Version TPH/3

EGT2
ENGINEERING TRIPOS PART IIA

Wednesday 25 April 2018 9.30 to 12.40

Module 3A3

FLUID MECHANICS II

*Answer not more than **five** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: Compressible Flow Data Book (38 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 A double-sided piston of finite mass is located in the middle of a long cylinder as shown in Fig. 1. The pressure of the air in the right-hand part of the cylinder is p_0 while the pressure of the air in the left-hand part is $3p_0$. The air temperature in both parts is T_0. Initially the piston is held at rest, and the air in both parts of the cylinder is stationary. At a certain initial instant of time the piston is released and begins to move without friction.

(a) Draw an x–t diagram to illustrate the motion of the piston and the resulting wave action in the air on both sides. [30%]

(b) Explain why the piston will reach a steady speed and determine the final Mach number of the piston relative to the initial conditions. [40%]

(c) How large does the initial pressure ratio across the piston have to be for the piston to reach the initial speed of sound? [30%]

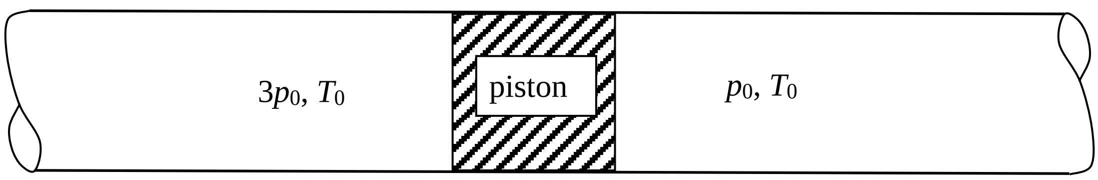

Fig. 1

The Riemann invariants are

$$V \pm \frac{2}{\gamma - 1} a = \text{const}$$

for left-running waves (positive sign) and right-running waves (negative sign).

2 (a) Consider flow with friction in a duct of constant cross-sectional area A. The variation of the impulse function F is governed by the relation

$$\delta\left(\frac{F}{A}\right) = p\left(M^2 - 1\right)\frac{\delta V}{V}$$

where p is the pressure and M is the Mach number. Explain how the velocity V varies along the duct in (i) subsonic flow and (ii) supersonic flow. What is the corresponding variation of the density ρ, the stagnation enthalpy h_0 and the stagnation pressure p_0? [30%]

(b) A large vessel discharges air through a frictionless convergent-divergent nozzle into a pipe of inside diameter 0.25 m and skin friction coefficient 0.005. The pressure and temperature in the vessel are 1 bar and 290 K respectively. The Mach number at the end of the convergent-divergent nozzle is 1.6. Determine the length of pipe required to reach a choked exit, and the mass flow rate at this condition. [40%]

(c) The length of the pipe is increased and the exit remains choked. A shock wave is formed at a distance of 0.41 m downstream from the entrance to the pipe. Calculate the strength of the shock wave and the new length of the pipe. [30%]

3 (a) A stationary hydraulic jump occurs in an otherwise uniform flow of water. Show that

$$V_1 V_2 = \frac{1}{2} g \left(h_1 + h_2 \right)$$

where g is the acceleration due to gravity, h is the depth, V is the velocity and subscripts 1 and 2 denote conditions upstream and downstream of the jump respectively. [20%]

(b) A channel contains a uniform flow of water with depth 2.5 m. The downstream end of the channel is suddenly blocked completely by a sluice gate. An hydraulic jump is formed and travels upstream. The water behind the hydraulic jump is found to have depth 4 m. What is the speed of the hydraulic jump and the speed of the upstream flow? [40%]

(c) On a separate occasion, the same initially uniform flow of depth 2.5 m is suddenly, but only partially, blocked by the sluice gate. The depth downstream of the resulting hydraulic jump is measured as 3.5 m. What is the speed of the jump in this case? [40%]

4 An aircraft flies at a true airspeed of 900 km/hour. For analysis purposes, the flow around the wings may be considered irrotational, isentropic and two-dimensional. The flow may be described in a Cartesian coordinate system (*x*,*y*) by a flow potential ϕ and velocity $\underline{V}$.

a) Write down, in differential form, equations for the conservation of:

(i) mass;

(ii) momentum; and

(iii) energy. [20%]

b) Hence derive the equation:

$$a^2 \nabla \cdot \underline{V} - \underline{V} \cdot \left(\underline{V} \cdot \nabla \underline{V} \right) = 0$$

where a is the local speed of sound. [20%]

c) By considering an appropriate perturbation to the flow, show that the equation:

$$\left(1 - M_\infty^2\right) \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

where M_∞ is the free stream Mach number, applies to *small* disturbances. [40%]

d) The aircraft is designed to operate at a lift coefficient $C_L = 0.5$ at an altitude of 11,000 m, where the ambient static air temperature is –60º C. If instead, the aircraft were to fly at the same true airspeed at low level over a desert, where the ambient static air temperature is +45º C, estimate the new lift coefficient. [20%]

5 A supersonic jet aircraft is designed for a cruise Mach number M=3.00. Two designs of engine intake are under consideration. The first, sketched in Fig. 2(a), is a conventional external compression design with a shock system focussed on the cowl lip. In the second design, a mixed internal/external compression design is proposed where the two external ramps are reduced in angle and the *upper* face of the cowl lip is placed at 10º to the free stream direction, as shown in Fig. 2(b). In this second design, the shock system is focussed just inside the cowl lip and reflects as a single oblique shock. The shock system is terminated by a normal shock at the throat position labelled in Fig. 2(b). The inner faces of the intake between the cowl lip and the throat may be treated as flat. In each case, the cowl lip is sharp with a wedge angle of 4º.

a) Write down the magnitude of the total flow turning due to the shocks in both cases. [10%]

b) For each design, draw carefully labelled sketches of the shock system. [20%]

c) For both designs, calculate the static pressure rise and pressure recovery (in terms of the ratio of exit to entry stagnation pressure) through the shock system. [25%]

d) Calculate the ratio of the pressure drag on the external face of the cowl lip for the two designs. The cowl lip has length L in both cases, as labelled in Figs. 2(a) and 2(b). [20%]

e) Discuss the relative benefits and drawbacks of each design for the proposed application. [25%]

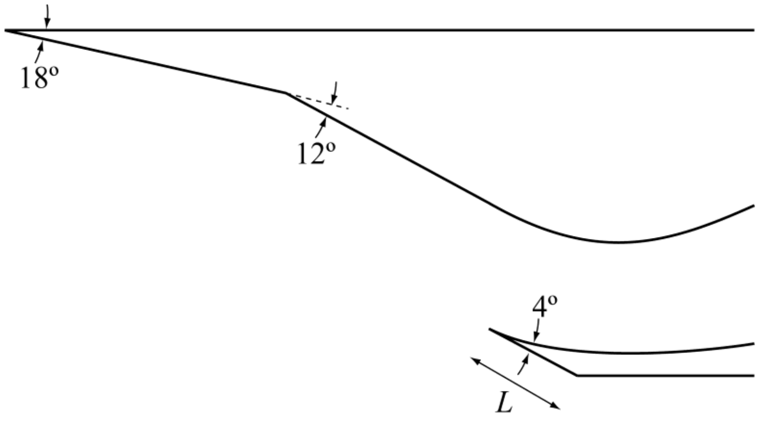

Fig. 2(a) (not to scale)

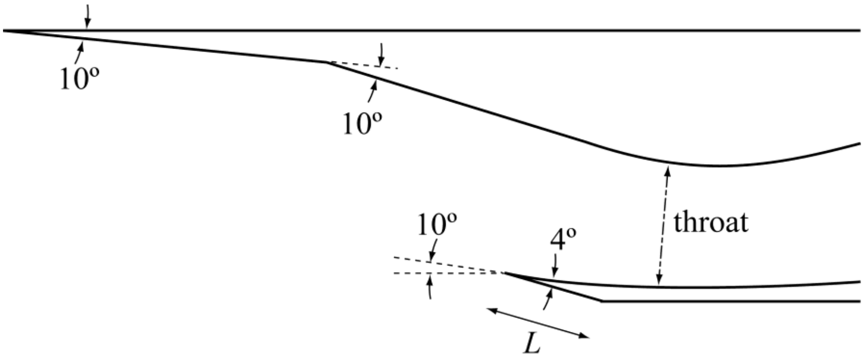

Fig. 2(b) (not to scale)

6 (a) Describe what is meant by hyperbolic, parabolic and elliptic in the context of partial differential equations and how this effects the way they are solved numerically. [20%]

(b) Viscous laminar flow close to a wall can be approximated by the differential equation

$$\frac{\partial u}{\partial t} - \nu \frac{\partial^2 u}{\partial x^2} = 0$$

where x denotes the distance normal to the wall, u denotes the speed, t the time and ν the kinematic viscosity. Into which category does this equation fall ? [10%]

(c) If the wall is impulsively started from a state of rest to move at a constant speed of 10 ms^{-1} at $t = 0$, sketch (qualitatively) the velocity profile at later times. [10%]

(d) Using a uniform grid with a spacing of Δt in time and Δx in space, derive a suitable explicit finite difference scheme for solving this equation. [20%]

(e) Describe the order of accuracy of your scheme, justifying your answer. [20%]

(f) By considering a sawtooth perturbation of small amplitude ε (the perturbation varies grid point to grid point $+\varepsilon$ to $-\varepsilon$), determine a condition for the stability of this scheme. [20%]

7 (a) Consider the mass conservation equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial f}{\partial x} = 0$$

and its discretisation by a finite volume scheme

$$\frac{\partial \overline{\rho}}{\partial t} + \frac{1}{\Delta x_i}\left(f_i^+ - f_i^- \right) = 0$$

where i is the index of a cell centre, $\Delta x_i = x_i^+ - x_i^-$, and $^+$ and $^-$ represent downstream and upstream edges of the cell, respectively. $\overline{\rho}$ represents the average density in the cell.

If the cell spacing is uniform and the edge fluxes (f) are approximated by averages from neighbouring cell centres, show that the discretisation in space and time is identical to a second order central finite difference scheme. [50%]

(b) A perfect gas entering an axial flow turbine has stagnation pressure p_{03} and stagnation temperature T_{03}. At exit from the turbine the stagnation pressure and stagnation temperature are p_{04} and T_{04} respectively and the overall total-to-total isentropic efficiency is η_{tt}.

(i) By considering $\Delta p_0 / p_{03}$ as a small quantity, show that:

$$\eta_{tt} \approx \frac{\Delta h_0}{\Delta p_0 / \rho_{03}} \left(1 + \frac{1}{2\gamma} \frac{\Delta p_0}{p_{03}} \right)^{-1}$$

where γ is the ratio of specific heat capacities, $\Delta p_0 = p_{03} - p_{04}$ and $\Delta h_0 = h_{03} - h_{04}$ are the changes in stagnation pressure and stagnation enthalpy across the turbine respectively and ρ_{03} is the inlet stagnation density. [40%]

(ii) Hence estimate an upper limit on $\Delta p_0 / p_{03}$ for the total-to-total isentropic efficiency determined assuming incompressible flow to be within 1% of the actual value η_{tt}. [10%]

8 The mid-span radius is constant throughout an eight-stage axial flow compressor. The inlet stagnation pressure is 1 bar, the inlet stagnation temperature is 288K and the flow is axial at inlet, between each stage and at compressor exit. At design operating conditions, the mean blade speed is 250 ms^{-1}, all stages have a stage loading coefficient of 0.4, a flow coefficient of 0.6 and each stage operates with the same axial velocity.

(a) Assuming isentropic flow through the compressor:

(i) What is the stagnation pressure ratio across the first stage? [10%]

(ii) What is the stagnation pressure ratio across the last stage? [10%]

(iii) Explain why your answers to (i) and (ii) are different. [5%]

(b) A more reliable estimate of the compressor performance can be obtained by assuming that for the first stage the rotor (relative) stagnation pressure loss coefficient is 0.05 at mid-span.

(i) Determine the axial velocity, relative tangential velocity, relative stagnation temperature and relative stagnation pressure at inlet to the first rotor. [25%]

(ii) Determine the relative tangential velocity, relative stagnation temperature and relative stagnation pressure at exit from the first rotor. [25%]

(iii) Determine the stagnation pressure ratio across the first rotor. [15%]

(iv) If the first stator has a stagnation pressure loss coefficient of 0.05 at mid-span determine the stagnation pressure ratio across the first stage. [10%]

You may assume that the working fluid is a perfect gas with:

$$R = 287\,\mathrm{J\,kg^{-1}K^{-1}}, \quad \gamma = 1.4 \quad \text{and} \quad c_p = 1005\,\mathrm{J\,kg^{-1}K^{-1}}.$$

END OF PAPER

Answers

1 (b) 0.392 (c) 17.09

2 (b) 2.155 m, 9.3 kg s^{-1} (c) $M = 1.51$, $M_s = .6976$, $L = 3.07$ m

3 (b) $v_{\text{jump}} = 4.464$ m s^{-1}, $v = 2.678$ m s^{-1} (c) 4.74 m s^{-1}

4 (d) 0.365

5 (a) 30° both cases (c) Intake a $\frac{p_2}{p_1} = 19.8$ $\frac{p_{02}}{p_{01}} = 71\%$, intake b $\frac{p_2}{p_1} = 19.8$ $\frac{p_{02}}{p_{01}} = 71\%$ (d) 7.2

6 (f) $\frac{v\Delta t}{\Delta x^2} \leq \frac{1}{2}$

7 (b) (ii) 3%

8 (a) (i) 1.336 (ii) 1.201

(b) (i) $v_x = 150$ m s^{-1}, $v_\theta^{rel} = -250 \text{ m s}^{-1}$, 919.1 K, 1.4318 bar

(ii) 100 m s^{-1}, 319.1 K, 1.4037 bar

(iii) 1.310

(iv) 1.299