EGT2
ENGINEERING TRIPOS PART IIA

Monday 23 April 2018 9.30 to 12.40

Module 3A1

FLUID MECHANICS I

Answer not more than ***five*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book
Attachments:

- Incompressible Flow Data Card (2 pages);
- Boundary Layer Theory Data Card (1 page);
- 3A1 Data Sheet for Applications to External Flows (2 pages).

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) The vorticity equation for an incompressible fluid is:

$$\frac{D\omega}{Dt} = (\omega \cdot \nabla)\mathbf{u} + \nu \nabla^2 \omega$$

Give the physical interpretation of each term in this equation. [20%]

(b) Consider an inviscid fluid containing two parallel and infinitely long line vortices, distance $2a$ apart, one with circulation Γ and the other with circulation $-\Gamma$. Far from the vortices, the fluid is stationary. Stating all your assumptions, write down the speed of this vortex pair. [10%]

(c) The vortex pair in part (b) approaches a flat wall that is perpendicular to its direction of travel. Calculate and sketch the paths traced by the individual vortices. [50%]

(d) Now consider an inviscid fluid containing a vortex ring with radius a, whose vorticity, ω, is spread uniformly over a vortex core of non-zero radius, as shown in Fig. 1. Far from the vortex ring, the fluid is stationary. The vortex ring approaches a flat stationary wall that is perpendicular to its direction of travel. Without further calculations, describe the motion of the vortex ring, paying particular attention to its circulation, Γ, and its vorticity, ω. [20%]

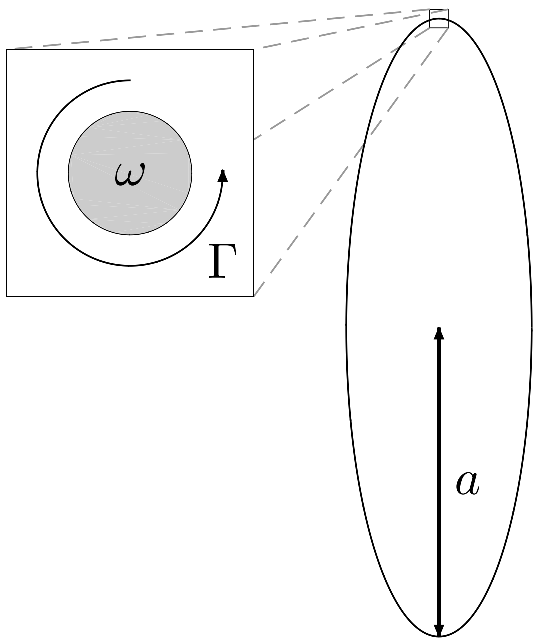

Fig. 1

2 An incompressible irrotational flow is taken to be the combination of a uniform flow with speed U, an x-wise doublet of strength $-2\pi a^2$ at $y = d$, and an x-wise doublet of strength $-2\pi a^2$ at $y = -d$, as shown in Fig. 2.

(a) Sketch the streamlines of this flow and explain why, for $y \ll d$, this is a reasonable model of the flow in a venturi. [20%]

(b) The entrance to the venturi is at $x \to -\infty$ and has half-height, h. The throat of the venturi is at $x = 0$ and has half-height t. Show that h and t are related by:

$$\frac{h}{t} = 1 + \frac{2a^2}{U(d^2 - t^2)}$$

and write down the volumetric flowrate through the venturi. [40%]

(c) Derive an expression for the velocity on the wall at the throat of the venturi $(x, y) = (0, t)$ in terms of U, a, t, and d. [10%]

(d) Calculate the velocity at $(x, y) = (0, t)$ that would be obtained by assuming that the velocity across the throat at $x = 0$ is uniform. Given that $t \ll d$ for the model to be valid, calculate how the error varies with a/d and t/d and comment on this result. [30%]

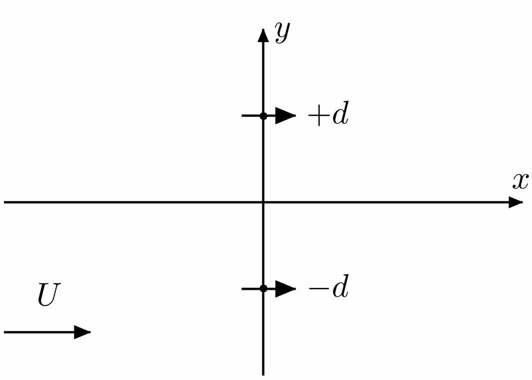

Fig. 2

3 The flapping wings of a micro air vehicle are to be modelled in two dimensions as two infinitely large plates hinged at the origin, as shown in Fig. 3. At time $t = 0$ the plates are adjacent at $\theta = 0$. At time $t > 0$, the left plate is at angle $\theta = \Omega t$, and the right plate is at angle $\theta = -\Omega t$. In this question cylindrical polar co-ordinates should be used, for which

$$\nabla\phi = \frac{\partial \phi}{\partial r}\mathbf{e_r} + \frac{1}{r}\frac{\partial \phi}{\partial \theta}\mathbf{e}_\theta \quad \text{and} \quad \nabla^2\phi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2}$$

(a) The flow between the plates is to be modelled as incompressible and irrotational. Show that the velocity potential

$$\phi = -\frac{\Omega r^2 \cos(2\theta)}{2\sin(2\Omega t)}$$

satisfies the conditions of irrotationality and incompressibility, as well as the boundary conditions on the plates. Why do these assumptions also require the fluid to be inviscid? [20%]

(b) Derive an expression for the slip velocity at the plates and the streamfunction, ψ, of the flow. Sketch the streamlines at time $t > 0$. [30%]

(c) Find the pressure on the wings as a function of r and t. Comment on what happens to the pressure as the angle between the wings, $2\Omega t$, approaches π. [30%]

(d) With the aid of sketches, describe qualitatively the flow that would occur if the wings had finite length and the fluid had small, but non-zero, viscosity. [20%]

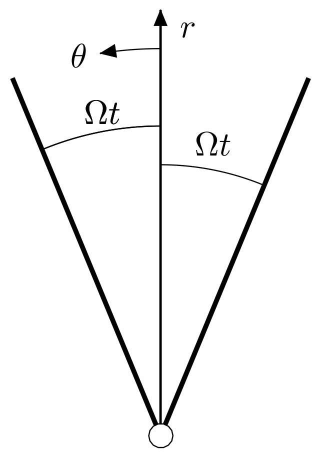

Fig. 3

4 When a jet forms parallel to a wall, as shown in Fig. 4, the jet origin is a source of momentum and the velocity profiles further downstream are self-similar.

(a) The jet spreads at uniform pressure and is thin so that the velocity $\mathbf{u}$ varies much more rapidly across the jet than along it. Starting from the Navier–Stokes equation

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u}$$

for an incompressible fluid with kinematic viscosity ν, apply the arguments of boundary layer theory to derive the approximate equation for the jet's velocity field (u, v):

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}$$

and state the boundary conditions for the x-velocity, u. [10%]

(b) Consider a similarity solution to this equation, with $u \propto x^a$, and the jet thickness $\delta \propto x^b$, where a and b are constants. Use the second equation in part (a) to deduce that $a + 2b = 1$. [20%]

(c) Assume the stream function to be of the form

$$\psi = F(x) f(\eta), \quad \text{where} \quad \eta = (1-b)\frac{y}{x^b}$$

Show that $\psi \propto x^{1-b}$. [15%]

(d) Now consider the stream function to be of the form $\psi = \nu x^{1-b} f(\eta)$.

(i) Calculate the two velocity components u and v from ψ. [15%]

(ii) Calculate the velocity derivatives $\partial u/\partial x$, $\partial u/\partial y$ and $\partial^2 u/\partial y^2$. [20%]

(iii) Deduce the equation for f and state the boundary conditions. [20%]

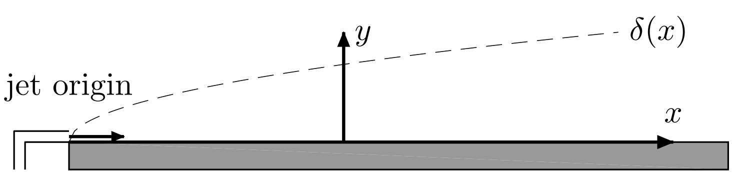

Fig. 4

5 Figure 5 shows a steady turbulent flow past a flat plate. The sharp edge of the plate is at $(x,y) = (0,0)$. The velocity just upstream of the sharp edge is uniform, $u = U =$ constant, and the angle of attack is zero. The boundary layer thickness is δ. A good approximation for the velocity profile at a downstream position, x, is given by Prandtl as

$$u = U\left(\frac{y}{\delta}\right)^{\frac{1}{7}} \text{ for } y < \delta\,, \text{ and } u = U \text{ for } y \geq \delta$$

(a) The above equation may be a good representation of the velocity profile of a turbulent boundary layer but is unphysical in at least one aspect. What is this? [10%]

(b) The control volume ABCD (the rectangle indicated by a wide broken line) in Fig. 5 encloses the whole boundary layer up to x. Calculate the mass flux on each edge of ABCD. The density of the fluid is ρ. [20%]

(c) Use balance of momentum in the control volume to determine the drag force $D(x)$ on the plate up to x and the friction drag coefficient $D_c = D(x)/(\frac{1}{2}\rho U^2 x)$. [30%]

(d) Prandtl also proposed the following relationship for the friction drag coefficient:

$$D_c \approx 0.05 Re_{\delta}^{-\frac{1}{4}}$$

Using this and the result in part (c) determine the boundary layer thickness δ as a function of Re_x, noting that $Re_\delta = U\delta/\nu$ and $Re_x = Ux/\nu$, where ν is the kinematic viscosity. [20%]

(e) Determine the skin friction coefficient C_f' as a function of Re_x. [20%]

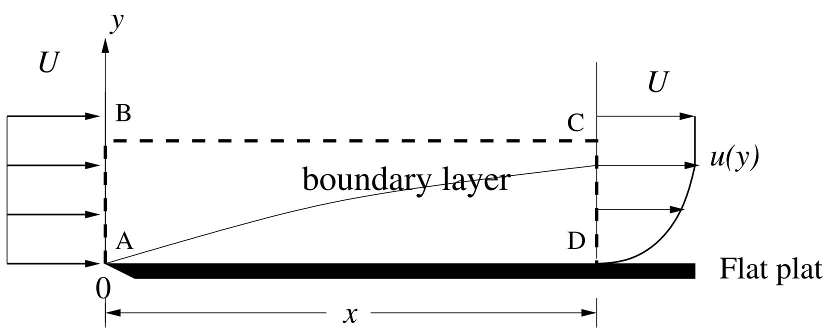

Fig. 5

6 (a) At small values of the incidence angle, α, a symmetric aerofoil in uniform flow has lift coefficient $c_l = 2\pi\alpha$. The corresponding lumped-parameter model has a bound vortex located at one quarter of the chord from the leading edge.

(i) Show that the lumped-parameter model's flow velocity is parallel to the aerofoil chord line at a point three quarters of the chord from the leading edge. [20%]

(ii) Explain the significance of the three-quarters-chord point for lumped-parameter modelling of configurations involving multiple aerofoils. [20%]

(b) Figure 6 shows the lumped-parameter representation of two symmetric aerofoils lying on the same line, subject to a uniform oncoming flow at speed U and incidence α. The first (forward) aerofoil has chord c_1 and circulation Γ_1 about the quarter-chord point. The second (aft) aerofoil has chord c_2 and circulation Γ_2 about the quarter-chord point. The distance between the trailing edge of the first aerofoil and the leading edge of the second aerofoil is negligible. The lift coefficient of the second aerofoil is c_{l2}. Find an expression independent of α for the lift coefficient of the first aerofoil. You can assume that departures from the mean flow are small at all relevant points. [40%]

(c) The first aerofoil is now rotated anticlockwise so that both aerofoils have the same lift coefficient.

(i) What is the lift coefficient of the aerofoils? [10%]

(ii) By what angle must the first aerofoil be rotated? [10%]

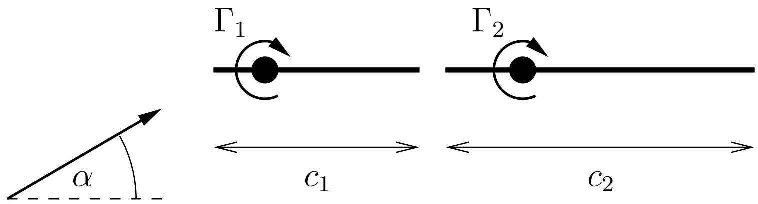

Fig. 6

7 (a) An aircraft with an elliptically loaded wing of semi-span s flies at speed U in air of density ρ. Its horseshoe-vortex representation has circulation Γ and semi-span $s' = (\pi/4)s$. Derive an expression for its induced drag. [10%]

(b) The aircraft of part (a) is joined by an identical partner, and the two fly parallel in the configuration shown in Fig. 7. On the basis of conditions at the horseshoe centres, estimate the proportional reduction in induced drag experienced by the aircraft. You may assume that the wing loadings remain elliptical. [20%]

(c)

(i) Find an expression for the flow velocity relative to the aircraft of part (b) at the position $(0, 0, h)$. (The origin of coordinates is indicated in Fig. 7 by O, and the axis system is right-handed.) [30%]

(ii) If an aircraft identical to those of part (b) flew with its wing centre at $(0, 0, h)$, how would its horseshoe-vortex circulation and induced drag differ from the free-flight values of part (a)? (Your answer need only be qualitative, but should be justified.) [20%]

(iii) Discuss briefly how your answer to (ii) would alter if the third aircraft flew instead at $(0, 0, -h)$. [20%]

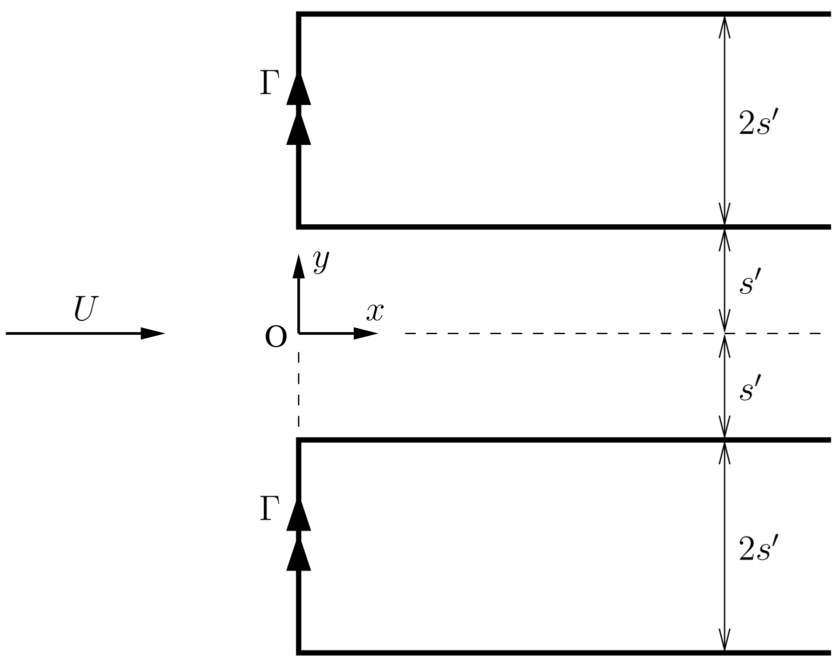

Fig. 7

8 (a) What defines an aerodynamically 'bluff body'? What are the options for reducing the aerodynamic drag of bluff bodies? [30%]

(b) Heavy goods vehicles are more likely to suffer from adverse effects due to cross-wind.

(i) Explain why the above statement is true, and why it is a particular concern in the UK. [10%]

(ii) Give an example of how cross-wind can increase the drag of a typical 'tractor-trailer' heavy goods vehicle. [10%]

(c) A car has frontal projected area, A, drag coefficient, C_D, and rolling resistance, R, all of which are independent of the car's speed, U.

(i) Write down an expression for the work done by the car when travelling a distance s at constant speed, U. [10%]

(ii) With $R = 180$ Newtons and with reasonable assumptions for other parameters, estimate the mechanical energy required to travel 120 kilometres at a constant speed of 120 kilometres per hour. [10%]

(iii) Estimate the percentage energy saving if ten self-driving cars were to travel in a convoy, one behind the other, rather than independently. What aerodynamic factors would most influence this energy saving? [30%]

END OF PAPER

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