Version CYB/5

EGT1
ENGINEERING TRIPOS PART IB

Tuesday 5 June 2018 9 to 11.40
9 to 10.40 Foreign Language Option

Paper 8

SELECTED TOPICS

*Answer **one** question from Section A. In addition:*

*If you are not taking the Foreign Language option, answer **four** questions, taken from only two of sections B–H. Not more than **two** questions from each section may be answered.*

*If you are taking the Foreign Language option, answer **two** questions from one of Sections B–H.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

Answers to questions in each section should be tied together and handed in separately.

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A: *Introductory Business Economics*

Answer not more than ***one*** *question from this section.*

1 (a) Suppose the market for beer is perfectly competitive. Supply is given by the function $Q^s = 3 + p$, where p denotes price. Demand is given by the function $Q^d = 9 - p$. Compute the equilibrium price and quantity, as well as *consumer surplus*. [6]

(b) Define and explain the different types of price discrimination. [6]

(c) Describe and compare the Cournot and the Bertrand models of oligopolistic competition. [6]

(d) Illustrate the fundamental principles of *Keynesian consumption theory*. [7]

2 (a) Consider a monopolistic supplier whose total production costs are described by the function $C(q) = 4 + q^2$, where q denotes the monopolist's quantity. Suppose that market demand for the monopolist's product is given by $q = 12 - \frac{p}{2}$, where p denotes price. Find the monopolist's profit-maximising price and quantity, and compute the *deadweight loss*. [6]

(b) Illustrate the Sweezy kinked-demand model of oligopoly. [6]

(c) Discuss the importance of the concept of Game Theory in making economic decisions by firms and explain the concept of the Prisoner's Dilemma. [6]

(d) Using either the Life Cycle Model or the Permanent Income Model, explain the potential impact of a temporary increase in income tax on the level of aggregate demand. [7]

SECTION B: ***Civil and Structural Engineering***

Answer not more than ***two*** *questions from this section.*

3 Figure 1 shows a symmetrical fabric-formed reinforced concrete beam, which has overhangs at each end. At the ultimate limit state, the beam carries a factored uniformly-distributed load of 30 kN m^{-1} all along its length, which includes all permanent and variable actions. In addition, a single factored concentrated load of 30 kN could be placed at any point along the length of the beam, and in the questions below it should be positioned so as to cause the most adverse effect in each case. The beam has a variable profile, but only three cross sections are of interest along the beam, namely at A, B and C, as shown in Fig. 1. Although the beam is cast in fabric, the cross sections at these points may be approximated by the rectangular sections shown in Fig. 2. The steel reinforcement near the top of the beam is uniform along the full length of the beam, while the reinforcement required to resist sagging bending is curved, as shown. The distance from the centre of all longitudinal reinforcing bars to the closest concrete surface is 50 mm. Assume a concrete class C30/37.

(a) Design the required longitudinal reinforcement over the supports (Cross Section A), choosing specific bar sizes for a singly-reinforced situation. [6]

(b) Design the required longitudinal reinforcement in sagging at midspan (Cross Section C), choosing specific bar sizes for a doubly-reinforced situation. [10]

(c) Is the shallowest section (Cross Section B) adequate for shear resistance, without the need to add shear reinforcement? Justify your answer. [6]

(d) Describe how shear reinforcement might be provided in a practical manner were it necessary. [3]

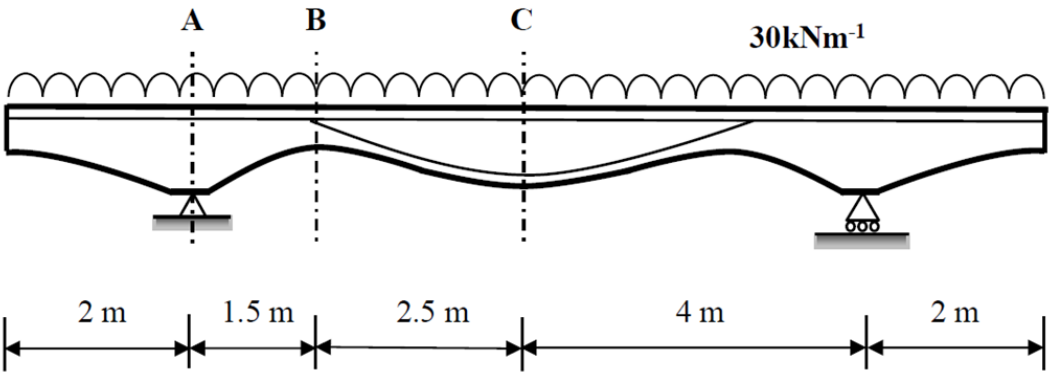
A
B
C
30kNm-1
2 m
1.5 m
2.5 m
4 m
2 m

Fig. 1

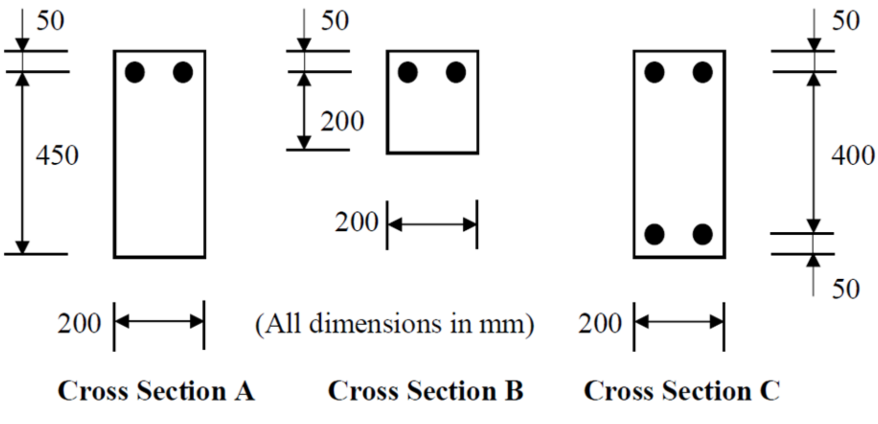
50
450
200
50
200
200
50
400
50
200
(All dimensions in mm)
Cross Section A
Cross Section B
Cross Section C

Fig. 2

4 A 4 m deep excavation in sand is supported by a cantilevered sheet pile wall (Fig. 3a). The specific gravity of the sand is $G_s = 2.75$ and it has a void ratio $e = 0.8$. The critical state friction angle of the sand is $\varphi' = 33°$. It is assumed that the wall moves sufficiently to mobilise full active pressure on the retained side, while the lateral earth pressure coefficient K_{mob} on the excavation side is considered to be a constant value.

(a) Compute the unit weight of the sand in dry and saturated conditions. [2]

(b) Assuming that the mobilised friction at the interface between the sheet pile wall and the sand on the passive side is $\delta = \varphi'/3$, compute the active and passive lateral earth pressure coefficients using Rankine's and Lancellotta's static solutions, respectively:

$$K_A = \frac{1 - \sin\varphi'}{1 + \sin\varphi'}$$

$$K_P = \frac{\cos\delta}{1 - \sin\varphi'}\left[\cos\delta + \sqrt{\sin^2\varphi' - \sin^2\delta}\right] e^{2\theta\tan\varphi'}$$

where

$$2\theta = \sin^{-1}\left[\frac{\sin\delta}{\sin\varphi'}\right] + \delta.$$

[3]

(c) Compute the depth of embedment, d, required to obtain $K_P/K_{mob} = 1.7$, and the maximum bending moment in the wall for this embedment depth. [10]

(d) If the water table is at the dredge level (Fig. 3b), compute the depth of embedment required to obtain the same ratio K_P/K_{mob} as in part (c) and the value of the maximum bending moment in the wall. [10]

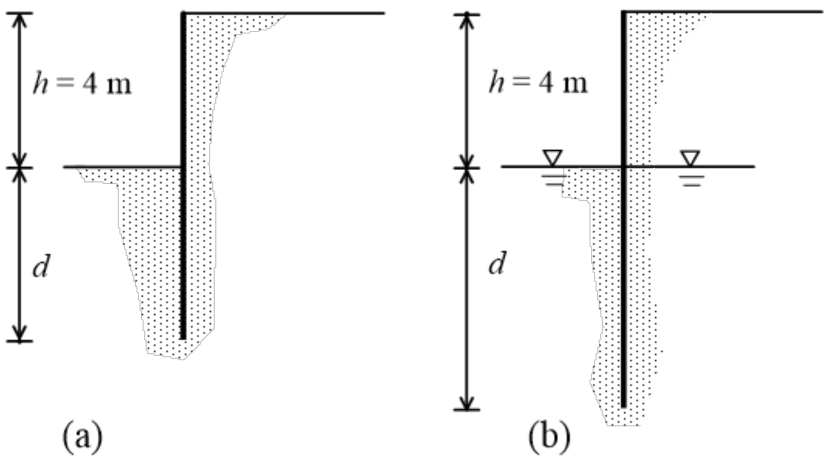

Fig. 3

 (TURN OVER

5 (a) A sample of dry sand with a weight $W_0 = 750$ g and an initial volume $V_0 = 0.5$ l, undergoes an increment of vertical stress $\Delta\sigma_v = 200$ kPa, and experiences a reduction of volume of 1%. An identical sample of sand, under vibration, experiences a reduction of volume of 10%. The specific gravity of the sand is $G_S = 2.65$.

(i) Determine the initial void ratio, e_0, and the dry unit weight, γ_{d0}, of the two samples;

(ii) Calculate the final void ratios, e_1 and e_2, and the dry unit weights, γ_{d1} and γ_{d2}, of the two samples.

[2]

(b) A soil sample with specific gravity $G_S = 2.75$ has an initial water content $w = 25\%$, and a void ratio $e = 0.86$. Compute the degree of saturation, S_r, and the bulk unit weight, γ, of the soil. [3]

(c) A fully saturated silty clay deposit has the following properties: specific gravity, $G_S = 2.70$, void ratio, $e = 1.20$, and friction angle $\varphi' = 27°$. The water table is 5 m below ground level (Fig. 4a).

(i) Compute the total and effective vertical and horizontal stresses at depths of 30 m and 50 m below ground level (points A and B in Fig. 4a). Assume that the silt is normally consolidated and that the soil is fully saturated to the surface.

(ii) If 25 m of clay are eroded (Fig. 4b), assuming that after erosion the ground water table is at the surface, what are the total and effective vertical and horizontal stresses at the same depths as in (i)? Draw qualitatively the variation of OCR and K_0 with depth.

[7]

(d) You are undertaking a feasibility study for a metro line, with bored tunnels crossing the historical centre of a city. The ground conditions can be summarised as 5 m of coarse-grained *made ground* followed by clay with undrained shear strength $s_u = 80$ kPa and unit weight $\gamma = 20$ kN m^{-3}.

(i) Define the stability ratio in terms of vertical total stress, tunnel support pressure and undrained shear strength of the clay, and explain its significance in the context of tunnel construction. [2]

 (cont.

(ii) What would be suitable construction techniques for the bored tunnels if the axes of the tunnels were at depths of 10 m or 25 m? Illustrate your answers, where appropriate, by consideration of the stability ratio for the tunnels. [3]

(iii) Settlements are likely to be significant because of the presence of historical masonry buildings. Explain the significance of ground movements caused by bored tunnelling. Why might the buildings be damaged? Describe the difference between the effects of differential and total settlement on buildings. How does compensation grouting prevent damage? [8]

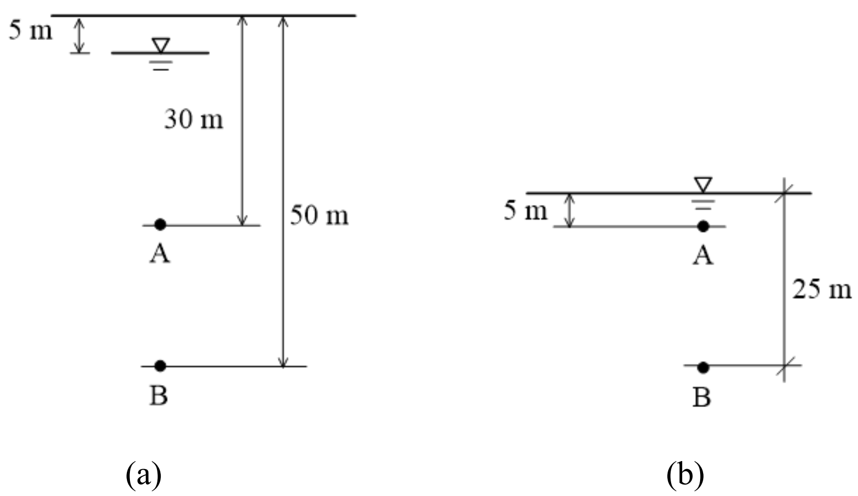

Fig. 4

 (TURN OVER

SECTION C: *Mechanics, Materials and Design*

*Answer not more than **two** questions from this section.*

6 (a) (i) Sketch the cross-section through a wind turbine blade and clearly label the relative wind speed V_{rel}, the angle of relative wind ϕ, the angle of attack α, the blade twist θ, and the lift and drag forces, F_L and F_D, acting on the blade. [5]

(ii) Mark on the sketch the axial and tangential velocities seen locally by the blade, having corrected for the slow down of air through the turbine using the axial induction factor a and the angular induction factor a'. Hence show that:

$$\tan\phi = \frac{(1-a)V_0}{(1+a')\omega r}$$

where V_0 is the initial axial velocity and ωr is the tangential velocity. [4]

(iii) Find expressions for F_N and F_T (the normal and tangential forces per unit length) as a function of the angle of relative wind ϕ, the lift force F_L and the drag force F_D. Normalise these forces to obtain expressions for the force coefficients C_N and C_T. [2]

(b) A horizontal axis wind turbine is designed with an upwind three-bladed rotor. For simplicity the blade is considered to be divided into two 25 m long sections, with the details given in Table 1.

Radial sections	r = 5 to 30 m	r = 30 to 55 m
Mid-point r	17.5 m	42.5 m
Chord c	4.5 m	3.6 m
Twist θ	20°	4.5°
Axial induction factor a	0.091	0.17
Angular induction factor a'	0.017	0.0024

Table 1

The wind turbine is designed to operate at a typical incident wind speed V_0 = 8 m s^{-1} and an angular velocity of 8 rpm. The lift and drag coefficients are given by:

$$C_L = 2\pi\alpha,\ C_D = 0.08 \text{ for } 0 < \alpha < 0.3 \text{ rad.}$$

(i) Show that the axial and angular induction factors given in the first column of Table 1 for the blade section with r from 5 to 30 m are consistent with Blade Element Momentum theory. [7]

(ii) Calculate the flapwise bending moment at the blade root (i.e. at r = 5 m) and the *total* axial thrust on the turbine. [4]

(iii) Calculate the tangential bending moment at the blade root (i.e. at r = 5 m) and the *total* torque at the turbine hub. [3]

Note that:

$$a = \left[\frac{4\sin^2\phi}{\sigma C_N} + 1\right]^{-1}, \quad a' = \left[\frac{4\sin\phi\cos\phi}{\sigma C_T} - 1\right]^{-1}, \quad \sigma = \frac{cB}{2\pi r}$$

where B is the number of blades, and σ is the rotor solidity. The density of air can be assumed to be 1.2 kg m^{-3}.

(TURN OVER

7 (a) Sketch a typical power versus wind speed characteristic for a wind turbine, labelling and defining the *cut-in*, *rated* and *stall* wind speeds. [3]

(b) Cut-in and stall wind speeds for a wind turbine are to be 2.5 m s^{-1} and 20 m s^{-1} respectively, and the turbine is to be designed to produce 1.8 MW of power at a wind speed of 12 m s^{-1}. Determine the turbine diameter required to meet this specification assuming a power coefficient $C_p = 0.35$. [2]

(c) Simplified wind data at the turbine location are given in Table 2. The turbine is to operate at variable speed between cut-in and rated wind speeds in order to optimise the tip-speed ratio and power coefficient to 8 and 0.35, respectively. Engineers are considering whether to choose a rated wind speed of 12 m s^{-1} or 16 m s^{-1} for the turbine of part (b). For both rated wind speeds:

(i) find the total annual energy produced, the capacity factor and the income from that energy assuming that it can be sold to the grid at £30 per MWh; [3]

(ii) find the payback period assuming an initial capital investment of £1,000 per kW of rated output power. You may ignore operation and maintenance costs. [3]

(iii) Use these calculations to choose an appropriate rated wind speed. [2]

Wind speed, v (m s^{-1})	Number of days per year
< 2	10
7	180
12	105
16	60
>21	10

Table 2

The following may be quoted without proof: $P = \frac{1}{2} C_p \rho A v^3$, $\lambda = \omega R / v$ and take ρ to be 1.23 kg m^{-3}.

(d) A horizontal-axis wind turbine is shown in Fig. 5. It is operating in air of density ρ and the wind speed is V. The swept area of the rotor is A and the axial induction factor is a as shown in the figure.

(i) Find expressions for the cross-sectional areas A_1 and A_2 shown in the figure in terms of A and a. [3]

(ii) By considering conservation of energy for the control volume shown in the figure, find an expression for the power output of the turbine in terms of ρ, V, A and a. [6]

(iii) Determine the efficiency of the turbine with $a = 0.4$ and comment on its value compared with the Betz limit of 59%. [3]

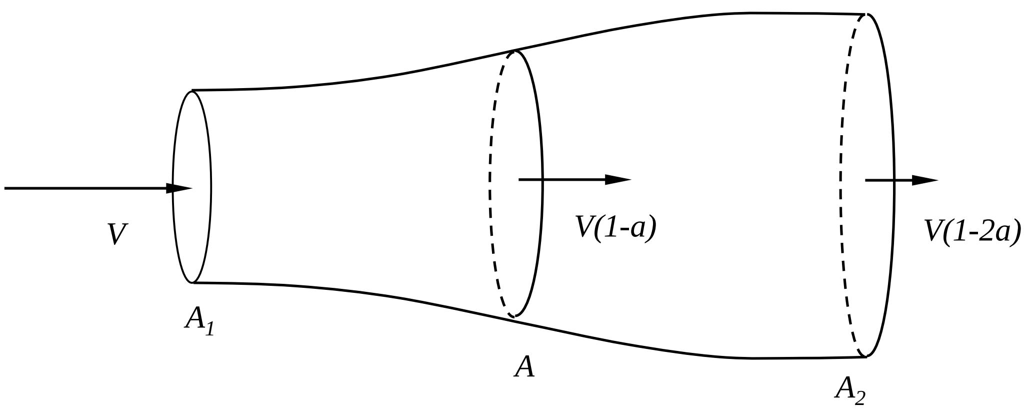

Fig. 5

8 (a) Describe how spectral analysis can be used to characterise wind loading and structural loading in wind turbines. [4]

(b) Describe likely causes of periodic loading on wind turbine blades. [4]

(c) Describe likely wind turbine blade failure mechanisms and outline ways that these can be avoided, including relevant design calculations. [4]

(d) The design lifetime of a composite wind turbine blade is 20 years. The turbine has 10^7 cycles of loading per year, with the stress range S following a probability distribution function ϕ given by

$$\phi(S) = \frac{1}{\overline{S}} \exp\left(\frac{-S}{\overline{S}}\right)$$

where $\overline{S}$ is the mean stress range. The number of cycles to failure N is related to the stress range S by the following expression

$$N = \left(\frac{S}{S_0}\right)^{-M}$$

with material constants $M = 10$ and $S_0 = 150$ MPa. Find the value of $\overline{S}$ required to meet the design life. Note that, for integer z,

$$\int_0^\infty t^z e^{-t} dt = z!$$

[10]

(e) In practice, what would be the main sources of uncertainty in your estimate for the allowable value of $\overline{S}$ in part (d)?

[3]

SECTION D: ***Aerothermal Engineering***

Answer not more than ***two*** *questions from this section.*

9 (a) Define the work coefficient, ψ, for a compressor or a turbine. Explain why the allowable work coefficient is much lower for a compressor stage than for a turbine stage. [5]

(b) A 12-stage compressor has a stagnation pressure ratio of 15. The compressor has an overall isentropic efficiency of 88%. All compressor stages are designed to have a mean radius of 0.55 m. The inlet stagnation temperature is 288 K. Calculate the design rotational speed of the compressor if all stages have a design stage work coefficient $\psi = 0.4$. [5]

(c) The multistage compressor in part (b) is used as the compressor in a single shaft turbojet engine. The turbojet is operated in a static testbed with an inlet stagnation temperature and pressure to the engine of 288 K and 1 bar, respectively. The turbine entry temperature is 1600 K. Assume that there is no drop in pressure across the combustor. If the turbine isentropic efficiency is 88% calculate the jet velocity. Neglect any losses in the propulsion nozzle. Assume that the mass flow rate of fuel is negligible. [10]

(d) The turbine has the same mean radius as the compressor. How many stages of turbine would you advise the designer to use? Justify your answer. What practicalities should the designer consider when choosing the number of turbine stages? [5]

Assume that the working fluid is air throughout with $\gamma = 1.4$ and $c_p = 1005$ J kg^{-1} K^{-1}.

10 (a) Stating all assumptions, show that the propulsive efficiency η_p can be written

$$\eta_p = \frac{2V}{V + V_j}$$

where V is the flight velocity and V_j is the propulsive jet velocity. Comment on the historic trend in bypass ratio and the factors which limit its value. [7]

(b) An aircraft powered by simple turbojet engines is flying at an altitude where the ambient pressure is p = 26 kPa. The turbojets have a turbine inlet stagnation pressure of 5 bar and a stagnation temperature of 1500 K. The turbine has an isentropic efficiency of 90%. The compressor requires a specific work input of 250 kJ kg^{-1}. Neglect any losses in the propulsion nozzle. Calculate the jet velocity at the exit of the propulsion nozzle. State any assumptions made. [8]

(c) At the same altitude, the aircraft is flying at a Mach number of 0.89 and at an ambient temperature of $T = 222$ K. Calculate the propulsive efficiency and the net thrust per unit mass flow of the turbojet. [4]

(d) With the aim of reducing noise, the turbojet engines on the aircraft are replaced with turbo-fan engines. The inlet area of the fan of the new engine is three times larger than the inlet area of the compressor of the old engine. Assume that the Mach number at the inlet plane of the fan in the new engine is the same as the Mach number at the inlet plane of the compressor in the old engine and that the net thrust is unchanged. Calculate the change in propulsive efficiency. State any assumptions made. [6]

Assume that the working fluid is air throughout with $\gamma = 1.4$ and $c_p = 1005$ J kg^{-1} K^{-1}.

11 (a) The Breguet range equation for the distance, s, between the start and end of cruise is given by

$$s = \frac{V}{g}\frac{L/D}{sfc}\ln\left[\frac{W_{start}}{W_{end}}\right]$$

where g is the acceleration due to gravity, V is the flight speed, sfc is the thrust specific fuel consumption, L/D is the aircraft lift-to-drag ratio and W_{start} and W_{end} are the total aircraft weights at the start and end of cruise respectively. Use this to derive a relationship for the fuel burn per kilometre payload $W_f/(s \times W_p)$, where W_f is the weight of the fuel and W_p is the weight of the payload. Comment on the meaning of this fuel burn parameter and comment on how a designer can make use of the parameter to improve the performance of an aircraft. [8]

(b) If the empty weight of an aircraft is 360 tonne, the weight of the payload is 70 tonne, the distance, s, between the start and end of cruise is 15,000 km, the aircraft lift-to-drag ratio $L/D = 20$, the flight speed $V = 260$ m s^{-1} and the thrust-specific fuel consumption $sfc = 0.016 \times 10^{-3}$ kg s^{-1} N^{-1} , calculate the fuel burn per kilometre payload $W_f/(s \times W_p)$. [5]

(c) The emission index, EI, is the mass of pollutant divided by the mass of fuel burned. Calculate the EI for carbon dioxide assuming that the combustion is complete. Assume that the fuel is jet fuel with a chemical formula C_8H_{18}. [4]

(d) Calculate the CO_2 produced per passenger kilometre. Assume the mass of each passenger with luggage is 100 kg. Comment on how this CO_2 figure compares with other types of transport. [4]

(e) The same aircraft is flown on a new route which has a distance, s, between the start and end of cruise of 6,000 km. Assume all other flight parameters are unchanged. Calculate the CO_2 produced per passenger kilometre on the new route. Comment on this figure compared to the figure in part (d). [4]

SECTION E: *Electrical Engineering*

Answer not more than **two** *questions from this section.*

12 (a) (i) What is meant by the term *mobility of carriers* in a semiconductor?

(ii) The electron mobility in silicon is 1000 cm^2/Vs. The effective mass of electrons in silicon is 0.33 times the electron mass in vacuum. Calculate the mean free collision time during scattering of electrons.

[5]

(b) Crystalline silicon has an electron and hole mobility of 1000 cm^2/Vs and 500 cm^2/Vs, respectively. The free carrier concentration in intrinsic silicon is 1×10^{10} cm^{-3}. This intrinsic silicon is then doped with $N_D = 1 \times 10^{15}$ cm^{-3} donor dopants. Answer the following:

(i) What is the resistivity of the intrinsic silicon?

(ii) What is the hole concentration (in cm^{-3}) in the silicon after doping?

(iii) What is the resistivity of the doped silicon?

[8]

(c) The intrinsic carrier concentration at temperature, $T = 300$ K is 1×10^{10} cm^{-3}. The band gap of silicon depends on the temperature according to

$$E_g = \left[1.17 - 5 \times 10^{-4} \left(\frac{T^2}{T + 636}\right)\right] \text{ eV}.$$

Find the electron concentration in the conduction band of intrinsic silicon at $T = 77$ K. [7]

(d) Consider the case of a particle in the finite potential well shown in Fig. 6. The total energy of the particle is less than V_0.

(i) Write down the time independent Schrodinger's equation for the particle in region B and region C.

(ii) Very briefly, explain the term *tunnelling* in this context.

[5]

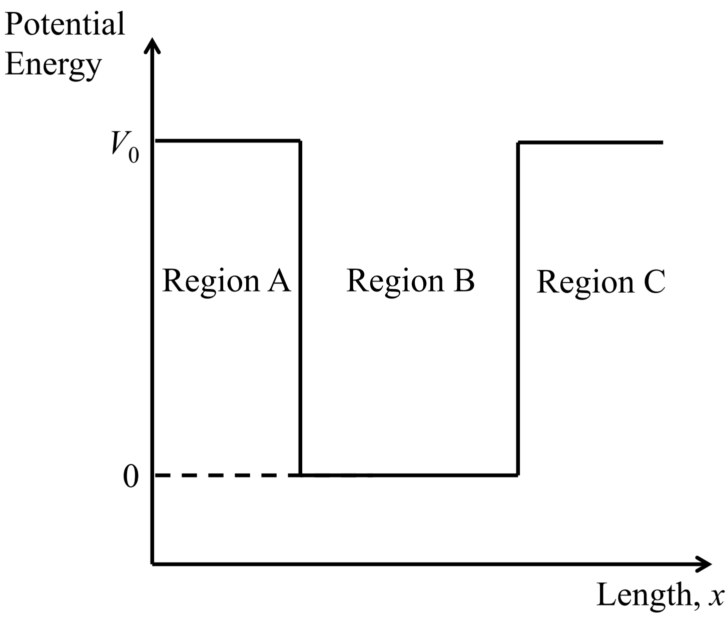

Fig. 6

13 (a) (i) With the help of a simple circuit diagram, explain the operation of a pixel driver circuit for active matrix organic light emitting diode displays.

(ii) A thin film transistor (TFT) with channel width 10 μm and channel length 5 μm is used as the current driver for an active matrix light emitting diode display. The field effect mobility is 1 cm^2/Vs, the gate oxide capacitance per unit area is 20 nF/cm^2 and the threshold voltage is 2 V. For a gate voltage applied to the driver of 5 V, calculate the current through the light emitting diode.

[9]

(b) (i) Briefly explain the difference in the design of a MOSFET and a MESFET.

(ii) Briefly explain the difference between an enhancement mode and depletion mode MOSFET.

[4]

(c) Upon the application of a suitable gate voltage, the semiconductor at a semiconductor-insulator interface of a MOSFET is depleted. The depletion region has a space charge of donor dopant ions of concentration N per unit volume and has a width of W. Assuming no field exists outside this depletion region, derive an expression for the electric field in the semiconductor as a function of the distance from the interface. [6]

(d) (i) What is meant by *constant field scaling* in the context of MOSFETs?

(ii) How does the area of the MOSFET scale if constant field scaling, with a scaling factor k, is implemented?

(iii) Why are dielectrics with high relative permittivity desirable?

[6]

14 (a) (i) X-ray diffraction is commonly used to analyse crystals. Derive the expression for the Bragg condition for constructive interference.

(ii) X-rays of wavelength 4×10^{-11} m undergo a first-order reflection at a glancing angle of 5° from a crystal. Find the spacing of the atomic planes in the crystal.

[7]

(b) (i) With the aid of diagrams, explain the basic process steps to pattern and dry etch silicon oxide using UV lithography.

(ii) What is the wavelength of an electron moving at 5×10^6 m s^{-1}?

(iii) An electron beam system of 30 keV is used for lithography. What is the approximate minimum feature size that one can pattern with this system?

[8]

(c) Using a simple diagram, explain the design and technique to inductively write and read binary information on a magnetic tape. [4]

(d) (i) Briefly list and explain two advantages of GaN based integrated electronics.

(ii) Sketch the plot of drift velocity of electrons versus electric field for GaAs. Explain the features.

[6]

 (TURN OVER

SECTION F: *Information Engineering*

Answer not more than **two** *questions from this section.*

15 (a) The most common colour representation in images is *RGB*. Two alternative colour spaces are *YUV* and *HSV*.

(i) Describe briefly the *YUV* and *HSV* colour spaces and explain possible advantages of working in these alternative representations. [4]

(ii) One widely used linear transformation between *RGB* and *YUV* spaces is given by:

$$\begin{aligned} Y &= 0.3R + 0.6G + 0.1B \\ U &= \alpha(B - Y)/8 \\ V &= \beta(R - Y)/8 \end{aligned}$$

If, for a given image pixel, the chrominance components, U and V, are both zero, show that the pixel is grey with $R = G = B$. [2]

(iii) If the values of U and V in (ii) are to be kept within the range $[-0.5, 0.5]$, find the maximum integer values of α and β that achieve this. Assume the R, G, B values range from 0 to 1. [5]

(iv) Find the 3×3 matrix, A, which maps the vector $[R, G, B]^T$ onto the vector $[Y, U, V]^T$. [3]

(b) We often wish to *high-pass* filter an image, for example, to enhance or detect the edges of objects in the image. One way of forming a high-pass filter is to subtract a *low-pass* filtered signal from the unfiltered signal. In the frequency domain we can illustrate this on 1-D signals as

$$Y(\omega) = \alpha X(\omega) - \beta G(\omega) X(\omega)$$

where α and β are scalars, $X(\omega)$ is the spectrum of the unfiltered signal, $G(\omega)$ is the low-pass filter (unit gain at low frequencies and zero gain at high frequencies), and $Y(\omega)$ is the spectrum of the output.

(i) Write down the frequency response, $H(\omega)$, of this high-pass filter. [2]

(ii) If we choose $\alpha = 2$ and $\beta = 1$, what are the gains of $H(\omega)$ at high and low frequencies? Sketch the effects of such a filter on a step function (edge) and hence describe the result of applying this filter to an image. [5]

(iii) If instead we choose $\alpha = \beta$, describe the gain of $H(\omega)$ at high and low frequencies and the effects of filtering an image with this type of high-pass filter. [4]

16 (a) A greyscale image, $I(x,y)$, is to be smoothed by *low-pass* filtering with a Gaussian filter, $G(x,y)$, and differentiated as part of the edge detection process.

(i) Explain why smoothing is required and why the Gaussian is a suitable low-pass filter. [2]

(ii) Give an expression for computing the intensity of a smoothed pixel, $S(x,y)$, in terms of two discrete 1-D convolutions. [3]

(iii) Derive expressions for computing the spatial derivatives after smoothing. Include details of the two 1-D convolutions required. [3]

(b) Image features such as *blob-like* shapes can be localised in an image by filtering with a *band-pass* filter over multiple scales.

(i) What is meant by *band-pass filtering* and how is such filtering implemented in this application? [4]

(ii) Show how the blob-like shapes can be localised in an image and describe how to determine an appropriate scale and feature size. [4]

(c) Image features are to be extracted in a window around each *keypoint* (blob centre). These are then to be matched in different images and over different viewpoints.

(i) Show how the neighbourhood of each keypoint can be normalised to a 16×16 patch of pixels. [2]

(ii) The SIFT (Scale-Invariant Feature Transform) descriptor is used to describe the 16×16 patch of pixels. Describe the main steps in computing this descriptor and how it achieves its invariance to lighting, image and viewpoint changes. [4]

(iii) How are these descriptors used to find correspondences in images from different viewpoints? [3]

17 (a) A *convolutional neural network* is to be used to map an input, $\mathbf{x}$, into an output, $y(\mathbf{x})$ by convolution with multiple filters and passing these intermediate outputs through *activation* and *pooling* functions.

(i) Draw a sketch of a single *artificial neuron*. Identify the D inputs, $\mathbf{x}$, the weights, $\mathbf{w}$, and the single output, y. [2]

(ii) What is the role of convolution in this context? [2]

(iii) Give details of two typical activation functions that can be used to produce the output. What properties must they have? [2]

(iv) What is the role of a pooling function? Give an example of one that is commonly used in convolutional neural networks. [2]

(b) A convolutional neural network is to be used to determine the category of an object in small 32×32 RGB images by training using a labelled dataset of 60,000 images downloaded from the internet. The network weights are to be estimated by *supervised learning* from a training dataset.

(i) What is meant by supervised learning? [2]

(ii) Explain the roles of training, validation and test datasets. [3]

(iii) Show how a training set of input and desired output pairs can be used to learn the network weights. Include details of a suitable training criterion or *loss function* that should be minimised during training. [4]

(iv) How is the loss function minimised? Give details of the algorithm used and the use of any *hyper-parameters*. [3]

(c) Describe the anatomy of a typical convolutional neural network that can be used for the classification task in part (b), and identify the number and role of each layer. Include details of the number and filter sizes used. [5]

SECTION G: *Bioengineering*

Answer not more than **two** *questions from this section.*

18 (a) Describe the main limitation of *pit eyes* and the three main improvements that have evolved to improve on it. [4]

(b) A neuron's response, $r = a\,s + \varepsilon$, is a linear function of a normally distributed stimulus, $s \sim \mathcal{N}(\mu_\mathrm{s}, \sigma_\mathrm{s}^2)$ with mean μ_s and variance σ_s^2, where the neuron's sensitivity is $a > 0$ and it has normally distributed response variability $\varepsilon \sim \mathcal{N}(0, \sigma_\mathrm{r}^2)$ with variance σ_r^2. (Note that, for simplicity, we assume the neural response can take on negative as well as positive values.)

(i) Using the parameters defined above, provide the formula for the information about the stimulus encoded by the response of the neuron. [3]

(ii) The energy consumed by the cell when emitting response r is $\beta\,r^2$, where $\beta > 0$ is a positive constant. Using the parameters defined above, provide the formula for the average energy consumed by the cell. [3]

(iii) Express the optimal value of a as a function of the other parameters for *efficient coding* under energy constraints when the average energy consumed by the cell cannot be more than E_max. [4]

(c) With regard to efficient coding in colour vision, explain the following.

(i) What mathematical technique can be used, and how can it be used, to apply the theory of efficient coding to colour vision? [5]

(ii) What aspects of colour vision does efficient coding account for? [4]

(iii) What experiments can be used to demonstrate these aspects? [2]

19 (a) For both the *fundus camera* and the *scanning laser ophthalmoscope*, describe the way in which the imaging data are constrained to a particular depth in the eye, also noting what properties affect the depth resolution. [5]

(b) Figure 7 shows a lens, with diameter D and focal length f, which focusses a light beam through an aperture of diameter d in a plate of thickness t. The aperture is located at the focal point of the lens, and the radial extent $r(x)$ of the light is given by:

$$r^2(x) = r_0^2 \left[1 + \left(\frac{\lambda x}{\pi r_0^2} \right)^2 \right]$$

where x is the horizontal distance measured from the focal point, r_0 the radius at the focal point, and λ is the wavelength of light. All components are in air.

(i) Define the Numerical Aperture (NA) of the lens, and show that it can be related to r_0 by:

$$r_0 \approx \frac{\lambda}{\pi \mathrm{NA}}$$

[6]

(ii) What is the maximum value of λ, in terms of NA, t and d, for which all the light within the radius $r(x)$ will pass through the aperture? [6]

(iii) What is the minimum thickness of the plate t, in terms of NA and d, for which there is no value of λ at which all light within the radius $r(x)$ would pass through the aperture? [3]

(iv) Suggest reasonable numerical values for d and t when using this aperture in an optical imaging device with a lens of NA $= 0.5$. [3]

(v) What are the consequences of your answer to part (b)(ii) for the spectrum of light passing through such an aperture? [2]

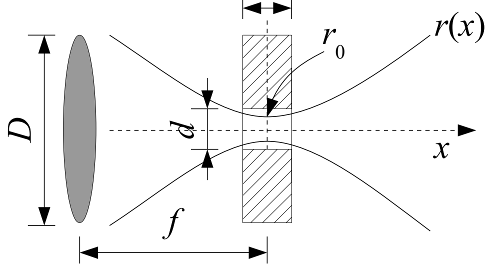

Fig. 7

20 (a) Compare the composition and structure of the *cornea* and *crystalline lens* tissues in the eye, detailing how their structure relates to their function. [8]

(b) Explain the individual contributions to visual focussing of the cornea and crystalline lens. [4]

(c) Describe how problems with the cornea and crystalline lens can interfere with vision and how these issues can be treated. [5]

(d) Figure 8 illustrates a standard linear material model, with parameters E_1, E_2 and η.

(i) How do the different elements of the model affect the response of the material to mechanical loading? Illustrate your answer with sketches of the response to different loading cases. [4]

(ii) Describe how the material properties E_1, E_2 and η for an eye lens could be measured. [4]

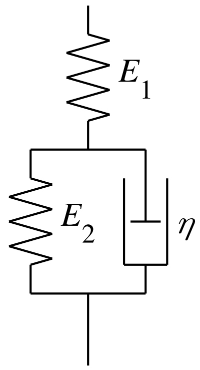

Fig. 8

SECTION H: *Manufacturing and Management*

Answer not more than **two** *questions from this section.*

21 (a) Describe what is meant by *technology push* and *market pull*. For each of these concepts, provide at least four examples. [6]

(b) Explain how *user observations* and *personas* could be used to understand the needs of potential buyers of a new type of bicycle lock. [9]

(c) Discuss the strengths and weaknesses of different forms of *prototyping*. Describe how each form of prototyping could be used in the development of a new type of bicycle lock. [10]

22 (a) Compare, using examples, the relative benefits of using business models based on:

(i) licensing of Intellectual Property Rights (IPR);

(ii) manufacturing and selling a product;

(iii) manufacturing and selling a product with consumables; and

(iv) selling a service.

[10]

(b) Discuss the differences between the following types of funding:

(i) grants;

(ii) debt; and

(iii) equity.

[6]

(c) Sketch a diagram that illustrates how different sources and types of funding are likely to be appropriate at different stages of the commercialisation of a new idea. [4]

(d) Describe the main elements of a business plan suitable for attracting investment from a Venture Capitalist (VC) for a business based upon the manufacturing and selling of a product with consumables. [5]

23 (a) Describe the three tests that an invention must satisfy in order for it to be considered patentable in the UK. [4]

(b) For a new invention, compare the advantages and disadvantages of:

(i) filing a patent; and

(ii) keeping the invention secret.

[6]

(c) Discuss the relationship between *product specification* and the design of a *manufacturing process* for that product. [10]

(d) Sketch a diagram to illustrate the trade-off between volume and variety for different production systems. Explain why it is so difficult to produce in high volume and high variety. [5]

END OF PAPER