EGT0

ENGINEERING TRIPOS PART IA

Wednesday 16 January 2019  9.00 to 12:10

CUED PART IA PROGRESS TEST

Answer all questions.

The approximate number of marks allocated to each part of a question is indicated in the right margin.

Tie up your answers to each section separately using the provided section cover sheets.

Make sure your name is on every sheet of script paper that you use.

Check that the CANDIDATE reference is the same on your supplementary booklet and all of your cover sheets.

STATIONERY REQUIREMENTS

Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

Section A: multiple choice supplementary booklet
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.
SECTION A: Mathematics - multiple choice questions

Questions 1-2: see multiple choice supplementary booklet.
SECTION B: Mathematics

3 (long) (a) Consider the differential equation

\[ \frac{d^2y}{dx^2} + (3 + a) \frac{dy}{dx} + 3ay = e^{-2x} \]

with boundary conditions

\[ \frac{dy}{dx} = y = 0 \text{ at } x = 0 \]

Assuming the constant \( a \) does not equal 2 or 3, derive the solution

\[ y = \frac{e^{-3x}}{3-a} + \frac{e^{-ax}}{(a-2)(a-3)} + \frac{e^{-2x}}{a-2} \]

You must derive the solution: do not simply check that the given solution satisfies the differential equation and the boundary conditions. [12]

(b) The solution is indeterminate when \( a = 2 \) or \( a = 3 \). By writing \( \varepsilon = a - 2 \) and taking the limit as \( \varepsilon \to 0 \), find the solution when \( a = 2 \). [12]

(c) What alternative technique could you use to find the solution when \( a = 2 \)? How would you go about finding the solution when \( a = 3 \)? Do not actually determine any further solutions, just describe the appropriate methods. [6]
SECTION C: Mechanical Engineering

4 (short) A mechanism consisting of two uniform bars is held in place by a linear spring of spring constant $k$. Points A and C are fixed, while B is constrained to move horizontally as shown in Fig. 1. Each bar has mass $m$ and length $L$. The spring is unstretched when the angle $\alpha$ is $0^\circ$, and the system is in equilibrium when $\alpha$ is $45^\circ$.

(a) Obtain an expression for the potential energy of the system as a function of the spring extension $x$. [5]

(b) Determine a value for the spring constant $k$. [5]
5 (short) Figure 2 shows a radar station tracking an unidentified flying object at a range of 1 km due south moving in a horizontal plane at a speed of $v = 200 \text{ m s}^{-1}$ heading west and with an acceleration $\alpha = 10 \text{ m s}^{-2}$ as shown.

(a) Write down the velocity and acceleration in polar coordinates with the origin located at the radar station, and hence find the radial component of the acceleration. [6]

(b) Calculate the instantaneous radius of curvature of the path. [4]
6 (long) A heavy chain of length $L$ and mass per unit length $\rho$ is resting on a table, as shown in Fig. 3.

(a) The chain is being pulled up by one of its ends with a force $F$ which varies with time $t$ so that the chain moves with constant velocity $v$. Determine $F$, as a function of time, and sketch it on a graph. [8]

(b) The chain is lifted up again, but this time by a constant force $F$.

(i) Derive the differential equation obeyed by the length of chain in the air, $z$, and use the following trial solution to find $z(t)$:

$$z(t) = a + bt + ct^2$$ [12]

(ii) Hence, find the time it takes for the whole chain to leave the table. [5]

(iii) Determine the minimum value of $F$ needed for the whole chain to leave the table, at least for an instant, and comment on your answer. [5]
SECTION D: Structural Mechanics

7 (short) Three identical circular cylinders are stacked as a triangular pile on a rough horizontal floor, as shown in Fig. 4. The lower cylinders may roll but not slip at their point of contact on the floor, A and C. Find the minimum coefficient of friction required at points of contact B and D, between pairs of cylinders, if the pile is to be stable under its own weight. [10]

Fig. 4
8 (long) (a) A plane pin-jointed truss is shown in Fig. 5(a). All members have the same cross-sectional area $A$ and are made of a linear-elastic material with Young’s modulus $E$. The self-weight can be neglected. The structure is loaded at joint B, as shown in the figure.

(i) Find the bar forces due to the applied loading. [8]

(ii) Find the horizontal displacement at C due to the applied loading. [10]
(b) An inverted, uniform, rigid L-shaped structure is pinned at one end and connected to a cable as shown in Fig. 5(b). The structure is tilted at an angle where ABC forms an equilateral triangle. The structure is made of two lengths of rod welded together, one with length $L$ and the other $L/2$ where $L = 5\, \text{m}$. The weight of a length $L$ of rod is $W = 30\, \text{kN}$. A linear triangular load varying from a value of $0\, \text{kNm}^{-1}$ at C to $10\, \text{kNm}^{-1}$ at D is applied in a direction perpendicular to CD, as shown. Find the tension in the cable. [12]
9 (short) A thin, flexible cable of weight \( w \) per unit length is draped over two small, rough circular pegs spaced \( L \) apart so that it has a sag of \( D \) in the middle and drops \( H \) at each end: see Fig. 6. The sag ratio \( D/L \) is sufficiently small that the weight of the cable between the pegs can be regarded as being uniformly distributed over the horizontal.

(a) Obtain an expression for the maximum tension in that part of the cable lying between the pegs. \[7\]

(b) What other calculation should be performed to determine whether the configuration is in static equilibrium? \[3\]

Fig. 6 (NOT drawn to scale)
SECTION E: Physical Principles of Electronics and Linear Circuits

10 (short) The charge density within a sphere varies as a constant $a$ times its radius. Find an expression for the direction and magnitude of the electric flux, $D$, within the sphere. [10]
### 11 (short)

(a) Explain what is meant by a Thevenin equivalent circuit. Draw the Thevenin equivalent circuit for the circuit shown in Fig. 7(a), and derive expressions for the Thevenin voltage and impedance.

![Fig. 7(a)](image)

(b) In the circuit of Fig. 7(b), $R = 200 \, \Omega$, $L = 40 \, \text{mH}$ and $C = 160 \, \mu\text{F}$. By applying Thevenin’s theorem, or otherwise, determine the RMS magnitude of the current flowing in capacitor $C$, its peak value, and its phase with respect to the 150 V voltage source.

![Fig. 7(b)](image)
12 (long)  (a) State Norton’s theorem. 

(b) Using Thevenin’s and/or Norton’s theorems, or otherwise, find the current in the resistor R shown in Fig. 8(a) for the two cases:

(i)  \( R = 13 \, \Omega \);  
(ii)  \( R = 3 \, \Omega \).  

(c) For the circuit of Fig. 8(b) find:

(i) the current drawn from the 240 V rms supply if the frequency is 50 Hz;  
(ii) the supply frequency which maximises the current drawn from the 240 V rms supply, the capacitor voltage at that frequency and the Q factor of the circuit.
CUED PART IA PROGRESS TEST

Section A: multiple choice supplementary booklet

Questions 1–2 can be found on subsequent pages of this booklet. The questions ask you to show your choice between options. Choose the one option you consider correct.

Your rough working will not be assessed. Only your responses on this answer sheet will be marked. There are no penalties for incorrect responses, only marks for correct answers.

When you have completed each question, fill in the appropriate circle on this answer sheet, e.g.

A   B   C   D   E
O   O   O   O   O

Use a soft pencil. If you make a mistake, erase thoroughly and try again.

Ignore the boxes for questions 3–8. Hand in this booklet at the end of the examination.

1(a) A B C D E

1(b) A B C D E

2(a) A B C D E

2(b) A B C D E

3(a) A B C D E

3(b) A B C D E

4(a) A B C D E

4(b) A B C D E

5(a) A B C D E

5(b) A B C D E

6(a) A B C D E F

6(b) A B C D E

7(a) A B C D E F

7(b) A B C D E F

8(a) A B C D E

8(b) A B C D E
1 (short)

(a) Evaluate $\cosh^{-1}(-2)$, giving your answer to two decimal places.

\[ \text{A} \] $(-1)^n 1.32 + 3.14i n$ where $n$ is an integer
\[ \text{B} \] $\pm 1.32 + 6.28i n$ where $n$ is an integer
\[ \text{C} \] $\pm 1.32 + 3.14i (2n + 1)$ where $n$ is an integer
\[ \text{D} \] $-1.32i$
\[ \text{E} \] $(-1)^n 1.32i + 3.14n$ where $n$ is an integer

(b) Which of the following is the power series expansion of
\[ \frac{\ln(1 + x)}{x - \frac{1}{2}x^2} \]
about the point $x = 0$?

\[ \text{A} \] $1 + \frac{1}{3}x^2 + O(x^4)$
\[ \text{B} \] $1 + \frac{1}{3}x^2 + O(x^3)$
\[ \text{C} \] $1 + x + \frac{5}{6}x^2 + O(x^3)$
\[ \text{D} \] $1 - x + \frac{5}{6}x^2 + O(x^3)$
\[ \text{E} \] $1 - x + \frac{1}{3}x^2 + O(x^3)$
2  (short)

(a) For the general solution of the linear difference equation

\[ a_{n+1} = a_{n-1} - 2a_n \]

what value, to one decimal place, does the ratio \( a_{n+1}/a_n \) tend towards for large, positive integer values of \( n \)?  

A  \( n \)  
B  0.4  
C  2.4  
D  \( -n \)  
E  \( -2.4 \)

(b) For what values of \( c \) do the simultaneous equations

\[
\begin{align*}
  x + ay & = 5 \\
-2x + 6y + az & = 2 \\
3y + z & = c
\end{align*}
\]

have the possibility of an infinite number of solutions if the value of \( a \) is appropriately chosen?  

A  all values of \( c \)  
B  \( c = \frac{7}{6} \)  
C  \( c = 2 \)  
D  \( c = -\frac{4}{3} \)  
E  no values of \( c \)