e \]
\[ 1 \cdot \delta_{ch} = (1 + 2\sqrt{2}) \left( \frac{WL}{EA} \right) \]
\[ \delta_{ch} = \left( 1 + 2\sqrt{2} \right) \left( \frac{WL}{EA} \right) \]
2b) - Solution using displacement diagram

Extensions calculated on previous page

D does not move vertically and as $T_{ab}=0$ it does not move horizontally relative to A

Scale 20mm: $\frac{WL}{EA}$

\[ \sigma_{ch} = 77 \text{mm} \frac{WL}{20 \text{mm} \ EA} = 3.85 \frac{WL}{EA} \]

- using trig

\[ \sigma_{ch} = a + b = \frac{EAC}{\cos 45^\circ} + \theta_{dc} = \sqrt{2} \cdot 2\frac{WL}{EA} + \frac{WL}{EA} \]

\[ = (1 + 2\sqrt{2}) \frac{WL}{EA} \]
\( F_1 = 10 \text{kN/m} \times 5 \times \frac{1}{2} \times \frac{1}{2} = 12.5 \text{kN} \)

\( F_2 = 30 \text{kN} \), \( F_3 = \frac{W}{2} = 15 \text{kN} \)

\[ \Sigma M_A = 0 \Rightarrow F_2 \times \frac{L}{2} \cos 30^\circ + F_3 \left( L \cos 30^\circ + \frac{L}{4} \cos 60^\circ \right) + F_1 \left( \frac{\sqrt{3}}{3} \times \frac{L}{2} \right) - T \times L \cos 30^\circ = 0 \]

\[ \therefore 30 \times \frac{\sqrt{3}}{2} + 15 \times \left( \frac{5\sqrt{3}}{2} + \frac{5}{2} \right) + 12.5 \times \frac{5}{3} - T \times 5 \times \sqrt{3} = 0 \]

\[ \therefore T = 36.98 \text{kN} \]

Many students recognised that the most direct way to solve the problem was to make a cut through the cable and take moments about A. Students who tried to resolve forces in the horizontal and vertical directions had a tendency to overlook the support reactions in their free body diagrams which lead to errors. The triangular load caused some difficulties in terms of finding the resultant force and location.
Section D: Structural Mechanics

Q9 (short): (2006: Q2)

\[ \text{Taking moments for the half-cable about peg A} \]
\[ \text{To D} = \frac{WL}{2} \]

(Using \( d \ll L \) to fix the centre of gravity at \( L/4 \) from the peg).

At the peg, the cable has to carry a horizontal component \( T_0 \) and a vertical component \( \frac{WL}{2} \)

\[ T_{max} = \sqrt{\left(\frac{WL}{2}\right)^2 + \left(\frac{WL}{2L}\right)^2} \]

\[ \text{so } T_{max} = \frac{WL}{2} \sqrt{1 + \left(\frac{L}{4D}\right)^2} \]

b) The cable wraps \( \frac{L}{2} + \beta \) around the peg, where

\[ \tan \beta = \frac{WL}{2L} = \frac{D}{L} \]

For slippage of a cable around a peg, we know

\[ \frac{T_{big}}{T_{small}} = \exp(\mu \theta) \quad \text{where } \theta = \frac{L}{2} + \beta \text{ in this case} \]

Considering the "drop" pulling down \( (T_{big} = WH) \)
and being pulled up \( (T_{small} = WH) \) and cancelling \( W \) we get, for equilibrium

\[ \frac{1}{2} \sqrt{1 + \left(\frac{L}{4D}\right)^2} \exp\left[-\mu\left(\frac{L}{2} + \tan^{-1}\frac{L}{4D}\right)\right] < H \]

\[ H < \frac{1}{2} \sqrt{1 + \left(\frac{L}{4D}\right)^2} \exp\left[-\mu\left(\frac{L}{2} + \tan^{-1}\frac{L}{4D}\right)\right] \]
2. Performance

average mark 5.6/10
6 perfect solutions / 284

3 marks were awarded for part (b) and no actual calculations were required. But for full marks, candidates should have mentioned that there is an exponential variation of tension in a cable slipping around a rough peg and that there were two extremes — H too large causing outward slip and the cable to pull straight, and H too small causing inward slip and the cable to sag and fall between the pegs.

2 marks were lost in part (a) if candidates only calculated the horizontal component of tension and mistook this for the maximum tension.

Unfortunately, some candidates first found the total vertical reaction on a peg and then used it in a moment equilibrium equation for a segment of cable on the central part. This violates vertical equilibrium for the central part. It is best to imagine cutting the cable just inside the point of contact with pegs to define the equilibrium of the central part of width L. The outer length L can then enter into the equilibrium of the central zone unless the cable is slipping.
By symmetry, the direction of the electric flux must be radial.

According to Gauss:

\[ D 4\pi r^2 = \int ar \times 4\pi r^2 \, dr \]
\[ D 4\pi r^2 = \pi ar^4 \]
\[ D = ar^2/4 \]

\[ D = \frac{ar^2}{4} \, e_r \]
Section E: Physical Principles of Electronics and Linear Circuits
Q11 (short): (2011: Q5)

(a)

Thevenin: Any linear circuit may be represented as:

\[ V_{Th} = V_{Open\, Circuit} \]
\[ R_{Th} = V_{Open\, Circuit} / I_{Short\, Circuit} \]

Thus, the Thevenin equivalent for the circuit in Fig. 3(a) has the following parameters:

\[ V_{Th} = V_{OC} = \frac{R_2}{R_1 + R_2} V \]
\[ I_{SC} = \frac{V}{R_1} \]
\[ R_{Th} = \frac{R_1 R_2}{R_1 + R_2} \]
\[ Z_L = j\omega L = j12.6 \Omega \]

\[ Z_C = \frac{1}{j\omega C} = -j19.9 \Omega \]

Regarding \( Z_C \) as load

\[ V_n = \frac{200}{200 + j12.6} \times 150V = (149.41 - j9.41)V = 149.71\angle -3.61^\circ V \]

\[ Z_{in} = \frac{200 \times j12.6}{200 + j12.6} = (0.79 + j12.55)\Omega = 12.57\angle 86.3^\circ \Omega \]

\[ I = \frac{149.71\angle -3.61^\circ}{(0.79 - j7.35)} = 20.26\angle 80.25^\circ A \]

\[ I_C = \sqrt{2}I_{rms} = 28.65A \]

5 (a) \[ V_{in} = V_{oc} = \frac{R_2}{R_1 + R_2} V \]

\[ R_{th} = \frac{R_1R_2}{R_1 + R_2} \]

(b) \[ I = 20.26\angle 80.25^\circ A \]

\[ I_C = 28.65A \]
Norton’s theorem: Any linear circuit, insofar as the load connected to it is concerned, may be represented as a current source, $I_N$, in parallel with a resistance $R_N$:

\[ I_N \quad R_N \]

where $I_N = I_{SC}$ and $R_N = V_{OC}/I_{SC}$
b) Convert left-hand side of the circuit to Thevenin equivalent, and do the same to the right-hand side:

![Circuit Diagram](image)

Now convert right-hand side to Norton to give a 1 A current source in parallel with a 3 Ω resistor, and combine this 1 A current source with the 2 A current source by adding (since they are in parallel):

![Circuit Diagram](image)

Finally, convert the right-hand side of the circuit back to Thevenin to give:

![Circuit Diagram](image)

i) \( R = 13 \Omega \quad I = (9-4)/20 = 0.25 \text{ A} \)

ii) \( R = 3 \Omega \quad I = (9-4)/10 = 0.5 \text{ A} \)
c) i) Find the total impedance as seen by the 240 V voltage source:

Impedance of the series combination of the capacitor, inductor and 1 Ω resistor is

\[ 1 + jωL + \frac{1}{jωC} = 1 + j(3.14 \times 6.77) = 1 - j3.63 \]

Combine this in parallel with the 1 Ω resistor and add the series 1 Ω resistor:

\[ Z = 1 + \frac{1}{1 - j3.63}/(1 + 1 - j3.63) = 1.884 - j0.211 \]

Finally \[ I = \frac{240}{Z} = 126.6 \angle -6.4^\circ \]

Examiner's comment: A common mistake here was to convert to Thevenin and find the current. This gives the current flowing in the inductor/capacitor i.e. the 'load' current, but not the supply current as asked for.

ii) Convert left-hand side of circuit to Thevenin:

![Circuit Diagram]

ii) Resonant frequency \[ ω_0 = 1/\sqrt{LC} \] and \[ f_0 = ω_0/2π \] giving 73.4 Hz

At resonance the impedance of the capacitor and inductor cancel out so that the only impedance is the 1.5 Ω of resistance. Therefore:

\[ I = \frac{120}{1.5} = 80 \; A \angle 0 \]

Capacitor voltage \[ V_C = X_C \times I \] where \[ X_C \] is the reactance of the capacitor

\[ = \frac{80}{ωC} = 369 \; V \angle -90 \]

\[ Q = \frac{ω_0L}{R} = 4.61/1.5 = 3.07 \]
2  b) i) $I = 0.25 \, \text{A}$  ii) $I = 0.5 \, \text{A}$

c)  i) $I = 126.6 \, \text{A} \angle +6.4^0$  ii) $f_0 = 73.4 \, \text{Hz}$  \( V_{\text{cap}} = 369 \, \text{V} \angle -90 \)  \( Q = 3.07 \)