EGT0
ENGINEERING TRIPOS PART IA

Thursday 7 June 2018 9.00 to 12.10

Paper 2

STRUCTURES AND MATERIALS

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper and graph paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

1 (**short**) Figure 1 shows an oar in plan view, in use. The oar is pivoted at B and the oarswoman pulls with a force of 1 kN at A. The blade of the oar from C to D is in the water, which applies a uniformly distributed force. At the instant shown, the oar is in equilibrium.

Sketch shear force and bending moment diagrams for the oar, marking all salient values.

[10]

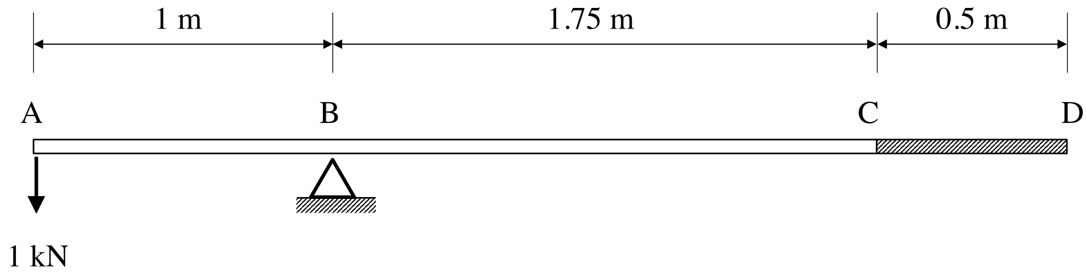

Fig. 1

2 (**short**) Figure 2 shows a light circular pulley of radius R mounted on a vertical mast by a frictionless pivot at point P. A mass M is suspended by a cable which is tethered to the mast at Q, a distance $2R$ below the pivot. At point X, halfway between P and Q, calculate the axial force, shear force and bending moment in the mast, making clear the direction in which they act. [10]

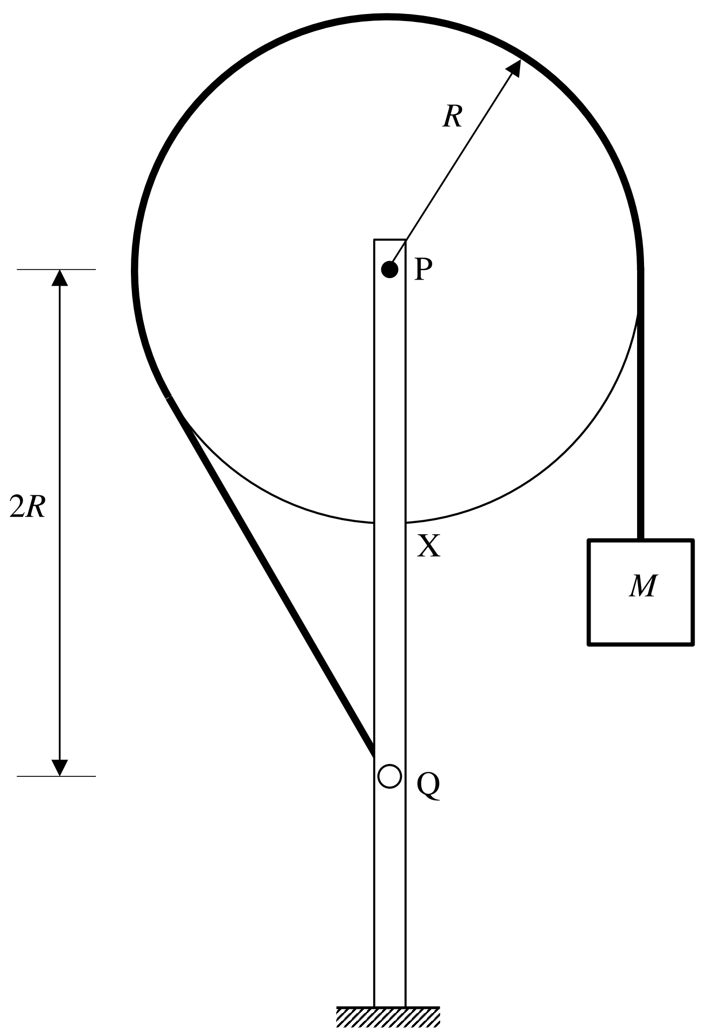

Fig. 2

3 (**short**) Figure 3 shows the cross-section of a solid beam. The cross-section is an isosceles triangle of width w and height h. Derive an expression for the second moment of area of the beam about its horizontal neutral axis. [10]

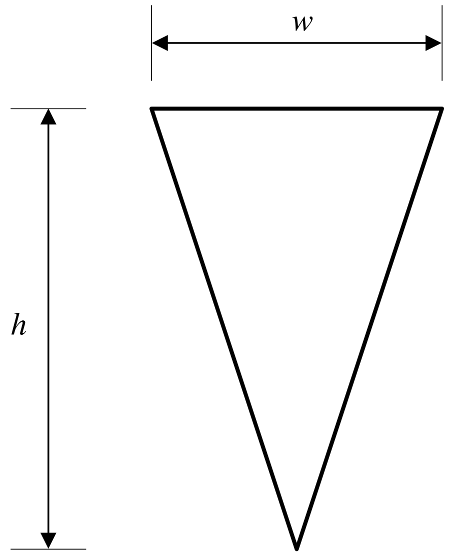

Fig. 3

4 (**short**) Figure 4 shows a semi-circular arch of radius R, with frictionless pin-joints at A, B and C. B is at the midpoint of the arch AC. A force F is applied to the arch as shown, at 45° to the vertical, mid-way between B and C. Explain why the reaction at A acts along the line AB and draw the bending moment diagram for this loading on a sketch of the arch, marking salient points and using the convention that the bending moment is plotted on the "tensile side" of a member. [10]

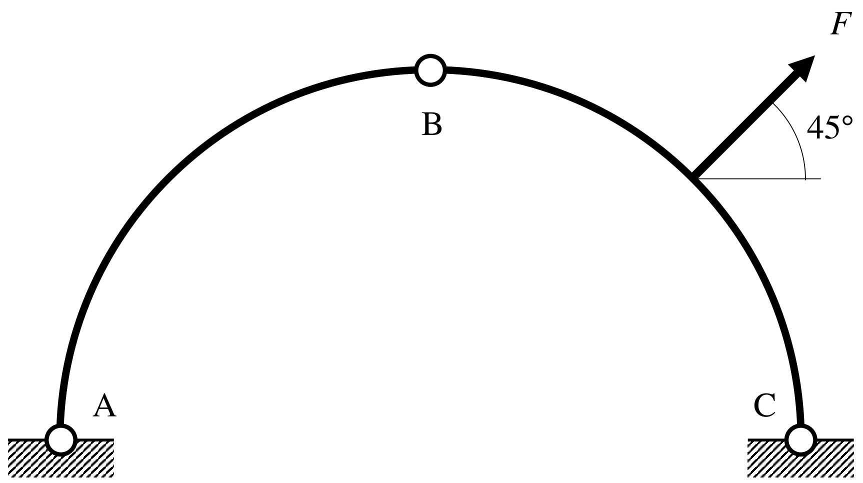

Fig. 4

5 (**long**) Figure 5 shows a cantilevered structure anchored to the ground at frictionless pin-joints A and B. A vertical load of 10 kN is applied downwards at joint D. The figure is drawn to scale and the bar lengths shown in the figure are in metres. All bars have the same cross-sectional area (A) and Young's modulus (E) so that $AE = 10^7$ N.

(a) Find the forces in all the bars of the structure. [10]

(b) Find the vertical displacement of joint E. [20]

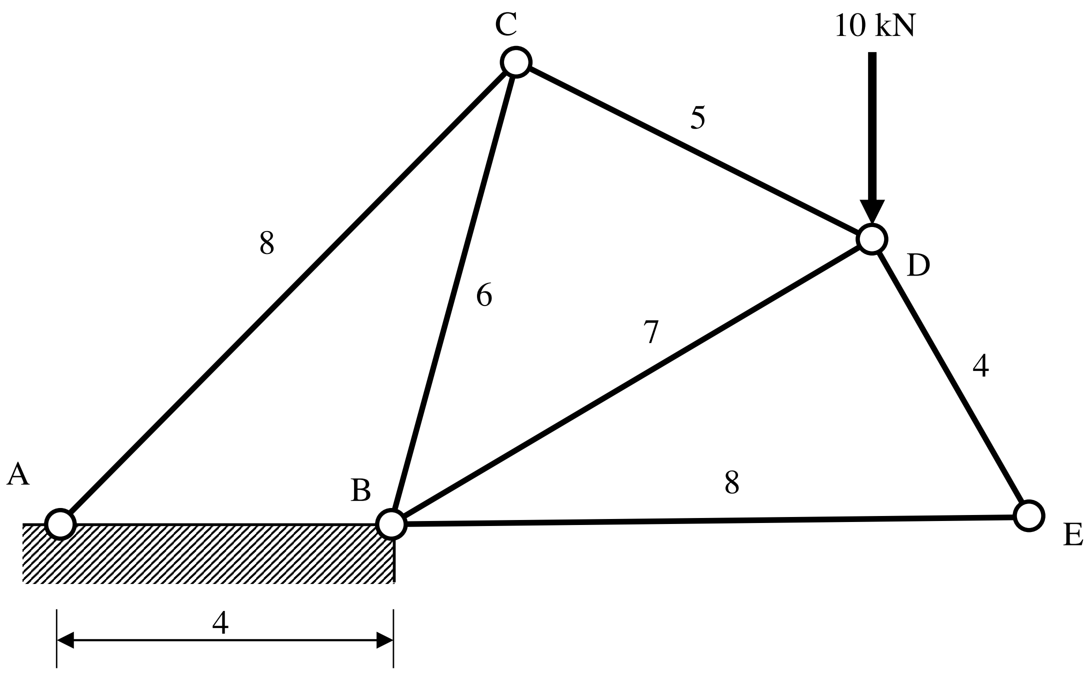

Fig. 5

6 (**long**) Figure 6 shows a composite beam comprising a steel I-beam and a concrete slab. The concrete is connected to the steel beam by a set of steel "shear studs" as illustrated. The dimensions given in the figure (which is not to scale) are given in cm. The steel beam has cross-sectional area 60 cm^2 and second moment of area 10,000 cm^4 about its own neutral axis. The Young's modulus of steel and cement are 450 GPa and 30 GPa respectively.

(a) Calculate the bending stiffness, *EI*, of the composite beam. [20]

(b) If the beam is subjected to a vertical shear force of 100 kN and each shear stud can withstand a maximum shear force of 50 kN, what is the maximum permissible stud spacing, *s*, to prevent slippage between the two components of the beam? [10]

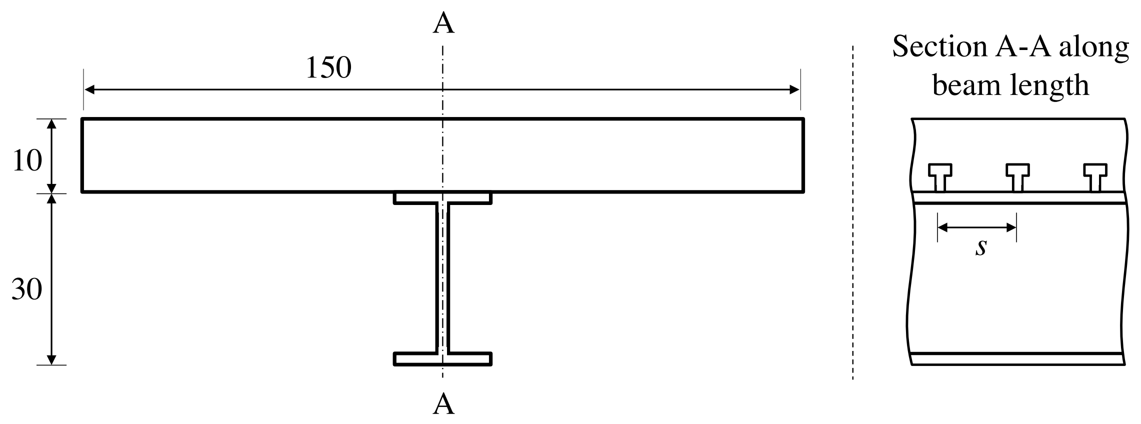

Fig. 6

SECTION B

7 **(short)**

(a) Define the bulk modulus. Starting with the generalisation of Hooke's Law relating linear stress and strain in 3D, show that the bulk modulus K of a linear elastic isotropic material is given by

$$K = \frac{E}{3(1-2\nu)}$$

where E is the Young's modulus and ν is the Poisson's ratio. Using this result, explain why it is improbable that a simple material could exist with $\nu > 0.5$. [5]

(b) A short cylindrical rod is made from a material with Young's modulus E and Poisson's ratio ν. The rod is compressed along its length so that the axial stress is $-\sigma$. The rod is constrained so that it cannot expand radially. Derive an expression for the axial strain in terms of E, ν and σ when the rod is compressed axially. [5]

8 **(short)**

(a) Sketch the unit cell for a Body Centred Cubic (BCC) material, showing the positions of the atoms in the cell. Identify an example close-packed direction and label the lattice parameter a. [4]

(b) The density of iron is $7870\ \mathrm{kg\,m^{-3}}$. Using this value, calculate the lattice parameter a. Hence, find the atomic radius R. [6]

9 (**short**)

(a) Figure 7 shows a sketch of a spanner which is to be manufactured in carbon steel, with an expected batch size of 5000. Selected dimensions of the spanner with the required precisions are shown in Fig. 7. The thickness is 4 ± 0.1 mm. Use the process attribute charts in the Materials Databook to choose between sand casting and forging, basing your selection on the following criteria: mass, tolerance, and economic batch size. Explain why the manufacturing process also involves machining. [6]

(b) The manufacturing process also involves a final heat treatment. State which hardening mechanism is exploited via heat treatment of carbon steel, and briefly summarise the microstructural mechanism. State with brief reasons whether the heat treatment is also expected to affect the Young's modulus and the fracture toughness of the steel. [4]

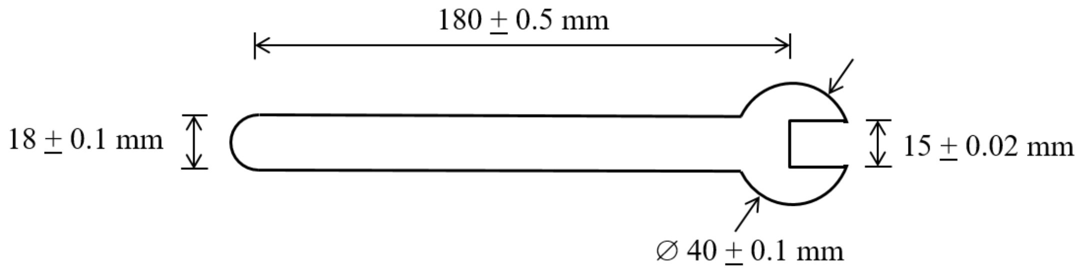

Fig. 7

10 (**short**)

(a) Sketch the stress-strain responses for a linear elastic material and for a viscoelastic material under cyclic loading with zero mean stress. Give an example of a viscoelastic material. [3]

(b) The upper part of Fig. 8 shows three models (labelled A-C) for the stress-strain responses of viscoelastic materials. The elements of the models are elastic springs and viscous dashpots, with the respective stiffnesses E and viscosities η shown. The lower part of the figure shows the instantaneous application of a constant stress σ_o at time $t = 0$, with three responses in the strain $\varepsilon(t)$ (labelled I-III). ***By inspection***, match the models A-C to the appropriate strain response, I-III, explaining your reasoning in each case. Give expressions for the strains indicated (ε_1, ε_2, ε_3, ε_4) in terms of the applied stress σ_o and the properties of the model elements. (**NB Solution of the full response $\boldsymbol{\varepsilon(t)}$ is not required**). [7]

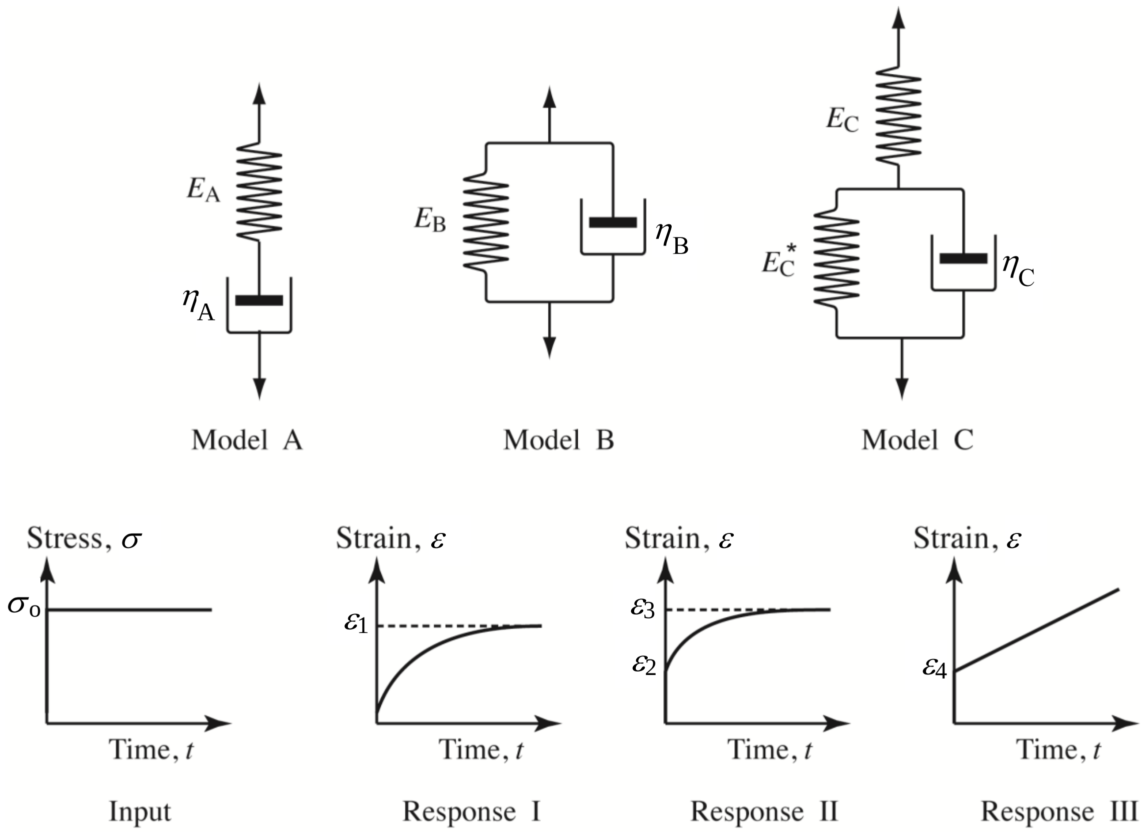

Fig. 8

11 **(long)**

(a) Sketch how the logarithm of crack growth per cycle under cyclic loading da/dN, varies with the logarithm of the stress intensity factor range ΔK. On your diagram, indicate the threshold ΔK_{th}, the Paris regime and the behaviour as K_{max} approaches the fracture toughness K_{IC}. [6]

(b) The fuselage of a prototype commercial jet aircraft has a radius R of 1.3 m. The aircraft is used typically for an average of six short-haul flights each day. During each flight, the cabin is pressurised to a pressure P of 50 kPa. The fuselage skin thickness t is 0.9 mm ($t << R$) and is made of an aluminium alloy with $K_{\text{IC}} = 30$ MPa m$^{1/2}$. In laboratory tests, the alloy exhibited the following crack growth rates:

$$\frac{da}{dN} = 10^{-8} \text{ m cycle}^{-1} \qquad \text{at} \qquad \Delta K = 5 \text{ MPa m}^{1/2}$$

$$\frac{da}{dN} = 10^{-7} \text{ m cycle}^{-1} \qquad \text{at} \qquad \Delta K = 8.9 \text{ MPa m}^{1/2}$$

(i) The aircraft fuselage and air pressurisation system are designed such that the cabin pressure can be sustained with a through-thickness crack. Determine the critical length of a through-thickness longitudinal crack of length $2a$ perpendicular to the hoop direction at the flight pressure. Assume that the dimensionless constant Y, in the expression for the stress intensity factor K, is equal to 1. [4]

(ii) Consider a through-thickness crack 75 mm long. Will fast fracture occur before there is a drop in cabin pressure? [4]

(iii) Assuming that the fatigue crack growth rate for the material satisfies Paris' Law, determine the constants n and A in Paris' Law (Materials Databook). [6]

(iv) The aircraft is subjected to a major maintenance check every 2 years. When a routine check was conducted on the aircraft, the presence of a 2 mm long through-thickness longitudinal crack in the fuselage was left undetected due to human error. Using the constants determined in part (iii), determine the remaining life of the aircraft following the maintenance check. Is the aircraft likely to fail or suffer a loss of cabin pressure before the next major maintenance check? [10]

12 (**long**) The strain energy stored in a linear elastic material of Young's modulus E under a tensile stress σ and strain ε is given by $\sigma\varepsilon / 2$, per unit volume.

(a) A sample of this material of length L and square cross-section $b \times b$, is loaded in tension to a stress σ_{max}. Show that the elastic energy stored in the sample is equal to $b^2 L \sigma_{\mathrm{max}}{}^2 / 2E$. [3]

(b) An identical sample is loaded in pure bending with a moment M, as shown in Fig. 9, with a maximum stress at the surface equal to σ_{max}. Use the Structures Databook to find the variation in stress $\sigma(y)$ and strain $\varepsilon(y)$, in terms of σ_{max} and the distance y from the neutral axis. Hence show that the elastic energy stored in pure bending is smaller than that in tension by a factor of 3, and explain qualitatively why this is the case. [10]

(c) Figure 10 shows two cross-sections of external dimensions $B \times B$. One section is solid, with an inner shaded region of dimensions $\alpha B \times \alpha B$. The second cross-section is hollow with internal dimensions $\alpha B \times \alpha B$. Samples of both sections, of length L, are loaded in pure bending until the maximum stress is equal to the elastic limit σ_y, as shown in Fig. 10. Use the result in (b) to calculate the elastic energy stored:

(i) in the sample with the solid cross-section;

(ii) in the material within the shaded region of the solid sample.

Use these results to show that the elastic energy stored in the hollow cross-section is equal to $(1 - \alpha^4)\ B^2 L \sigma_y{}^2 / 6E$. [9]

(d) Find the dimensions of a solid square cross-section of equal mass per unit length as the hollow cross-section in Fig. 10. Use the result in (b) to find the elastic energy stored in a sample of length L with this cross-section. What is the ratio of the elastic energy stored in the hollow cross-section, to the energy stored in the solid section of equal mass per unit length? What is the maximum possible value of this ratio as α varies? [8]

(cont.

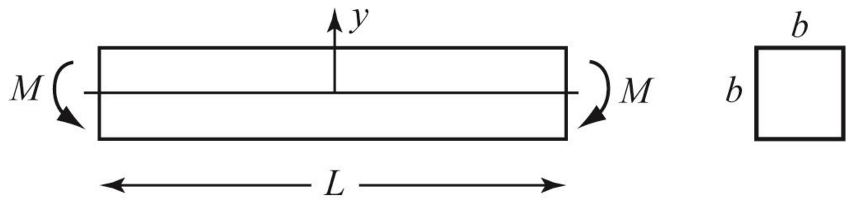

Fig. 9

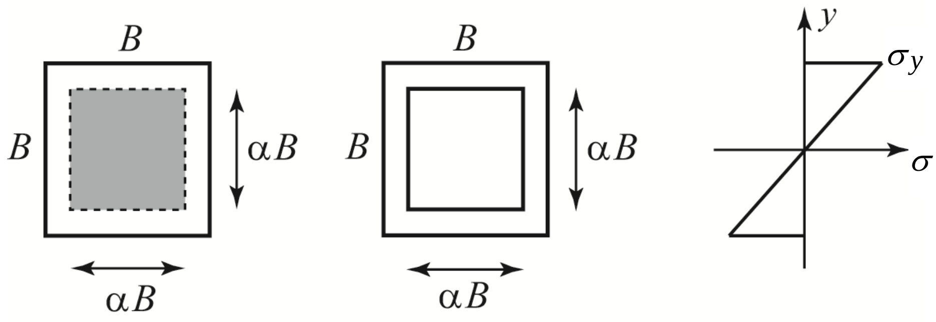

Fig. 10

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ENGINEERING TRIPOS PART IA, 2018
Paper 2
Numerical Answers

Section A (Structures)

2. $F_X = Mg\left(1 + \frac{\sqrt{3}}{2}\right)$, $S_X = \frac{Mg}{2}$, $M_X = \frac{MgR}{2}$

3. $I = \frac{wh^3}{36}$

5. (a) AC: 20.6 *kN*; BC –20.0 *kN*, CD 10.2 *kN*; BD –10.7 *kN*.

(b) 91 *mm* downwards

6. (a) 116 $MN\ m^2$

(b) $s \leq 0.17\ m$.

Section B (Materials)

8. (b) $a \approx 2.87\times10^{-10}$ m, $R \approx 1.24\times10^{-10}$ m

11. (b) (i) $a_{\text{crit}} \approx 54.9$ mm, total crack length $2a_{\text{crit}} \approx 109.8$ mm

(iii) $n = 4$ and $A = 1.6\times10^{-11} \dfrac{\text{m/cycle}}{(\text{MPa}\sqrt{\text{m}})^4}$.

(iv) $N_{\text{f}} \approx 229\times10^3$ cycles