EGT0

ENGINEERING TRIPOS PART IA

Wednesday 17 January 2018	Reading time: 9.00am to 9:10am	
	Writing time: 9:10am to 12.10pm	LT1/2

CUED PART IA PROGRESS TEST

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

*Tie up your answers to **each section separately using the provided section cover sheets**.*

*Make sure your **name** is on every page.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Supplementary page: Table for Question 4 (two pages)
Engineering Data Book

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A: Mechanical Engineering

1 (short) A racing car is instrumented to measure the magnitude and direction of the acceleration and velocity in a horizontal plane. At one instant the velocity and acceleration are as shown in Fig. 1, where the directions are defined with respect to the longitudinal axis of the car.

(a) Express the velocity and acceleration in intrinsic coordinates. [5]

(b) Calculate the radius of curvature of the path of the vehicle. [5]

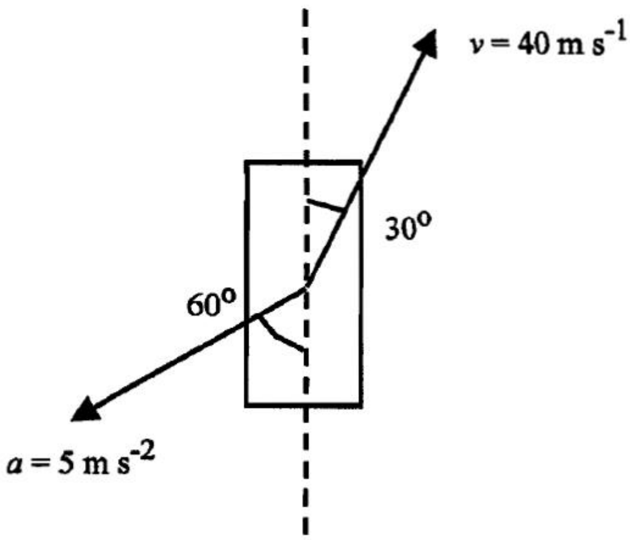


Fig. 1

2 (short) Figure 2 shows a smooth rod of length L rotating anti-clockwise in a horizontal plane about O at a constant angular velocity Ω. A particle P, which is free to slide along the rod, is attached to a light inextensible wire of length $L/2$ whose other end is connected to O. At the instant shown in Fig. 2, the wire snaps and the particle P slides towards the end of the rod.

(a) Show that the radial position r of P while P is still in contact with the rod, expressed as a function of time t after the wire snapped, is given by $r = A\cosh bt$ and determine A and b in terms of L and Ω. [6]

(b) Derive an expression for the time taken for P to reach the end of the rod. [4]

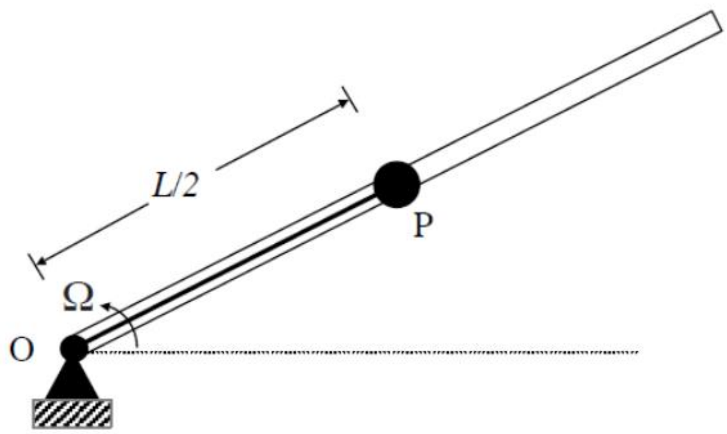


Fig. 2

3 (long) A particle P, of mass 250 g, is attached to one end of a light elastic string having an unstretched length of 300 mm and a stiffness of 40 N/m. The other end of the string is fixed to a point O at the centre of a horizontal frictionless table, as shown in plan in Fig. 3, so that the particle is free to orbit about the fixed point.

(a) At what speed must the particle travel so that it orbits around O in a circular path whose radius, *r*, is 500 mm? [6]

(b) The particle is orbiting in this manner when a second particle Q, also of mass 250 g, is placed on the table in its path. The two particles collide, and remain locked together after the collision. What are the radial and tangential components of the velocity and acceleration of the new combined particle, immediately after the collision? [9]

(c) Describe the subsequent motion of the combined particle qualitatively, and obtain a fourth-order expression in *r* which will determine the minimum and maximum values of *r*. Show that one solution is approximately *r* = 391 mm, and state whether you would expect this to be the minimum or the maximum value of *r*. What will be the other limiting value of *r* ? [15]

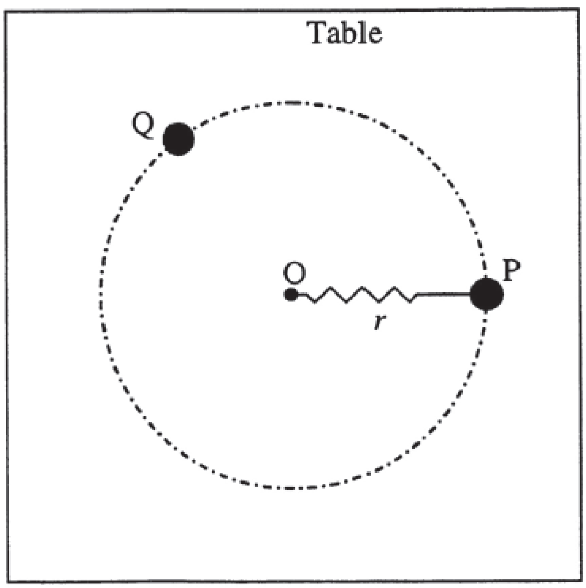


Fig. 3

SECTION B: Structural Mechanics

4 (short) The plane truss shown in Fig. 4 forms part of a gantry, and is to be designed for a single loading condition shown – a vertical force W applied at K. Members are of lengths L, L√2 or 2L as shown. Calculate all the bar forces T due to the imposition of load W. List all the results in the table given on the attached sheet marked *Table for Question 4*. A number of the bars are found to carry zero force. Explain why they may nevertheless serve a valuable function. [10]

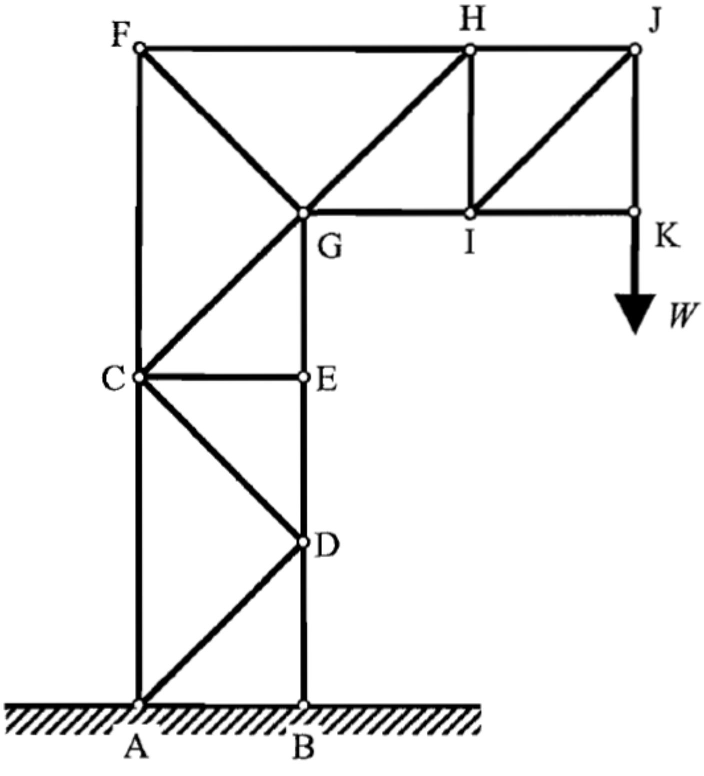


Fig. 4

5 (long)

(a) The cable shown in Fig. 5(a) is one of the two main cables of a suspension bridge during construction. The partially constructed roadway imposes a load on this cable of 100 kN m^{-1} over the central half of the span, while elsewhere the cable is unloaded. The dip of the cable at the centre of the span is 100 m.

(i) Calculate the horizontal and vertical reactions at the supports. [5]

(ii) Derive an analytical expression for the shape of the cable, $y(x)$ for the portions: $0 \le x \le 250\,\text{m}$ and for $250 \le x \le 500\,\text{m}$. [9]

(iii) Show that the total length of the cable is approximately 1023 m. [6]

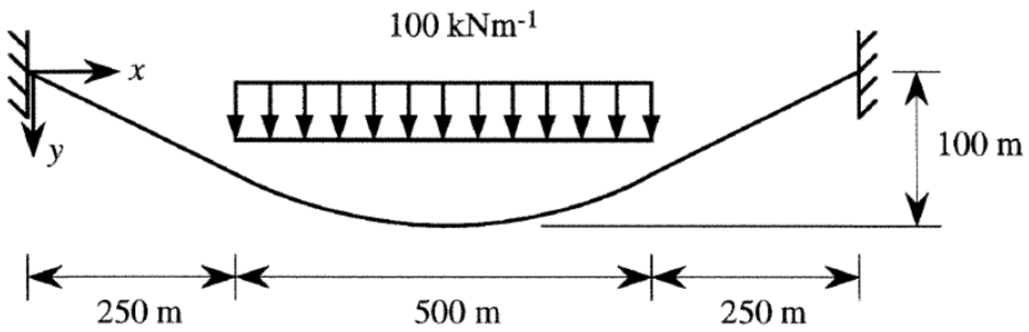


Fig. 5(a)

Cont.

(b) A three pin arch structure of height 12.5 m and width 10 m is show in Fig. 5(b). The structure is symmetric and the shape of the structure for positive x is given by $y = (1/10)\,x^3$, where the origin is at the middle pin C. A vertical load of 20 kN is applied to the structure at location B shown in the figure.

(i) Find the reaction forces at pin A and D. [5]

(ii) Find the location and magnitude of the maximum moment in section AB. [5]

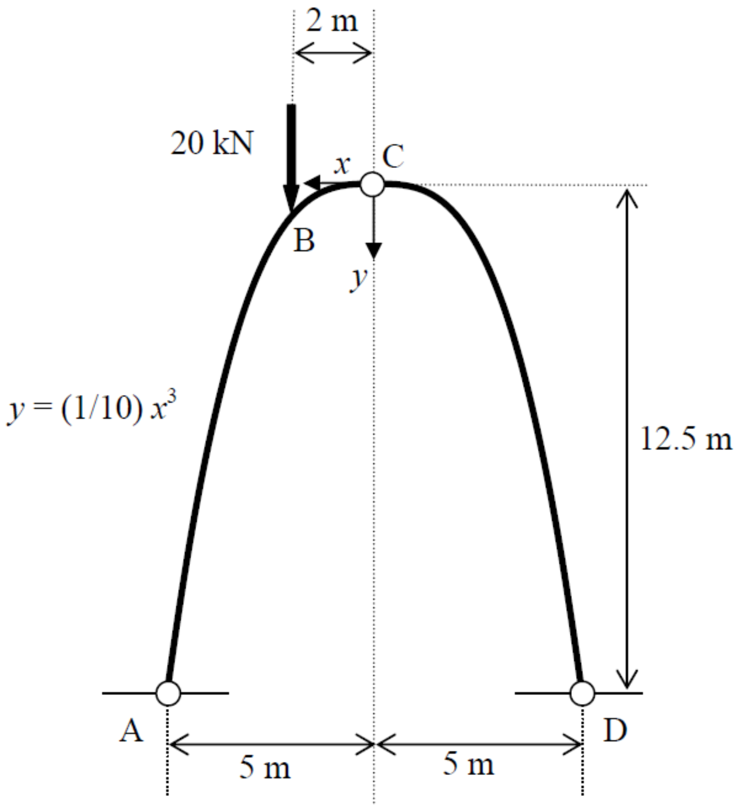


Fig 5(b)

6 (short) An inextensible cable is loaded between two supports as shown in Fig 6(a).

(a) Find the support reactions. [4]

(b) The left hand support is now replaced with a block on a plane as shown in Fig 6(b). The coefficient of friction between the block and the plane is 0.3. Assuming there is an even pressure distribution under the block, find the minimum mass, *M*, of the block required in order to ensure that the block will not slide. [6]

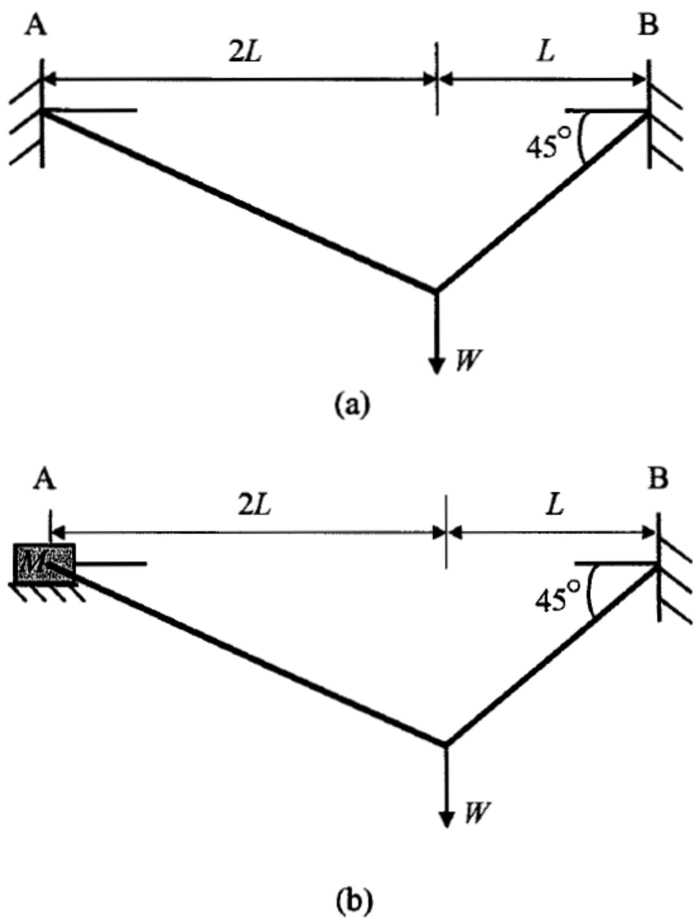


Fig. 6

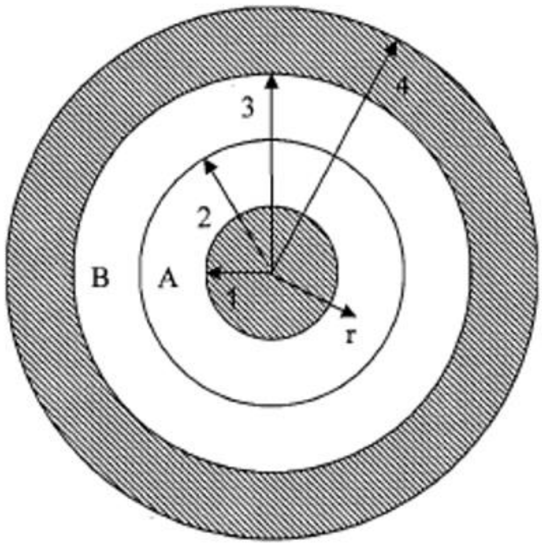
SECTION C: Physical Principles of Electronics and Linear Circuits

7 (short) The coaxial cable shown in Fig. 7 consists of an inner conductor of circular cross-section, radius 1 mm and an outer conductor which is an annulus concentric with the inner conductor of inner radius 3 mm, outer radius 4 mm. The gap between the inner and outer conductors is filled with two dielectric materials: annulus A has inner radius 1 mm, outer radius 2 mm and a relative permittivity of 5; annulus B has inner radius 2 mm, outer radius 3 mm and a relative permittivity of 2. The cable may be assumed to be very long, and has a charge per unit length on the inner and outer conductors of +1 pC/m and -1 pC/m respectively.

(a) State Gauss' Law. [2]

(b) Sketch a graph of the magnitude of the electric field strength vs radius r, taking $r = 0$ as the centre of the inner conductor, from $0 < r < 5$ mm. Annotate your sketch with the values of the electric field strength at $r = 1$ mm, 2 mm and 3 mm. [5]

(c) Determine the capacitance per unit length of the cable. [3]



All dimensions are in mm

Fig. 7

8 (short)

(a) State Thevenin's and Norton's theorems. [3]

(b) Calculate the Thevenin and Norton equivalents of the circuit shown in Fig. 8. [7]

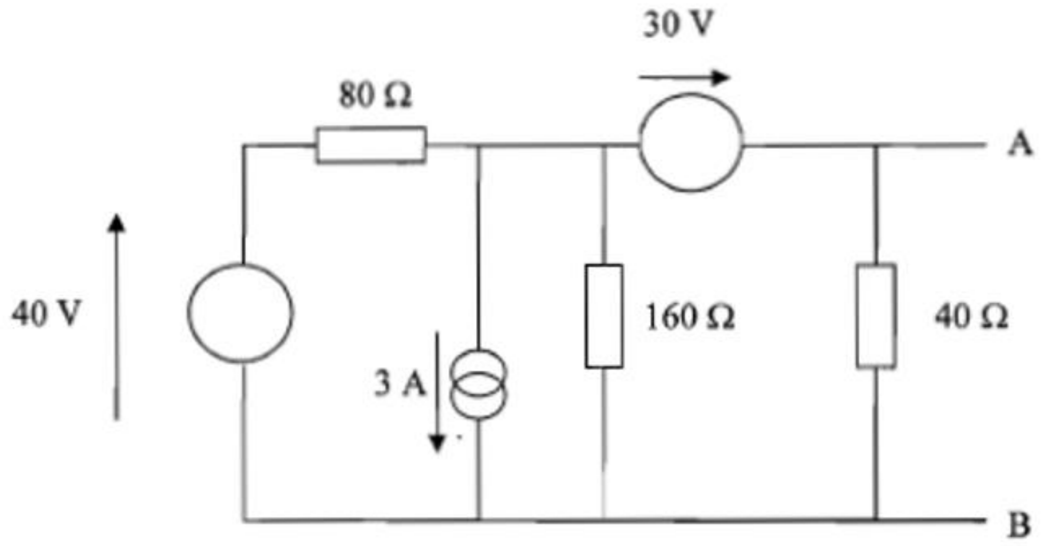


Fig. 8

9 (long)

(a) Explain briefly how the techniques of *mesh current analysis* and *loop current analysis* are used in d.c. and a.c. electrical circuits. [5]

(b) Figure 9(a) shows the circuit for an a.c. bridge. The voltage source supplies a sinusoidal waveform of frequency ω. At balance the current through the meter M is zero. Find the conditions for balance in terms of R_1, R_2, R_3, R_4 and the ratio L/C. [17]

(c) Consider the bridge in Fig. 9(b). Find an expression for the frequency ω at which the bridge balances, in terms of R_1, R_2, R_3, R_4 and C. [8]

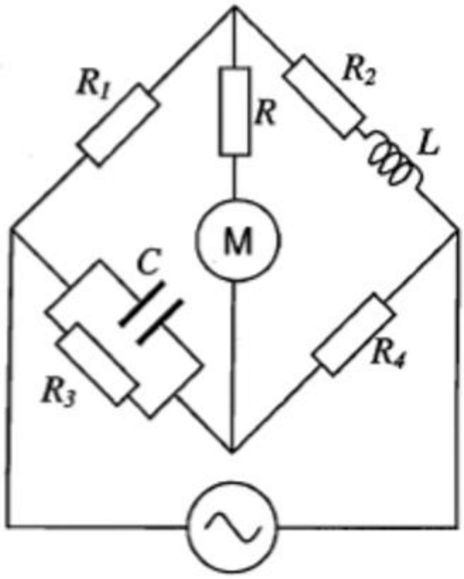


Fig. 9(a)

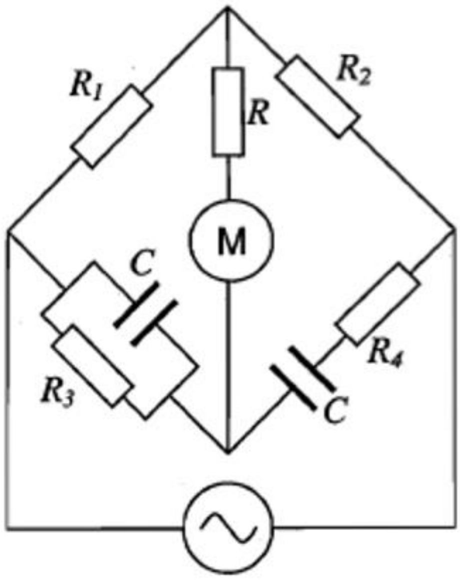


Fig. 9(b)

SECTION D: Mathematical Methods

10 (short) Evaluate z where

$$\left(\sin^{-1} z\right)^2 = 2\pi^2 i$$

and show all solutions on an Argand diagram. [10]

11 (short) Solve

$$\sin x \frac{dy}{dx} + y\cos x = x\cos x$$

with $y = 1$ when $x = \pi/2$. [10]

12 (long)

(a) Find the shortest distance between the two lines given by

$$\mathbf{r} = \begin{bmatrix} 4 \\ -2 \\ 3 \end{bmatrix} + t\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \quad \text{and} \quad \mathbf{r} = \begin{bmatrix} -7 \\ -2 \\ 1 \end{bmatrix} + s\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$ [10]

(b) A line is given by the parametric equation

$$\mathbf{r} = \mathbf{a} + s\mathbf{d}$$

show that the perpendicular distance of this line from the origin can be written in two forms:

$$p = \frac{|\mathbf{a} \times \mathbf{d}|}{|\mathbf{d}|} \quad \text{or} \quad p = \left|\mathbf{a} - \frac{\mathbf{a}.\mathbf{d}}{\mathbf{d}.\mathbf{d}}\mathbf{d}\right|$$ [10]

(c) By considering $\mathbf{d} \times (\mathbf{a} \times \mathbf{d})$ and sketching the result along with the line $\mathbf{r} = \mathbf{a} + s\mathbf{d}$, explain why the two expressions for p in part (b) are equivalent. [10]

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Wednesday 17 January 2018

Candidate:

College:

Table for Question 4

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