EGT1
ENGINEERING TRIPOS PART IB

Wednesday 6 June $2018 \quad 2$ to 4.10

## Paper 5

## ELECTRICAL ENGINEERING

Answer not more than four questions.
Answer not more than two questions from any one section and not more than one question from each of the other two sections.

All questions carry the same number of marks.
The approximate percentage of marks allocated to each part of a question is indicated in the right margin.

Answers to questions in each section should be tied together and handed in separately

Write your candidate number not your name on the cover sheet.
STATIONERY REQUIREMENTS
Single-sided script paper

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

## SECTION A

## Answer not more than two questions from this section

1 The circuit in Fig. 1 is the input stage for an operational amplifier. The circuit is based on a pair of field effect transistors (FETs) $T_{1}$ and $T_{2}$ in the long-tailed pair configuration.
(a) Explain why the FET input circuit might be preferred to the bipolar transistor version. Why is the voltage at point $P$ in the circuit critical to the performance of the circuit?
(b) Using the small signal model for the FET, derive an expression for the common mode rejection ratio (CMRR) of the circuit, assuming $r_{\mathrm{d}}$ can be neglected and the FETs have identical $g_{\mathrm{m}}$. State any assumptions made.

The expression for the CMRR depends greatly on the value of the resistor $R \mathrm{~s}$, which also sets the bias current for the two FETs. The circuit shown in Fig. 2 is often used to replace $R_{\mathrm{S}}$ in the circuit of Fig. 1.
(c) Show and briefly explain how the circuit in Fig. 2 allows a suitable bias voltage to be set at point $P$ in Fig.1. What assumptions are made about the transistors $Q_{1}$ and $Q_{2}$ ? Assume $V_{\text {be }}=0.7 \mathrm{~V}$.
(d) Sketch the small signal model for the circuit in Fig. 2 and calculate the output resistance $R_{0}$ at point $K$. The transistors have small signal parameters $h_{\mathrm{iel}}, h_{\mathrm{ie} 2}, h_{\mathrm{oe} 1}, h_{\mathrm{oe} 2}$, $h_{\mathrm{fe} 1}$ and $h_{\mathrm{fe} 2}$, while $h_{\mathrm{re}}$ can be neglected for both transistors.
(e) Why is the circuit in Fig. 2 preferred over the Zener diode current sink when being used in an operational amplifier?

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Fig. 1


Fig. 2

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2 An ideal operational amplifier (OpAmp) is assumed to have infinite open-loop gain. This is not the case in a real OpAmp as the open-loop gain will reduce as a function of frequency and is often controlled deliberately by gain compensation.
(a) Sketch a graph showing how gain compensation is set in an OpAmp with a d.c. gain of $A_{0}$ and a gain compensation frequency cut-off of 100 Hz . Show the equation for this gain characteristic $A(\omega)$. What is the maximum closed-loop gain at 100 kHz ?
(b) The circuit shown in Fig. 3 is a negative impedance convertor (NIC) operating on an impedance $Z$, based on an OpAmp. Derive an expression for the input impedance of the NIC assuming the OpAmp is ideal.
(c) A pair of NICs can be used to create a gyrator as shown by the circuit in Fig. 4, where each NIC is described by the contents of the dashed box in Fig. 3.
Show that $Z_{\text {in }}=R^{2} / Z$ in Fig. 4.
(d) A timing oscillator requires a frequency of 10 mHz and is to be generated by an $L C$ resonant circuit. What value of inductance $L$ is required if a $100 \mu \mathrm{~F}$ capacitor is available? What value of $R$ would be required to simulate the inductor using a NIC assuming the same capacitor was available?

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Fig. 3


Fig. 4

## SECTION B

Answer not more than two questions from this section
3 (a) A balanced star-connected three-phase voltage supply is connected to a balanced star-connected load with impedance $Z_{\text {ph }}$ at each phase. Derive, using Kirchhoff's laws, the impedance at each phase of an equivalent delta-connected load, stating your answer in terms of $Z_{\text {ph }}$.
(b) The star point of the supply and the star-connected load are connected with a conductor. Explain if there is any current flow through the conductor.
(c) A balanced three-phase supply provides a constant line voltage of 11 kV at 50 Hz . This is connected to a balanced star-connected load and a balanced delta-connected load. Each phase of the star-connected load consists of a $500 \Omega$ resistor in series with a 4 H inductor. Each phase of the delta-connected load consists of a $1 \mathrm{k} \Omega$ resistor.
(i) Find a single star-connected load that is equivalent to the two loads, i.e. when connected to the supply in place of the two loads the line currents from the supply remain unchanged.
(ii) Use your answer from (i) to find the line current drawn from the supply and the total real and reactive power provided.
(iii) An unbalanced star-connected load is now additionally connected to the supply. The three phases of the load consist of resistors given by $Z_{A}=1.5 \mathrm{k} \Omega, \quad Z_{B}$ $=1.0 \mathrm{k} \Omega$ and $Z_{\mathrm{C}}=1.2 \mathrm{k} \Omega$. There is also a conductor between the star point of the source and load. Now find the real and reactive power provided by the supply. Explain if your answer would be different if the conductor between the star points was not present.

4 (a) Draw a phasor diagram for a synchronous generator where the following electrical quantities are illustrated: excitation voltage $E$, voltage $V_{g}$ at the machine terminals, synchronous reactance $X_{\mathrm{s}}$.
(b) A synchronous generator is connected to an infinite bus.
(i) What is an infinite bus?
(ii) Explain how the phasor diagram changes when the torque of the prime mover increases with the excitation voltage remaining unchanged.
(iii) Explain by means of a phasor diagram how a synchronous generator can control reactive power. Discuss its advantages relative to power factor correction using capacitors.
(c) A synchronous star-connected generator with synchronous reactance $X_{\mathrm{S}}=1.1 \Omega$, is connected to a 50 Hz infinite bus with line voltage 22 kV .
(i) If the prime mover is set to 400 MW and the power factor at the terminals of the machine is 0.9 lagging, find the excitation voltage.
(ii) If the excitation voltage is the same as in part (i), find the maximum real power that can be provided by the generator, and the corresponding reactive power when this real power is delivered.

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5 (a) Explain why an increased connectivity is important in a power system. Explain also the relative merits of gas turbines versus coal-fired power stations.
(b) Describe three types of faults that can occur in transmission systems.
(c) A 100 MVA 11 kV generator with a synchronous reactance of $X_{\mathrm{s}}=0.1$ pu supplies, via a 132 kV transmission line, a 22 kV bus which has an equivalent 60 MW load with power factor 0.9 lagging. The transmission line has an impedance of $8+\mathrm{j} 40 \Omega$. The generator is connected to the transmission line via an $11 \mathrm{kV} / 132 \mathrm{kV}$ transformer with rating 150 MVA and reactance of 0.2 pu . The 22 kV bus is connected to the transmission line via a $132 \mathrm{kV} / 22 \mathrm{kV}$ transformer with rating 200 MVA and reactance of 0.1 pu .
(i) Find the real and reactive power supplied by the generator.
(ii) A symmetrical three-phase fault to earth occurs at the 22 kV bus. Assuming that the excitation voltage of the generator is 1 pu , find the fault current (measured in Ampere) in the transmission line. Find also an appropriate rating for a circuit breaker to be placed between the 22 kV bus and the load. Suggest a way the fault current could be decreased.

## SECTION C

Answer not more than two questions from this section
6 (a) A plane polarised electromagnetic wave has an rms electric field given by:

$$
E_{y}=E_{0} e^{j(\omega t-\beta z)}
$$

Obtain an expression for the corresponding magnetic field $H(z, t)$ and for the average power density in $\mathrm{Wm}^{-2}$ carried by the wave in free space.
(b) A loop-antenna orientated in the $y-z$ plane having a diameter $a$ is immersed in the wave. Obtain an expression for the emf induced in the loop.
(c) The Sandy Heath transmitter produces a $1,000 \mathrm{~kW}$ signal. BBC 1 is transmitted on channel 31 which has a carrier wave frequency of 554 MHz . Channel 4 is transmitted on channel 21 which has a carrier wave frequency of 474 MHz . Calculate the ratio of the power of the BBC1 and the Channel 4 signal in order for identical emfs to be induced in a loop antenna.
(d) A loop-antenna with a diameter of 150 mm is connected to a receiver, which requires a minimum power $P_{\max }=4 \times 10^{-9} \mathrm{~W}$ for adequate reception. The receiver has an input resistance of $R_{\mathrm{in}}=75 \Omega$ and is power-matched to the transmitter. Assuming that the transmitter at Sandy Heath produces an isotropic signal with a gain of 2, calculate the maximum distance the receiver can be placed from Sandy Heath, assuming that the antenna is perfectly aligned and that 1 kW of the BBC 1 signal is being watched.

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7 (a) Figure 5 shows the equivalent circuit for an infinitesimally short length, $\delta z$ of a lossless transmission line which has inductance and capacitance of $L$ and $C$ per unit length. By considering the voltage across the inductor and the current through the capacitor, derive the Telegrapher's equations. Then, using these two equations eliminate $I(x, t)$ and derive the second partial differential equation for $V(x, t)$ :

$$
L C \frac{\partial^{2} V}{\partial t^{2}}=\frac{\partial^{2} V}{\partial x^{2}}
$$

(b) Show that the equation derived in part (a) can be satisfied by an equation of the form:

$$
V(x, t)=f_{+}(x-v t)+f_{-}(x+v t)
$$

where $f_{+}$and $f_{-}$are forward and backward travelling functions and $v$ represents the velocity of the wave.


Fig. 5

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(c) Starting from the expressions for the forward and backward voltage and current waves:

$$
\begin{aligned}
& V(x, t)=\left(V_{F} e^{-j \beta x}+V_{B} e^{j \beta x}\right) e^{j \omega t} \\
& I(x, t)=\left(I_{F} e^{-j \beta x}+I_{B} e^{j \beta x}\right) e^{j \omega t}
\end{aligned}
$$

show that the impedance of a line, of characteristic impedance $Z_{0}$, which is terminated by a line of impedance $Z_{\mathrm{L}}$ at a distance $d$ from the load is given by:

$$
Z(x)=Z_{0} \frac{Z_{L} \cos (\beta x)-j Z_{0} \sin (\beta x)}{-j Z_{L} \sin (\beta x)+Z_{0} \cos (\beta x)}
$$

[Hint: use Euler's theorem and the expressions for the exponential and the voltage reflection coefficient given in the databook].
(d) A transmission line of characteristic impedance $Z_{0}$ carries a signal of wavelength $\lambda$ and is connected to a load $Z_{\mathrm{L}}$ at its end. The joining section is $\lambda / 4$ long and has an impedance of:

$$
Z_{b}=\frac{Z_{0}^{2}}{Z_{L}}
$$

Sketch the impedance along the length of the joining section and explain the significance of this result.

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