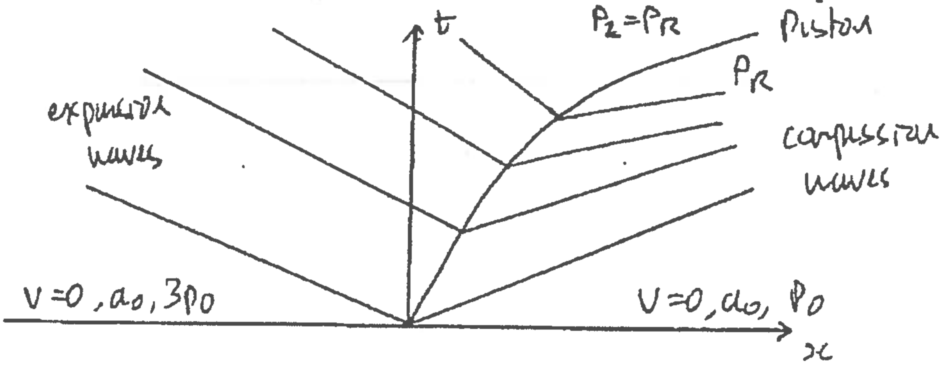
2018 3A3

Examiner's comments at back

1\. (a)



(b) The pressures on each side of the piston will gradually equalise due to compression (RHS) and expansion (LHS). Then the force accelerating the piston will fall to zero.

For RR waves $\quad V_R - \frac{2}{\gamma-1} a_R = -\frac{2}{\gamma-1} a_0 \; ; \; a_0 = \sqrt{\gamma R T_0}$

LR waves $\quad V_L + \frac{2}{\gamma-1} a_L = \frac{2}{\gamma-1} a_0$

also $\frac{p}{p_0} = \left(\frac{a}{a_0}\right)^{\frac{2\gamma}{\gamma-1}} \Rightarrow \frac{p_R}{p_0} = \left(\frac{a_R}{a_0}\right)^{\frac{2\gamma}{\gamma-1}}$

$$\frac{p_L}{3p_0} = \left(\frac{a_L}{a_0}\right)^{\frac{2\gamma}{\gamma-1}}$$

$\therefore \frac{p_R}{p_0} = \left(1 + \frac{(\gamma-1)}{2}\frac{V_R}{a_0}\right)^{\frac{2\gamma}{\gamma-1}} ; \quad \frac{p_L}{3p_0} = \left(1 - \frac{(\gamma-1)}{2}\frac{V_L}{a_0}\right)^{\frac{2\gamma}{\gamma-1}}$

Steady state is reached when $p_R = p_L$; $V_R = V_L = V_p$

$$3 = \frac{\left(1 + \frac{\gamma-1}{2} \frac{V_p}{a_0}\right)^{\frac{2\gamma}{\gamma-1}}}{\left(1 - \frac{\gamma-1}{2} \frac{V_p}{a_0}\right)^{\frac{2\gamma}{\gamma-1}}}$$

$\gamma = 1.4$

$$\therefore \quad \frac{V_p}{a_0} = M_p = \frac{2}{\gamma-1} \left[\frac{3^{\frac{\gamma-1}{2\gamma}} - 1}{3^{\frac{\gamma-1}{2\gamma}} + 1} \right]$$

$$= \frac{2}{0.4} \; \frac{1.17 - 1}{1.17 + 1} = \underline{\underline{0.392}}$$

(c) $$M_p = \frac{V_p}{a_0} = \frac{2}{\gamma-1} \left[\frac{r^{\frac{\gamma-1}{2\gamma}} - 1}{r^{\frac{\gamma-1}{2\gamma}} + 1} \right]$$ where r is the initial pressure ratio

For $M_p = 1$ have $\frac{\gamma-1}{2}\left(r^{\frac{\gamma-1}{2\gamma}} + 1\right) = r^{\frac{\gamma-1}{2\gamma}} - 1$

$$\therefore \; r^{\frac{\gamma-1}{2\gamma}} = \frac{\gamma+1}{3-\gamma} = 1.5$$

$$\therefore \; \underline{\underline{r = 17.086}}$$

2. (a) $\delta\left(\frac{F}{A}\right) = p(M^2-1)\frac{\delta V}{V}$; $\delta\left(\frac{F}{A}\right)$ is negative due to friction

(i) subsonic flow $M<1 \therefore M^2-1$ is -ve $\Rightarrow \delta V$ is +ve

flow accelerates

(ii) supersonic flow $M>1 \therefore M^2-1$ is +ve $\Rightarrow \delta V$ is -ve

flow decelerates.

subsonic flow ρVA is constant with V increasing, A constant

$\Rightarrow \rho$ is decreasing

supersonic flow, same argument, ρ is increasing

Stagnation enthalpy is constant $\Rightarrow$ no heat transfer.

Stagnation pressure is falling due to frictional losses

(b) Inlet Mach no is 1.6; tables $\frac{\dot{m}\sqrt{c_pT_0}}{AP_0} = 1.0246$

$c_f = 0.005$, $D = 0.25$ m $\qquad \frac{4c_fL_{max}}{D} = 0.1724$

$P_0 = 1$ bar, $T_0 = 290$ K

$\frac{4c_f}{D} = 0.08 \quad \therefore L_{max} = 2.155$ m

$\dot{m} = 1.0246 \times \frac{AP_0}{\sqrt{c_pT_0}} = 9.3$ kg/s

with $A = \frac{1}{4}\pi D^2$; $c_p = 1005$ J/kgK

(c) Shock at 0.41 m downstream from pipe entrance.

$$\frac{4C_f L_{12}}{D} = \frac{4C_f L_{max}}{D} - \frac{4C_f L_2}{D} \; ; \quad \frac{4C_f}{D} = 0.08$$

$$\therefore L_2 = 2.155 - 0.41 = 1.745\,m$$

$$\frac{4C_f L_2}{D} = 0.1396 \Rightarrow \text{tables} \quad M = 1.51$$

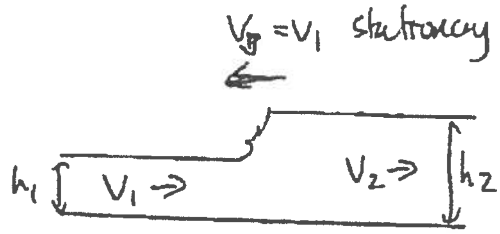
$$M_s = 0.6976$$

Tables again $\frac{4C_f L_{2'max}}{D} = 0.2129$ (interpolated)

$$\therefore L_{2'max} = 2.66\,m$$

$\therefore$ total length is $L_{2'max} + L_{12} = \underline{\underline{3.07\,m}}$

3. (a) Hydraulic jump :



Continuity $h_1 V_1 = h_2 V_2$

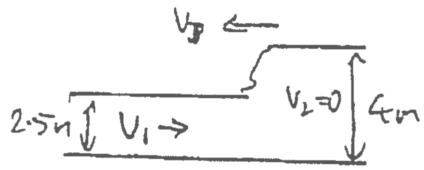
momentum $\frac{1}{2}\rho g h_1^2 - \frac{1}{2}\rho g h_2^2 = \rho h_2 V_2^2 - \rho h_1 V_1^2$

$$= \rho h_1 V_1 (V_2 - V_1)$$

$$\frac{1}{2} g (h_1 - h_2)(h_1 + h_2) = h_1 V_1 V_2 \left(1 - h_2/h_1\right)$$

$$\therefore \quad \underline{\underline{V_1 V_2 = \frac{1}{2} g (h_1 + h_2)}}$$

(b)



$\Rightarrow$ change of frame

in stationary frame $(V_1 + V_J) V_J = \frac{1}{2} g (4 + 2.5)$

also $(V_1 + V_J)\, 2.5 = V_J \cdot 4$

$$\therefore \quad 1.6 V_J^2 = 3.25 g = 31.88$$

$$\therefore \underline{\underline{V_J = 4.464 m/s}}$$

$$\underline{\underline{V_1 = 2.678 m/s}}$$

(c) Partial blockage, $h_2 = 3.5m$

$$(V_1+V_J)(V_2+V_J) = \frac{1}{2}g(h_1+h_2) \quad \text{stationary frame}$$

$$(V_2+V_J) \cdot 3.5 = (V_1+V_J)\,2.5$$

$$(V_1+V_J)^2 = \frac{1}{2} \times 9.81 \times (3.5+2.5)\ \frac{3.5}{2.5}$$

$$= 41.202$$

$$\therefore V_1+V_J = 6.42\,m/s$$

$$\therefore \underline{\underline{V_J = 3.74\,m/s}}$$

4 a i) $\nabla.(\rho V)=0 \Rightarrow \nabla.V+\frac{1}{\rho}V.\nabla\rho=0$

ii) $-\frac{1}{\rho}\nabla p = V.\nabla V$

iii) $h_0 = \text{const} = c_pT+\left(\frac{u^2+v^2}{2}\right)=\frac{a^2}{\gamma-1}+\frac{1}{2}(u^2+v^2)$

b) Isentropic $\therefore p=k\rho^\gamma \Rightarrow dp = k\gamma\rho^{\gamma-1}d\rho \Rightarrow \nabla p = a^2\nabla\rho$

$\therefore \nabla.V = \frac{-1}{\rho}V.\nabla\rho = \frac{-1}{\rho a^2}V.\nabla p = \frac{1}{a^2}V.(V.\nabla V)$ thus $a^2\nabla.V - V(V.\nabla V)=0$

c) Consider velocity components:

$$a^2\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)-\left[u\underline{e_x}+v\underline{e_y}\right].\left[\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)\underline{e_x}+\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)\underline{e_y}\right]=0$$

$$\Rightarrow (a^2-u^2)\frac{\partial u}{\partial x}-uv\frac{\partial u}{\partial y}-vu\frac{\partial v}{\partial x}+(a^2-v^2)\frac{\partial v}{\partial y}=0$$

consider a small perturbation to oncoming flow velocity U_∞

$\therefore$ subs $u \Rightarrow U_\infty + \frac{\partial\phi}{\partial x}$, $v \Rightarrow \frac{\partial\phi}{\partial y}$ & neglect H.O.T.

$$\Rightarrow (1-M^2)\frac{\partial^2\phi}{\partial x^2}+\frac{\partial^2\phi}{\partial y^2}=0$$

and, to this order,

$$a^2 = a_\infty^2 + \frac{\gamma-1}{2}(U_\infty^2-u^2)$$

H.O.T.

$$\Rightarrow M^2 = M_\infty^2 + \text{H.O.T.}$$

d) 900 km/h = 250 m/s $\quad a=\sqrt{\gamma RT}$

for $T = 45°C$ $a = 357$ m/s $\Rightarrow M = 0.70$

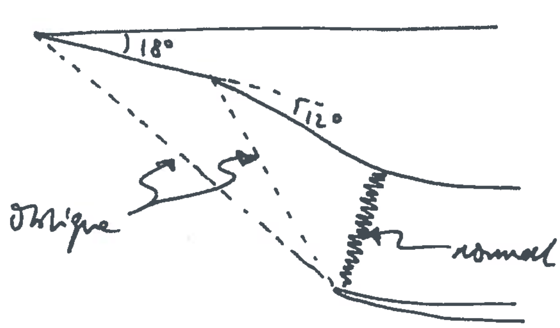
for $T=-60°C$ $a = 293$ m/s $\Rightarrow M = 0.85$

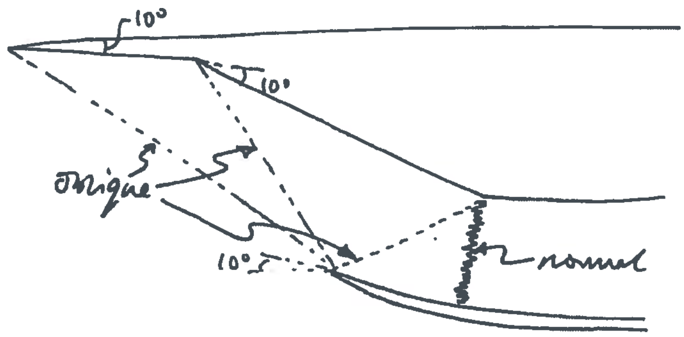
Glauert gives: $C_L = \frac{C_{L_0}}{\sqrt{1-M_\infty^2}}$

$$C_L = 0.5 \times \frac{\sqrt{1-0.85^2}}{\sqrt{1-0.70^2}} = 0.365$$

5 a) $30°$ in both cases

b)





c)

M	θ	β	p_2/p_1	M_2	p_{02}/p_{01}
3.00	18°	35.5	3.368	2.10	0.839
3.00	10°	27.4	2.055	2.505	0.963
2.10	12°		1.923	1.66	0.972
2.50	10°		1.864	2.10	0.976
2.10	10°		1.73	1.73	0.983
1.66	Normal		3.05		0.872
1.73	Normal		3.33		0.843

For external only:

$p_2/p_1 = 19.8 \quad p_{02}/p_{01} = 71\%$

For mixed compression

$p_2/p_1 = 22 \quad p_{02}/p_{01} = 78\%$

d)

M	θ	p_2/p_1
3.00	34°	8.27
3.00	14°	2.65

$\therefore$ pressure drag ratio $= \dfrac{8.27}{2.65} \times \dfrac{\sin 34°}{\sin 14°} = 7.2$

e) Mixed gives more pressure rise and better pressure recovery. But harder to start, more likely to unstart and β values of front shocks, plus shock system, shows that (for a given capture area) mixed is likely to be longer (and heavier) but with lower cowl drag.

6.a) Hyperbolic equations are like wave equations and can be solved by following wave directions (characteristics

e.g. $\frac{\partial^2 u}{\partial t^2} - c^2\frac{\partial^2 u}{\partial x^2} = 0$

Instability usually results from allowing "information" in the numerical scheme to travel faster than the wave speed & b.c. must exhibit these properties.

Elliptic equations, like Laplace's equation $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\varphi = 0$, have the property that solution at any point is influenced by solution everywhere else. Numerical scheme & boundary conditions must reflect this. E.g. time marching to steady solution.

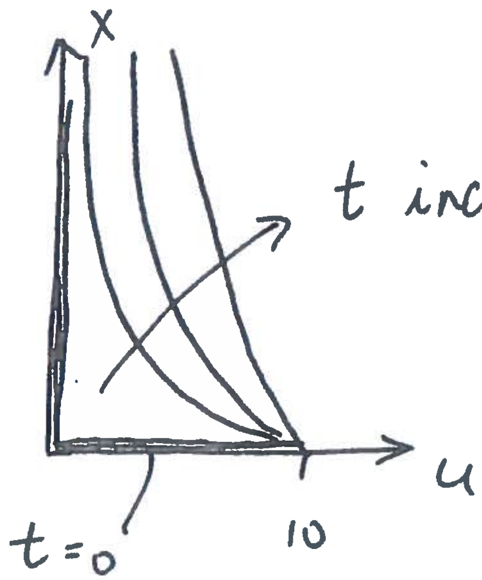
Parabolic equations have an unbalanced order in the highest derivatives for each independent variable

e.g. $\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial x^2}$ the diffusion equation.

These are usually solved by marching forward in time.

b) Equation is parabolic.

c)



d) $$\frac{u_i^{n+1} - u_i^n}{\Delta t} = \nu \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\Delta x^2}$$

(e) $$u_i^{n+1} = u(x, t+\Delta t) = \underset{\overset{\shortparallel}{u_i^n}}{u(x,t)} + \Delta t \frac{\partial u}{\partial t} + \frac{\Delta t^2}{2}\frac{\partial^2 u}{\partial t^2} + \cdots$$

$$\therefore \quad \frac{u_i^{n+1} - u_i^n}{\Delta t} = \frac{\partial u}{\partial t} + \underbrace{\frac{\Delta t}{2}\frac{\partial^2 u}{\partial t^2}}_{\text{neglect}} + \cdots$$

$\Rightarrow$ <u>1st order in time</u>

$$u(x+\Delta x, t) = u(x,t) + \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2!}\frac{\partial^2 u}{\partial x^3} + \frac{\Delta x^3}{3!}\frac{\partial^3 u}{\partial x^3} + \cdots$$

$$u(x-\Delta x, t) = u(x,t) - \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2!}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^3}{3!}\frac{\partial^3 u}{\partial x^3} + \cdots$$

$$\therefore \quad \frac{u_{i+1}^n - 2u_i^n + u_{i-1}^n}{\Delta x^2} = \frac{\partial^2 u}{\partial x^2} + O(\Delta x^2)$$ $\Rightarrow$ <u>2nd order in space</u>

f) Finite difference scheme is linear $\Rightarrow$ equation for errors is the same as that for u

$$\Rightarrow \quad \frac{\varepsilon_i^{n+1} - \varepsilon_i^n}{\Delta t} = \nu \frac{\varepsilon_{i+1}^n - 2\varepsilon_i^n + \varepsilon_{i-1}^n}{\Delta x^2}$$

For the case $\{\varepsilon_i^n\} = \{\ldots -\varepsilon, \varepsilon, -\varepsilon, \varepsilon, -\varepsilon \ldots\}$

$$\Rightarrow \quad \varepsilon_i^{n+1} = \epsilon + \frac{\nu \Delta t}{\Delta x^2}\left(-\varepsilon - 2\varepsilon - \varepsilon\right)$$

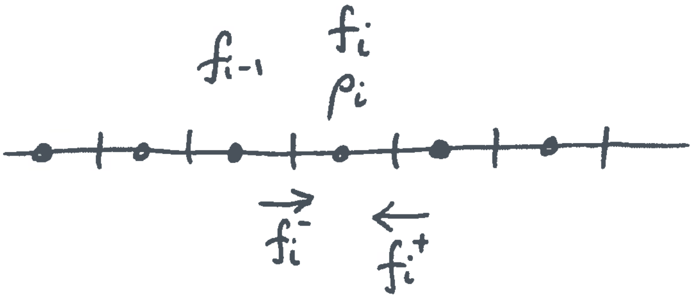
$$= \epsilon \left[1 - \frac{4\nu\Delta t}{\Delta x^2}\right]$$

Scheme will be stable only if

$$-1 \leq 1 - \frac{4\nu\Delta t}{\Delta x^2} \leq 1$$

i.e. $\quad \dfrac{4\nu\Delta t}{\Delta x^2} \leq 2 \quad \Rightarrow \quad \dfrac{\nu\Delta t}{\Delta x^2} \leq \dfrac{1}{2}$

7 a)



$f_i = \bar{f}\big|_{i\text{th cell}}$

$\rho_i = \bar{\rho}\big|_{i\text{th cell}}$

Conservation equation in finite volume form

$$\Rightarrow \quad \frac{\partial(\rho_i \Delta x_i)}{\partial t} = f_i^- - f_i^+ \qquad \Delta x_i = x_i^+ - x_i^-$$

with $f_i^- = \dfrac{f_{i-1} + f_i}{2}$ $\qquad f_i^+ = \dfrac{f_{i+1} + f_i}{2}$ $\Rightarrow f_i^- - f_i^+ = \dfrac{f_{i-1} - f_{i+1}}{2}$

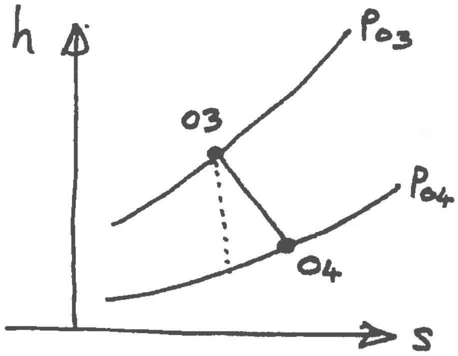
Now
$$f(x - \Delta x, t) = f(x,t) - \Delta x \frac{\partial f}{\partial x} + \frac{\Delta x^2}{2!}\frac{\partial^2 f}{\partial x^2} + O(\Delta x^3)$$
$$f(x + \Delta x, t) = f(x,t) + \Delta x \frac{\partial f}{\partial x} + \frac{\Delta x^2}{2!}\frac{\partial^2 f}{\partial x^2} + O(\Delta x^3)$$

$$\Rightarrow \frac{f_{i-1} - f_{i+1}}{2} = -\Delta x \frac{\partial f}{\partial x} + O(\Delta x^3) = f_i^- - f_i^+$$

$$\Rightarrow \frac{\partial \rho_i}{\partial t} + \frac{1}{\Delta x_i}\left(f_i^+ - f_i^-\right) = O(\Delta x^2)$$

i.e. is identical to a second order central finite difference scheme.

7b(i) PERFECT GAS



$$\eta_{tt} = \frac{T_{03} - T_{04}}{T_{03} - T_{04}^{ISEN}} = \frac{T_{03} - T_{04}}{T_{03}\left(1 - \frac{T_{04}^{ISEN}}{T_{03}}\right)}$$

$$\frac{T_{04}^{ISEN}}{T_{03}} = \left(\frac{p_{04}}{p_{03}}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_{03} - \Delta p_0}{p_{03}}\right)^{\frac{\gamma-1}{\gamma}} = \left(1 - \frac{\Delta p_0}{p_{03}}\right)^{\frac{\gamma-1}{\gamma}}$$

$$\approx 1 + \left(\frac{\gamma-1}{\gamma}\right)\left(-\frac{\Delta p_0}{p_{03}}\right) + \left(\frac{\gamma-1}{\gamma}\right)\left(\frac{\gamma-1}{\gamma} - 1\right)\frac{1}{2}\left(-\frac{\Delta p_0}{p_{03}}\right)^2$$

$$= 1 - \left(\frac{\gamma-1}{\gamma}\right)\frac{\Delta p_0}{p_{03}} - \left(\frac{\gamma-1}{\gamma}\right)\frac{1}{\gamma}\frac{1}{2}\left(\frac{\Delta p_0}{p_{03}}\right)^2$$

$$1 - \frac{T_{04}^{ISEN}}{T_{03}} \approx \left(\frac{\gamma-1}{\gamma}\right)\frac{\Delta p_0}{p_{03}}\left(1 + \frac{1}{2\gamma}\frac{\Delta p_0}{p_{03}}\right)$$

$$R = c_p - c_v$$

$$\frac{R}{c_p} = 1 - \frac{c_v}{c_p}$$

$$\frac{R}{c_p} = 1 - \frac{1}{\gamma} = \frac{\gamma-1}{\gamma}$$

$$\Rightarrow \eta_{tt} \approx \frac{\Delta h_0 / c_p}{T_{03}\left(\frac{\gamma-1}{\gamma}\right)\frac{\Delta p_0}{p_{03}}\left(1 + \frac{1}{2\gamma}\frac{\Delta p_0}{p_{03}}\right)}$$

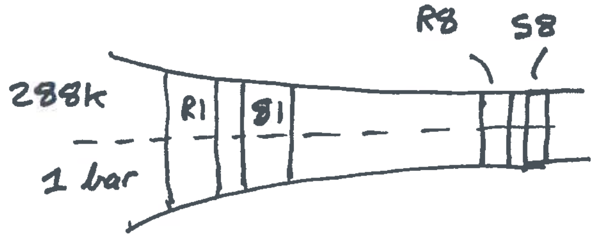
$$= \frac{\Delta h_0}{T_{03}\frac{R}{c_p} c_p \frac{\Delta p_0}{p_{03}}}\left(1 + \frac{1}{2\gamma}\frac{\Delta p_0}{p_{03}}\right)^{-1}$$

$$\eta_{tt} \approx \frac{\Delta h_0}{\Delta p_0 / p_{03}}\left(1 + \frac{1}{2\gamma}\frac{\Delta p_0}{p_{03}}\right)^{-1}$$

(ii) REQUIRE $\left|1 + \frac{1}{2\gamma}\frac{\Delta p_0}{p_{03}}\right| < 1.01 \Rightarrow \frac{1}{2\gamma}\frac{\Delta p_0}{p_{03}} < 0.01$

$2\gamma \approx 3 \Rightarrow \frac{\Delta p_0}{p_{03}} < 3\%$

8(a) $\dfrac{\Delta h_0}{U^2} = 0.4$



$$\Delta T_0 = \frac{0.4 \times 250^2}{1005} = \underline{24.88\ K}$$

(i) <u>STAGE 1</u>

$$T_{0\,INLET} = 288\ K$$

$$T_{0\,EXIT} = 288 + 24.88 = 312.88\ K$$

$$\frac{P_{0\,EXIT}}{P_{0\,INLET}} = \left(\frac{312.88}{288}\right)^{\frac{\gamma}{\gamma-1}} = \underline{\underline{1.336}} \quad \text{(STAGE 1)}$$

(ii) <u>STAGE 8</u>

$$T_{0\,INLET} = 288 + 7 \times 24.88 = 462.16\ K$$

$$T_{0\,EXIT} = 288 + 8 \times 24.88 = 487.04\ K$$

$$\frac{P_{0\,EXIT}}{P_{0\,INLET}} = \left(\frac{487.04}{462.16}\right)^{\frac{\gamma}{\gamma-1}} = \underline{\underline{1.201}} \quad \text{(STAGE 8)}$$

(iii) Stage 8 has a higher inlet stagnation temperature so has a <u>lower</u> blade Mach number so will only produce a <u>lower</u> pressure rise.
(Aerodynamically slower).

(b)(i) $\phi = 0.6 \Rightarrow v_x = 0.6U = 0.6 \times 250 = \underline{\underline{150\ m/s}}$

$$v_{\theta 1}^{rel} = v_{\theta 1} - U = 0 - 250 = \underline{\underline{-250\ m/s}}$$

$$T_{1} = T_{01} - \frac{(v_x^2 + v_\theta^2)}{2c_p} = 288 - \left(\frac{150^2 + 0^2}{2 \times 1005}\right) = \underline{276.81\ K}$$

$$T_{01}^{rel} = T_1 + \left(\frac{v_x^2 + v_\theta^{rel\,2}}{2c_p}\right) = 276.81 + \left(\frac{150^2 + (-250)^2}{2 \times 1005}\right) = \underline{\underline{319.10\ K}}$$

$$P_{01}^{rel} = \left(\frac{T_{01}^{rel}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} . P_{01} = 1 \left(\frac{319.10}{288}\right)^{3.5} = \underline{\underline{1.4318\ bar}}$$

(ii) Stage 1, rotor exit

$$\Delta h_0 = U(V_{\theta 2} - V_{\theta 1}) \Rightarrow \psi = \frac{v_{\theta 2}}{U} - \underbrace{\frac{v_{\theta 1}}{U}}_{=0}$$

$$\Rightarrow v_{\theta 2} = U\psi = 250 \times 0.4 = \underline{\underline{100\ m/s}}$$

CONSTANT RADIUS $\Rightarrow T_{02}^{REL} = T_{01}^{REL} = \underline{\underline{319.10\ K}}$

$$Y_p = \frac{P_{02}^{REL} - P_{02}^{REL}}{P_{01}^{REL} - P_1} \qquad \left(P_{01}^{REL} = P_{02}^{REL,\,ISEN} \text{ FIXED RADIUS}\right)$$

$$P_{02}^{REL} = P_{01}^{REL} - Y_p\left(P_{01}^{REL} - P_1\right)$$

$$P_1 = P_{01}\left(\frac{T_1}{T_a}\right)^{\frac{\gamma}{\gamma-1}} = 1\left(\frac{276.81}{288}\right)^{3.5} = \underline{\underline{0.8705\ bar}}$$

$$P_{02}^{REL} = 1.4318 - 0.05(1.4318 - 0.8705) = \underline{\underline{1.4037\ bar}}$$

(iii) $P_{02} = P_{02}^{REL}\left(\frac{T_{02}}{T_{02}^{REL}}\right)^{\frac{\gamma}{\gamma-1}}$

$$P_{02} = 1.4037\left(\frac{312.88}{319.10}\right)^{\frac{\gamma}{\gamma-1}} = \underline{\underline{1.3102 \text{ bar}}}$$

STAGE 1 $\quad \frac{P_{02}}{P_{01}} = \frac{1.3102}{1.0} = \underline{\underline{1.3102}}$

(IV) NEED P_2

$$T_2 = T_{02} - \frac{(V_x^2 + V_{\theta 2}^2)}{2c_p} = 312.88 - \frac{(150^2 + 100^2)}{2 \times 1005}$$

$$\underline{T_2 = 296.71 \text{ K}}$$

$$P_2 = P_{02}\left(\frac{T_2}{T_{02}}\right)^{\frac{\gamma}{\gamma-1}} = 1.3102\left(\frac{296.71}{312.88}\right)^{3.5} = \underline{1.0881 \text{ bar}}$$

STATOR 1 : $\quad Y_P = \frac{P_{02} - P_{03}}{P_{02} - P_2}$

$$P_{03} = P_{02} - Y_P(P_{02} - P_2)$$
$$= 1.3102 - 0.05(1.3102 - 1.0881) = \underline{1.2990 \text{ bar}}$$

$$\left.\frac{P_{03}}{P_{01}}\right|_{\substack{\text{STAGE 1} \\ \text{ACTUAL}}} = \frac{1.2990}{1.0} = \underline{\underline{1.2990}}$$

Engineering Tripos Part IIA 2018

Assessor's Comments

Q1 Shock Tube Problem. Attempts 27, Mean 10.7

Poorly done by most candidates and the average mark was only achieved after changing the mark scheme to increase the weighting of the introductory parts of the question. Common errors were: to consider only one side of the piston and then find an erroneous equation to complete the set; to not use the data about the initial temperature; to get hopelessly bogged down in the algebra.

Q2 Fanno flow. Attempts 51. Mean 16.5

Probably too straightforward as a question. Well done by every candidate. Least satisfactory part was, surprisingly, the introductory discussion of variation of basic parameters. Even those who got this badly wrong, answered the later parts correctly, by correct use of tables.

Q3 Hydraulic Jumps. Attempts 45. Mean 16.5

Another question the candidates murdered, with most producing "crib" quality solutions.

Q4 Prandtl-Glauert Scaling. Attempts 7. Mean 11.1

The question involved quite a bit of derivation of standard theory. Interestingly, every candidate forgot that the speed of sound is a function of position in compressible flow and that some comment about why it can be replaced by the stagnation value (result of linearization) should have been included.

Q5 Design of Supersonic Intakes. Attempts 45. Mean 12.8

All candidates had a clear idea of the shock systems implied by the two intake designs. Most struggled with the pressure drag on the intake lip, tending to use free-stream pressure on the outside. Discussion of the relative merits of the designs was good, except when it involved the knock-on effects for any engine. Some of the statements made about this can only be described as bizarre e.g. (a) Intake has a higher static pressure recovery, therefore does more work meaning the engine has less work to do or (b) Engine requires highest static pressure recovery as possible.

N.B. The engine requires a given Mach Number (or rather a given value of $\dfrac{\dot{m}\sqrt{c_p T_0}}{p_0 A}$) and the flow to be turned to engine axial direction. If a higher Mach number than this is delivered, there will need to be further diffusion (and probably therefore more stagnation pressure loss). If the flow is not delivered at the engine axial angle, then further turning is required (again involving an intake extension and probably more loss). The only real figures of merit for the engine are: stagnation pressure recovery (as high as possible), flow uniformity and zero flow angle.

Q6 Finite Difference Schemes. Attempts 44. Mean 14.1

All candidates knew exactly what to do and did it well. Only hard marking on the discussion part kept the average to a reasonable level.

Q7 (a) Finite Volume Methods and (b) Axial Turbines. Attempts 10. Mean 11.5

Not many attempts. Those that did attempt it did well on one half of the question (approximately equally split between the parts), suggesting that this was a last resort question for those running out of other questions they thought they could do. The half attempts were quite good.

Q8 Axial Compressors. Attempts 26. Mean 11.8

All candidates did well on the early parts, with most trouble caused by the change of frame for the rotor and stator and in particular which frame to apply the loss coefficients (relative frame for the rotor, absolute frame for the stator). Only 3 sightings of Bernoulli, which is a record low.

Tom Hynes

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