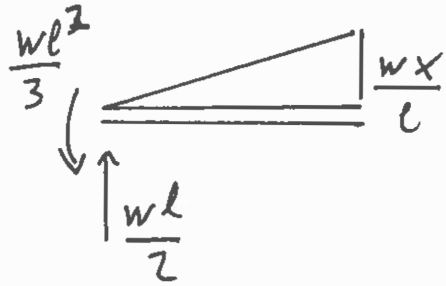
Section A

1a) i) $v_x = \dfrac{wx}{l}$



ii) $M = \dfrac{wl^2}{3} + \dfrac{wx^3}{6l} - \dfrac{wlx}{2}$

$EIv'' = M$

$EIv' = \dfrac{wl^2}{3}x + \dfrac{wx^4}{24l} - \dfrac{wlx^2}{4} + c_1$

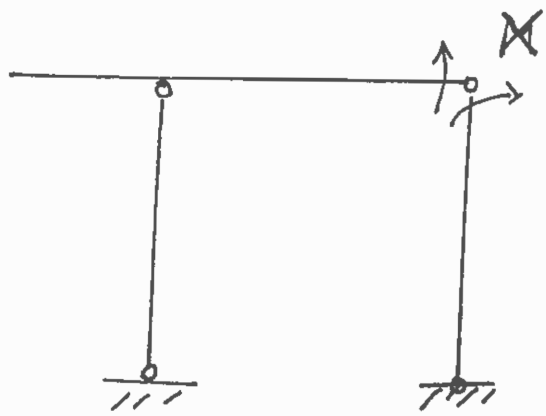
$EIv = \dfrac{wl^2}{6}x^2 + \dfrac{wx^5}{120l} - \dfrac{wlx^3}{12} + c_1 x + c_2$

Boundary conditions: $v(0)=0 \quad v'(0)=0$

$\Rightarrow EIv = \dfrac{wx^5}{120l} - \dfrac{wlx^3}{12} + \dfrac{wl^2x^2}{6}$

$EIv(l) = \dfrac{11wl^4}{120} \qquad EIv'(l) = \dfrac{wl^3}{8}$

b) i)



$EI\theta_1 = \dfrac{l}{3} + l = \dfrac{4}{3}l$

$EI\theta_0 = \dfrac{wl^3}{8}$

$\Rightarrow \cancel{X}\theta_1 = \theta_0$

$X = \dfrac{3}{32}wl^2$

Moment at C : 93.75 kNm

Axial forces

$V_E = \dfrac{93{,}75}{10} = 9.375\ kN \qquad V_D = -9{,}375\ kN$

ii) Horizontal deflection

$$\delta_{H,A} = \frac{11 w \ell^4}{120 EI} - \frac{M \ell^2}{2 EI} = 45.83 - 23.48 = 22.40 \text{ mm}$$

Vertical deflection

$$\delta_{V,A} = \frac{M \ell}{6 EI} \cdot 5 = \frac{93{,}75 \cdot 5 \cdot 10}{6 \cdot 200 \cdot 10^6} = 3{,}9 \text{ mm}$$

iii) Deformation at B only vertical and 100 mm

$$\theta_0 = \frac{0{,}1}{10} \qquad EI\,\theta_1 = \frac{4}{3}\ell$$

$$\Rightarrow \quad X = 0{,}01 \cdot \frac{3}{4\ell} EI = 150 \text{ kNm} \qquad \text{Moment at } C$$

2a) i) $I = \pi r^3 t = 3.14 \cdot 10^{-5}$

$$\sigma_A = \frac{My}{I} = -\frac{25 \cdot 10^3 \cdot 0{,}1}{3.14 \cdot 10^{-5}} = -79.6 \text{ MPa}$$

$\sigma_B = 0$

ii) Due to torque

$$\tau_t = \frac{T}{2 A_e t} = \frac{25 \cdot 10^3 \cdot 0{,}1}{2\pi \cdot 0{,}1^2 \cdot 0{,}01} = 3.98 \text{ MPa}$$

Due to shear stress

at A is zero

at B is $\tau_s = -\frac{S A \bar{y}}{I \cdot 2t} =$

$$= \frac{25 \cdot 10^3 \cdot \pi \cdot 0{,}1 \cdot 0{,}01 \cdot 0{,}064}{3.14 \cdot 10^{-5} \cdot 0{,}02} = -7.95 \text{MPa}$$

Total at A $\tau = 3.98$ MPa

at B $\tau = -12$ MPa

b) i) Mohr's circle at B $\sigma_1 = -3.98$ MPa and $\sigma_2 = 3.98$ MPa

Mohr's circle at A

$\sigma_A = -79.6$ MPa $\tau = 3.98$ MPa

$\sigma_1 = -80$ MPa $\sigma_2 = 0.2$ MPa

A is clearly critical, so using

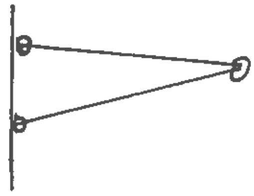
$$\lambda^2 \left[(\sigma_1 - \sigma_2)^2 + \sigma_2^2 + \sigma_1^2 \right] = 2Y^2$$

$$\lambda^2 [80.2^2 + 0.2^2 + 80^2] = 2.275$$

$$\Rightarrow \lambda = 3.41$$

ii) too high

iii) tapering tube struts box cross section etc.



. . . .

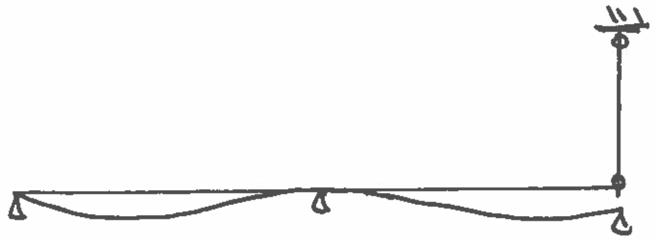
For instance tapering tube

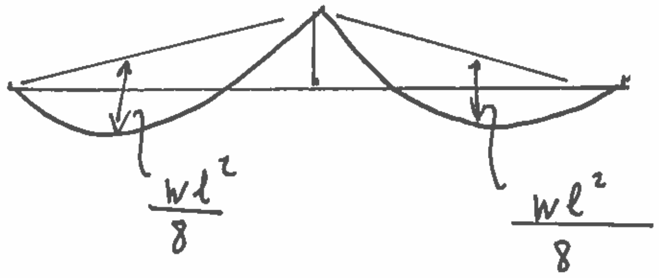
$$\sigma = \frac{Mr}{2\pi r^3 t} = \frac{M}{2\pi r^2 t}$$

Need to reduce stress by a factor of ~4.

Increase radius by ~2.

3) a)

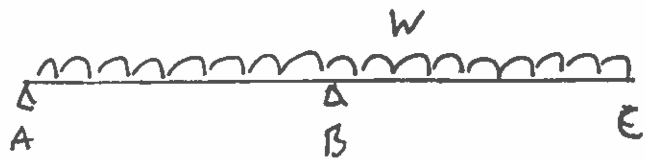




b) i) Number of redundancies is 1

- Remove support A
- Introduce pin at B
- Cut cable
- ....

ii)



Moment at B: $M_B = \frac{q l^2}{2}$

rotation at B

$$EI\,\theta_B = -\frac{w l^3}{24} + \frac{w l^3}{6} = \frac{w l^3}{8} \Rightarrow EI\,\delta_1 = \frac{w l^4}{8}$$

BC as cantilever

$$EI\,\delta_2 = \frac{w l^4}{8}$$

Total disp:

$$\delta = \delta_1 + \delta_2 = \frac{w l^4}{4\,EI}$$

iii)

rotation at B

$$EI\,\theta_B = -\frac{Tl^2}{3} \Rightarrow EI\,\delta_1 = -\frac{Tl^3}{3}$$

BC as cantilever

$$EI\,\delta_2 = -\frac{Tl^3}{3}$$

$$\delta = \delta_1 + \delta_2 = -\frac{2Tl^3}{3EI}$$

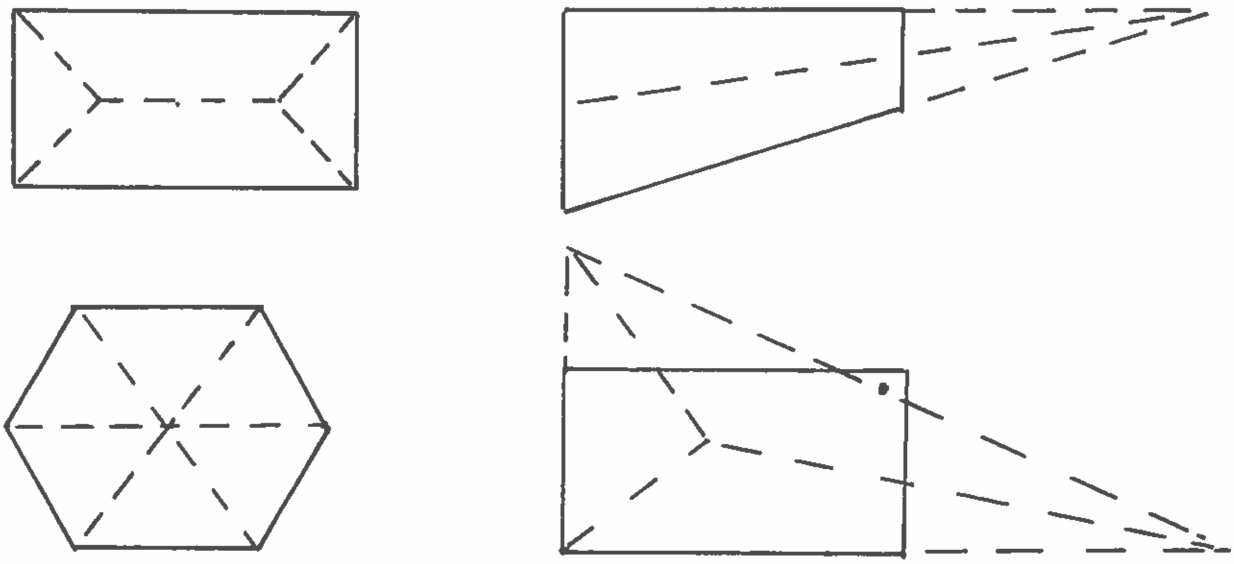
Compatibility

$$\frac{2l^3T}{3EI} + \frac{T}{EA}\frac{l}{2} = \frac{wl^4}{4EI}$$

$$\Longrightarrow T = \frac{3wl^4EA}{8l^3EA + 6lEI}$$

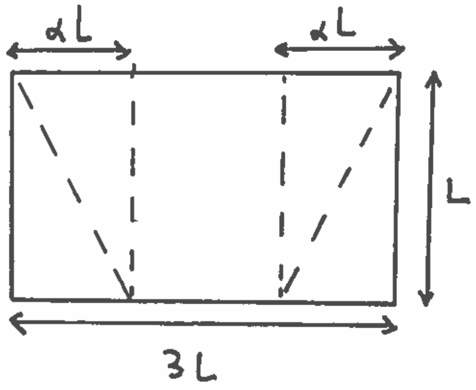
SECTION B

4 a)



b)

i)



Hodograph

Max disp. $\delta = \alpha L$

Internal work

$$W_i = 2\sqrt{\alpha^2 L^2 + L^2}\; m \sqrt{1+\alpha^2} = 2L(1+\alpha^2)m$$

External work

$$\frac{W_e}{p} = 4\,\frac{L}{2}\,\alpha L\,\frac{\delta}{3} + (3L - 2\alpha L)\,L\,\frac{\delta}{2}$$

$$= \frac{2}{3}\alpha L^2 \delta + 3L^2\,\frac{\delta}{2} - \alpha L^2 \delta$$

$$= -\frac{1}{3}\alpha L^2 \alpha L + \frac{3}{2}\alpha L^3 = \frac{1}{6}L^3 \alpha\,(9 - 2\alpha)$$

$W_e = W_i$

$$p = \frac{2L(1+\alpha^2)m}{\frac{1}{6}L^3\alpha(9-2\alpha)} = \frac{12(1+\alpha^2)m}{L^2\alpha(9-2\alpha)}$$

ii) $\frac{dp}{d\alpha} = 0$

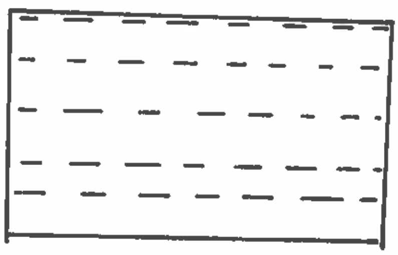
$$\frac{dp}{d\alpha} = \frac{24\alpha m L^2\alpha(9-2\alpha) - 12(1+\alpha^2)m L^2(9-4\alpha)}{\ldots} = 0$$

$$= 2\alpha^2(9-2\alpha) - (1+\alpha^2)(9-4\alpha) = 0$$

$$= 18\alpha^2 - 4\alpha^3 - 9 + 4\alpha - 9\alpha^2 + 4\alpha^3 = 0$$

$$= 9\alpha^2 + 4\alpha - 9 = 0 \implies \alpha = 0{,}802$$

iii)

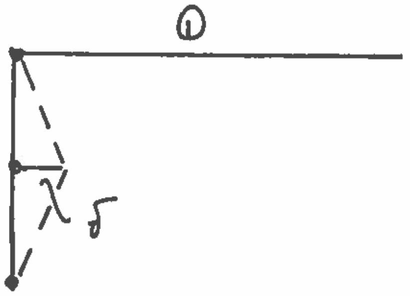


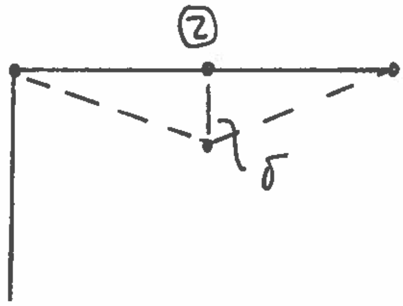
$$m = \frac{pL^2 9}{8} \Rightarrow p = m\frac{8}{9L^2}$$

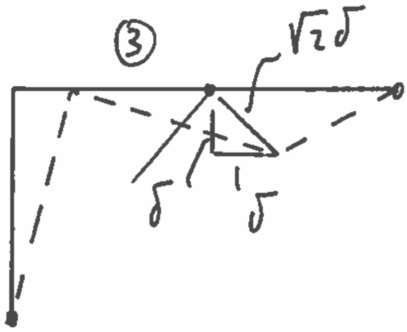
Think of strips/beams in horizontal direction; compatibility not important, however equilibrium important.

5)

a)







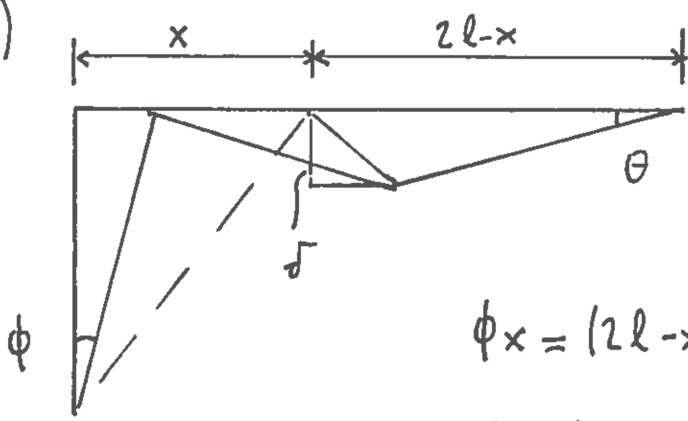
b) Mechanism 1: $H\cdot\delta = 2\cdot\left(\frac{2\delta}{\ell}\right)M_p + 4\,\frac{\delta}{\ell}\,M_p \implies H = M_p\,\frac{8}{\ell}$

Mechanism 2: $w\cdot 2\ell\cdot\frac{\delta}{2} = 4\,\frac{\delta}{\ell}\,M_p \implies w = M_p\,\frac{4}{\ell^2}$

Mechanism 3: $H\cdot\frac{\delta}{2} + 2w\ell\,\frac{\delta}{2} = M_p\,\frac{\delta}{\ell} + M_p\,\frac{3\delta}{\ell}$

$$\implies H + 2w\ell = \frac{8M_p}{\ell}$$

c)



$$\phi x = (2\ell - x)\,\theta$$

$$\delta = \phi x = (2\ell - x)\,\theta$$

$$\implies \phi = \frac{\delta}{x} \qquad \theta = \frac{\delta}{2\ell - x}$$

Internal virtual work

$$W_{int} = M_p \phi + M_p \theta + M_p (\theta + \phi) = 2 M_p (\theta + \phi)$$

$$= 2 M_p \delta \left(\frac{1}{x} + \frac{1}{2l - x} \right) = 2 M_p \delta \frac{2l}{x(2l-x)} = \frac{4 M_p \delta l}{x(2l-x)}$$

External virtual work

$$W_{ext} = H \frac{\delta l}{2x} + w \left(x \frac{\delta}{2} + (2l - x) \frac{\delta}{2} \right)$$

$$= H \frac{\delta l}{2x} + \delta \frac{H}{2} = H \delta \frac{l + x}{2x}$$

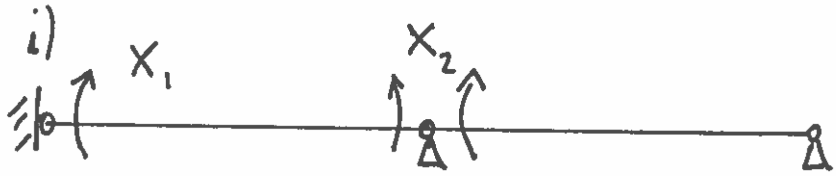
Work equilibrium

$$H = \frac{4 M_p l}{x(2l-x)} \frac{2x}{(l+x)} = \frac{8 M_p l}{2l^2 + lx - x^2}$$

$$\frac{dH}{dx} = - \frac{8 M_p l (l - 2x)}{(2l^2 + lx - x^2)^2} = 0 \implies x = \frac{l}{2}$$

6 a) equilibrium + material law

b) i)



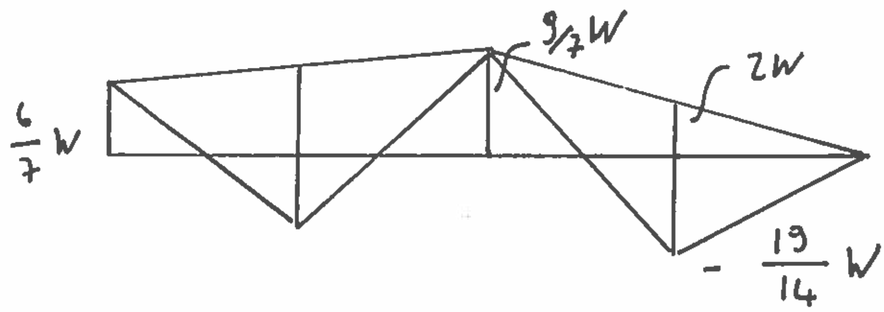
from Data Book

$EI\,\delta_{11} = \frac{8}{3} X_1$ $\qquad EI\,\delta_{12} = \frac{4}{3} X_1$

$EI\,\delta_{22} = \frac{16}{3} X_2$ $\qquad \delta_{21} = \delta_{12}$

$\delta_{10} = \frac{W\,64}{16\,EI} \implies EI\,\delta_{10} = 4W \quad EI\,\delta_{20} = 8W$

$$\begin{pmatrix} \frac{8}{3} & \frac{4}{3} \\ \frac{4}{3} & \frac{16}{3} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} 4 \\ 8 \end{pmatrix} W \implies \begin{array}{l} X_1 = \frac{6}{7} W \\ X_2 = \frac{9}{7} W \end{array}$$

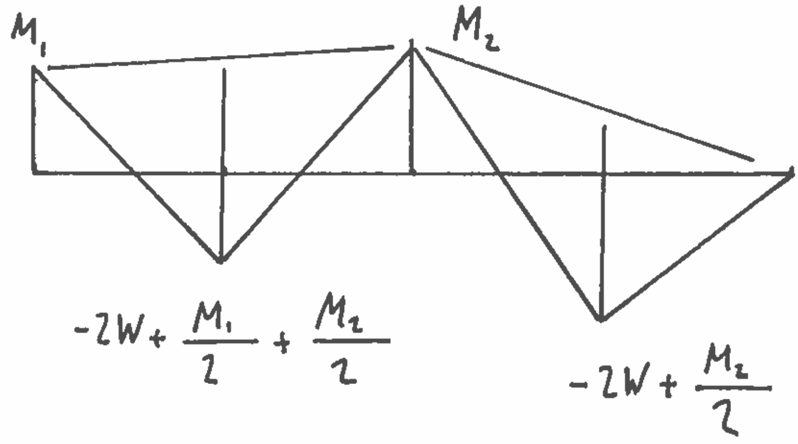


UB 356 × 127 × 39 $\quad z_e = 576\ cm^3 \quad z_p = 659\ cm^3$

$M_y = 355 \cdot 10^6 \cdot 576\ 10^{-6} = 204.5\ kNm$

$\frac{19}{14} W = 206.5 \implies 150.68\ kN$

ii)



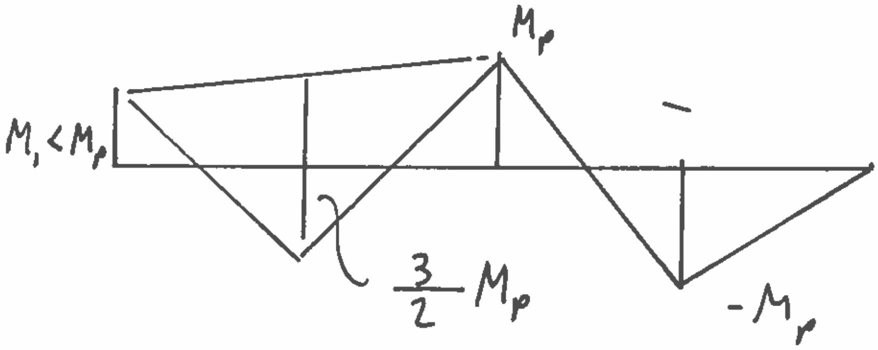
Optimum solution with (moments in right span)

$$M_2 = M_p \quad \text{and} \quad -2W + \frac{M_2}{2} = -M_p$$

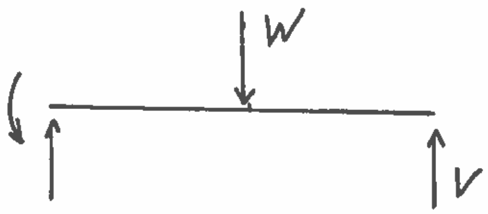
$$\Longrightarrow \quad W = \frac{3}{4} M_p$$

$$M_p = 355 \cdot 10^6 \cdot 659 \cdot 10^{-3} = 233.9 \text{ kNm}$$

$$\Longrightarrow \quad W = 175.45 \text{ kN}$$



iii)



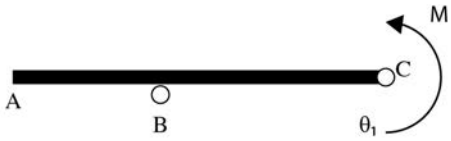
$$M_p - W \cdot 4 + V \cdot 8 = 0 \quad \Longrightarrow \quad V = \frac{M_p}{4}$$

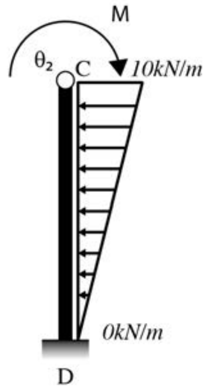
$$= 58.5$$

(b) (i)

Students must make the frame determinate. Addition of a pin at *C* is the method given in the question text. There is no horizontal reaction at *E* therefore:

$$H_D = \frac{10(10)}{2} = 50kN$$





$$\theta_1 = \theta_2$$

$$\theta_1 = \frac{ML}{3EI}$$

$$\theta_2 = \left[\frac{dv}{dx} @ x = L\right] - \frac{ML}{EI}$$

$$\frac{dv}{dx} = \frac{wL^3}{24} - \frac{wL^3}{4} + \frac{wL^3}{3} = \frac{wL^3}{8EI}$$

$$\theta_2 = \frac{wL^3}{8EI} - \frac{ML}{EI}$$

$$M = 93.75kNm$$

Therefore vertical reactions (and thus maximum axial force in the frame) are:

$$V_E = 93.75 \div 10 = 9.375kN$$

$$V_D = -9.375kN$$