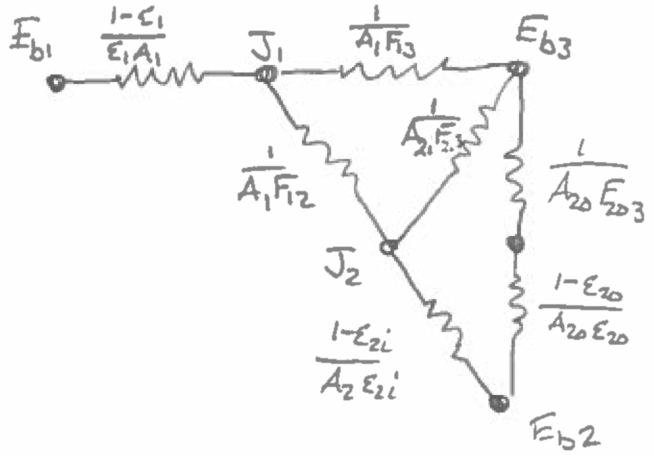
Radiation

1 a)

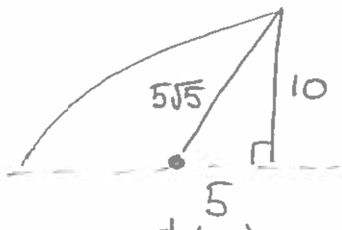


7 resistors
6 nodes

b)

$$F_{13} = \frac{127^\circ}{360^\circ} = 0.35$$

$$F_{12i} = 1 - F_{13} - \cancel{F_{11}}^{0} = 0.65$$



$\tan^{-1}(2/1) = 63.4^\circ$

$$F_{2i1} = \frac{A_1}{A_2} F_{12i} = \frac{1}{50} 0.65 = 0.013$$

$$F_{2i3} = \frac{A_3}{A_2} F_{32i} = \frac{20}{50} = 0.4$$

$$F_{2i2i} = 1 - 0.4 - 0.13 = 0.586$$

$F_{32i} = 1$

$F_{2o3} = 1$

c)

$$R_{E_{b1}J_1} = \frac{1-\varepsilon_1}{A_1\varepsilon_1} = \frac{1-0.8}{0.01\cdot 0.8} = 25\ m^{-1}$$

$$R_{J_1E_{b3}} = \frac{1}{A_1 F_{13}} = \frac{1}{0.01\cdot 0.35} = 283.5\ m^{-1}$$

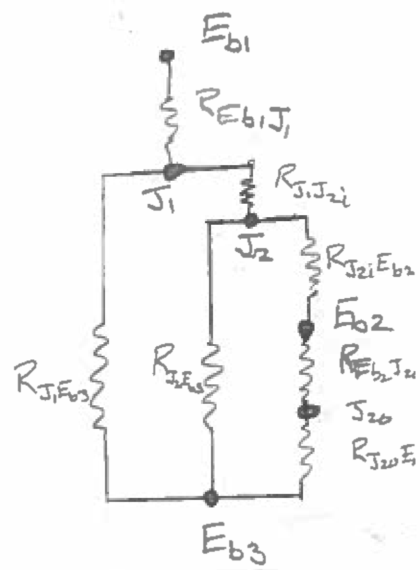
$$R_{J_1J_{2i}} = \frac{1}{A_1 F_{12}} = \frac{1}{0.01\cdot .65} = 154.5\ m^{-1}$$

$$R_{J_{2i}E_{b3}} = \frac{1}{A_2 F_{2i3}} = \frac{1}{0.5\cdot 0.4} = 5\ m^{-1}$$

$$R_{E_{b2}J_{2o}} = \frac{1-\varepsilon_{2o}}{A_2\varepsilon_{2o}} = \frac{1-0.8}{0.5\cdot 0.8} = 0.5\ m^{-1}$$

$$R_{J_{2o}E_{b3}} = \frac{1}{A_2 F_{2o3}} = \frac{1}{0.5\cdot 1} = 2\ m^{-1}$$

$$R_{J_{2i}E_{b2}} = \frac{1-\varepsilon_{2i}}{A_{2i}\varepsilon_{2i}} = \frac{1-0.1}{0.5\cdot .1} = 18\ m^{-1}$$



6 nodes
7 resistors ✓

$$R_{eq} = 25 + \left(283.5^{-1} + \left(154.5 + \left(5^{-1} + (18+2+0.5)^{-1}\right)^{-1}\right)^{-1}\right)^{-1}$$

$$R_{eq} = 126.6\ m^{-1}$$

1d) $$\dot{q}_1 - \dot{q}_c = \frac{\sigma(T_1^4 - T_3^4)}{R_{eq}} = \dot{q}_{rad} = \frac{5.67\cdot10^{-8}\frac{W}{m^2K^4}(1800^4 - 300^4)}{126.6\,m^{-1}}$$

$$\boxed{\dot{q}_{rad} = 4695\,W/m}$$

$$\dot{q}_{cond} = \frac{T_1 - T_3}{R_c} = \frac{(1800-300)K}{10\,Km/W} = \boxed{150\,W/m}$$

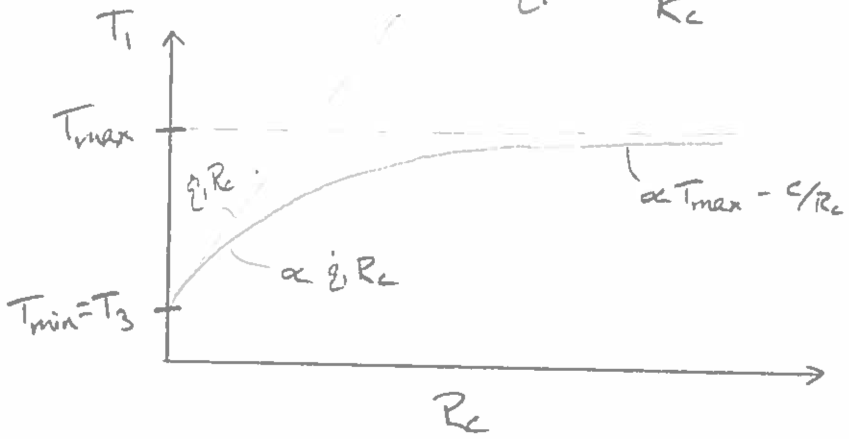
e) $$\dot{q}_1 = \frac{\sigma(T_1^4 - T_3^4)}{R_{eq_{1,3}}} - \frac{T_1 - T_3}{R_c}$$

for $R_c \to \infty$ all heat txfr radiative

$$T_{1,max} = \left(\frac{\dot{q}_1 R_{eq_{1,3}}}{\sigma} + T_3^4\right) \Rightarrow T_1 \propto T_{max} - \frac{c}{R_c} \text{ as } R_c \text{ near } \infty$$

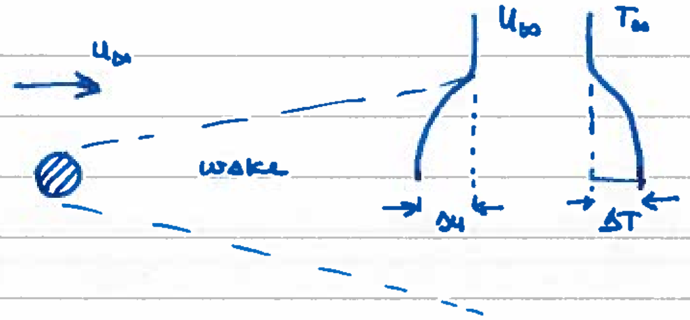
for $R_c \to 0$ all heat txfr conductive

$$T_{1,min} = T_3 \qquad \dot{q}_1 \simeq \frac{T_1 - T_3}{R_c} \qquad T_1 \propto R_c\dot{q}_1 + T_3 \text{ as } R_c \text{ near } 0$$

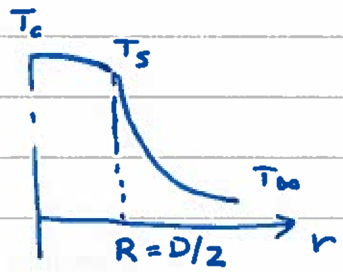


$\boxed{\text{WAKE}}$

(a)



(b) TEMPERATURE WITHIN THE SPHERE



$$\frac{T_c - T_s}{T_s - T_\infty} \ll 1$$

STEADY HEAT TRANSFER : $\quad Q = h\,(T_s - T_\infty) \approx \frac{\lambda_m}{D}(T_c - T_s)$

$$\therefore \quad \frac{T_s - T_\infty}{T_c - T_s} \gg 1 \quad \rightarrow \quad \frac{\lambda}{hD} \gg 1 \quad \text{or} \quad \frac{hD}{\lambda_m} \ll 1$$

BIOT NUMBER.

(c) HEAT TRANSFER FROM SPHERE TO SURROUNDINGS $\boxed{2}$

$$\frac{d}{dt}\left(\rho_m c_m V T_s\right) = -hA\left(T_s - T_\infty\right)$$

$$\frac{d \ln\left(T_s - T_\infty\right)}{dt} = -\frac{hA}{\rho_m c_m V} = -\frac{h\pi D^2}{\rho_m c_m \pi D^3/6} = -\frac{6h}{\rho_m c_m D}$$

$$\frac{T_s - T_\infty}{T_0 - T_\infty} = \exp\left(-\frac{6h}{\rho_m c_m D}t\right) = \exp\left(-t/\tau\right)$$

$$\tau = \frac{\rho_m c_m D}{6h} = \frac{\rho_m c_m D^2}{6\, Nu_D \lambda_\infty}$$

(d)

$$\dot H = \int_A \rho h \underline{u} \cdot d\underline{A}$$

$$\dot H(z) = \int_0^\infty \rho u h \; 2\pi r \, dr$$

$$\dot H(z) - \dot H_\infty = \int_0^\infty \left(\rho u h - \rho_\infty u_\infty h_\infty\right) 2\pi r \, dr$$

(d) (cont)

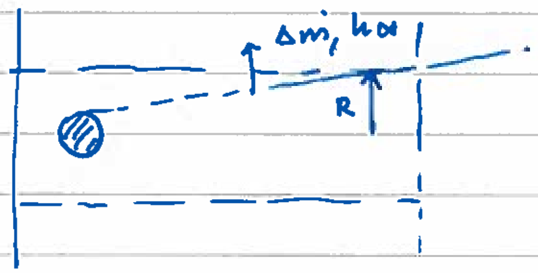
$$\dot{H} - \dot{H}_\infty = \int_0^\infty \rho_\infty C_{p\infty} \Big\{ \left[u_\infty - \Delta u \exp(-\beta r^2) \right] \left[T_\infty + \Delta T \exp(-\beta r^2) \right] - u_\infty T_\infty \Big] \cdot 2\pi r \, dr$$

$$= \rho_\infty C_{p\infty} \int_0^\infty \Big[\cancel{u_\infty T_\infty} - \Delta u \, T_\infty \exp(-\beta r^2) + \Delta T \, u_\infty \exp(-\beta r^2) - \Delta u \, \Delta T \exp(-2\beta r^2) - \cancel{u_\infty T_\infty} \Big] 2\pi r \, dr$$

$$= \rho_\infty C_{p\infty} u_\infty T_\infty \left[\frac{\Delta T}{T_\infty} - \frac{\Delta u}{u_\infty} - \frac{1}{2} \frac{\Delta u}{u_\infty} \frac{\Delta T}{T_\infty} \right] \frac{\pi}{\beta}$$

//

(e) We apply energy conservation + mass under steady conditions to the following CV:



Mass: $\dot{m}_\infty - \dot{m} = \Delta \dot{m} = \int_0^R \rho_\infty (u_\infty - u) \, 2\pi r \, dr$

$$= \rho_\infty \Delta u \int_0^R e^{-\beta r^2} 2\pi r \, dr =$$

$r \to \infty$ $\quad \Delta \dot{m} = \rho_\infty \Delta u \int_0^\infty e^{-\beta r^2} 2\pi r \, dr =$

$$\Delta \dot{m} = \rho_\infty u_\infty \frac{\Delta u}{u_\infty} \frac{\pi}{\beta}$$

ENERGY:

$$\underbrace{\dot{H}_\infty - \dot{H}} - \underbrace{\Delta\dot{m}\, h_\infty} + \dot{Q} = 0$$

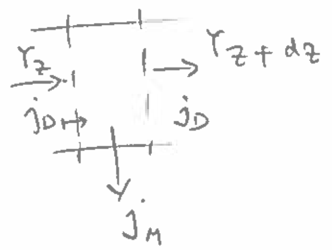
$\dot{Q}$: HEAT TRANSFER FROM SPHERE = energy change in sphere

$$\rho_\infty c_\infty u_\infty T_\infty \frac{\pi}{\beta}\left[\frac{\Delta T}{T_\infty} - \frac{\Delta u}{u_\infty} - \frac{1}{2}\frac{\Delta u}{u_\infty}\frac{\Delta T}{T_\infty} + \frac{\Delta u}{u_\infty}\right] = h\pi D^2 (T_s - T_\infty)$$

$$\frac{\Delta T}{T_s - T_\infty}\left(1 - \frac{1}{2}\frac{\Delta u}{u_\infty}\right) = \frac{h\beta D^2}{\rho_\infty c_\infty u_\infty} = Nu_D \frac{\alpha_\infty \beta D}{u_\infty}$$

3

(a) THE PROBLEM IS PURELY DIFFUSIVE, WITH NO RADIAL GRADIENTS, AND ONLY A RESISTANCE AT THE SURFACE :



$$\dot{j}_D = -\rho D \frac{dY_z}{dz}$$

$$\pi R^2 \left\{ -\rho D \frac{dY}{dz}\Big|_z - \left[-\rho D \frac{dY}{dz} \right]_{z+dz} \right\} - \dot{j}_M 2\pi R = 0$$

$$\pi R^2 \rho D \frac{d^2 Y}{dz^2} - \dot{j}_M 2\pi R = 0$$

$$\frac{d^2 Y}{dz^2} - \frac{2\dot{j}_M}{\rho D R} = 0$$

(b) MODEL $\dot{j}_M$ AS STEADY-STATE RESISTANCE ACROSS THE MEMBRANE :

$$\dot{j}_M = \rho \frac{Y - Y_\infty}{R_M + 1/h_M}$$

$$R = R_M + \frac{1}{h_M}$$

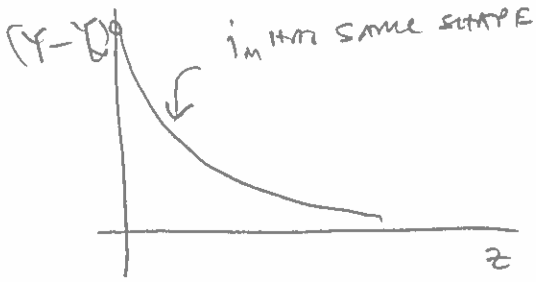
$$\frac{d^2(Y - Y_\infty)}{dz^2} - \frac{1}{RD}\frac{1}{R_M + 1/h_M}(Y - Y_\infty)$$

$$\frac{Y - Y_\infty}{Y_0 - Y_\infty} = \xi \rightarrow \frac{d^2\xi}{dz^2} - m^2 \xi = 0 \qquad m^2 = \frac{1}{2D}\frac{1}{R_M + 1/h_M}$$

$$\xi = A \exp(-mx) + B \exp(mx)$$

BC : $\xi(0) = 1$
$\xi(\infty) = 0$ $\Big\}$ $\xi = \exp(-mx)$

3 (b) (cont)



$$\frac{j_M}{\rho D} = \left(\frac{Y - Y_\infty}{R_M + 1/h_m}\right)$$

(c) THE CONDITION FOR THE RADIAL GRADIENTS TO BE SMALL ACROSS THE CYLINDER IS

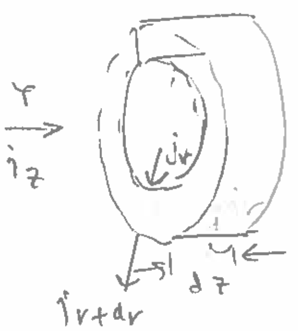
$$\frac{Y_c - Y_{s,i}}{Y_c - Y_\infty} \ll 1$$

THE RADIAL FLUX ACROSS r IS

$$j_M \sim \rho D\left(\frac{Y_c - Y_{s,i}}{R}\right) \sim \rho \frac{Y_{s,i} - Y_\infty}{R_M + 1/h_m}$$

$$\frac{Y_c - Y_{s,i}}{Y_{s,i} - Y_\infty} \ll 1 \rightarrow \boxed{\frac{R/D}{R_M + 1/h_m} \ll 1}$$

(d) CONSIDER A SHELL OF DIMENSIONS $r + dr$ AND $z + dz$



$$\left[-\rho D \frac{\partial Y}{\partial z} - \left[-\rho D \frac{\partial Y}{\partial z}\right]_{z+dz}\right] 2\pi r\, dr \; +$$

$$\left[-\rho D\, 2\pi r \frac{\partial Y}{\partial r} - \left[-\rho D\, 2\pi r \frac{\partial Y}{\partial r}\right]_{r+dr}\right] dz$$

$$+\left[\rho u_z Y_z - \rho u_z Y\big|_{z+dz}\right] 2\pi r\, dr = 0$$

EXPANDING THE 1ST ORDER TERMS IN r AND z:

$$\left[2\pi \rho D \frac{\partial^2 Y}{\partial z^2} + \rho D \frac{\partial}{\partial r}\left(2\pi r \frac{\partial Y}{\partial r} \right) + \rho u_z \frac{\partial Y}{\partial z} \right] = 0$$

$$\frac{\partial^2 Y}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial Y}{\partial r} \right) - \frac{u_z}{D}\frac{\partial Y}{\partial z} = 0$$

(e) BOUNDARY CONDITIONS:

(i) $Y(0) = Y_0$

THE FLUX AT THE BOUNDARY IS DETERMINED BY THE RESISTANCE OF THE MEMBRANE AND OUTER FLOW:

(ii) $$\rho D \left.\frac{\partial Y}{\partial r}\right|_{r=R} = \rho \frac{Y(R) - Y_\infty}{R_M + 1/h_m}$$

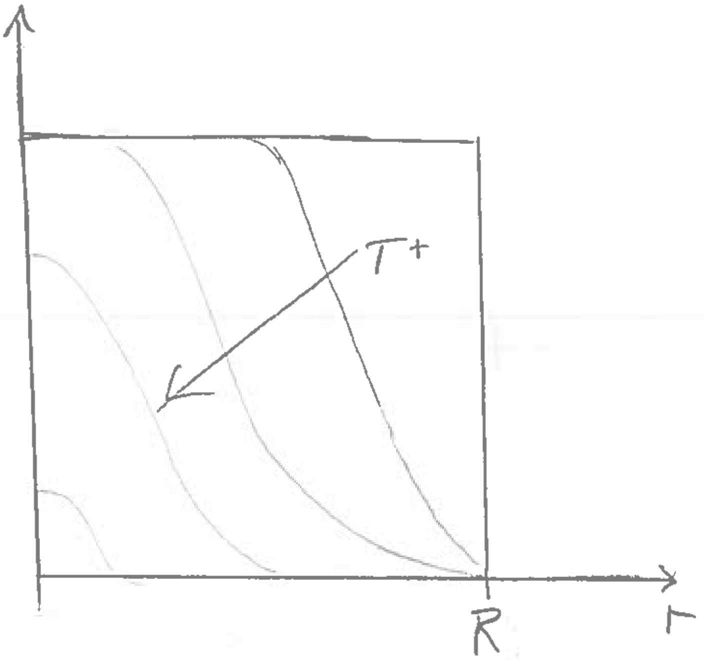
CONDITIONS FOR A very long FIN MEAN THAT THE MASS FRACTION Y GOES to Y_∞ AT $x \to L$..

(iii) $Y(z \to L) = Y_\infty$

(iv) SYMMETRY CONDITIONS DICTATE THAT $\left.\frac{\partial Y}{\partial r}\right|_{r=0} = 0.$

REDUNDANTLY, DIFFUSIVE FLUXES AT THE LONG END ARE ALSO ZERO, AT $Y = Y_\infty \rightarrow \left.\frac{\partial Y}{\partial z}\right|_{z \to L} = 0.$

4a) Cooling Sphere



b) I.C. $T(r, t=0) = T_0$

B.C. $\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0$

$T_{r=R} = T_\infty$

c) Non-Dimensional analysis

$\Theta = \frac{T - T_\infty}{T_0 - T_\infty}$ - given

α [m^2/s]

r [m]

R_0 [m]

t [s]

$\hat{r} = r/R$

$\hat{t} = \frac{\alpha}{R^2} t$

4 variables - 2 dimensions = 2 new non-D variables

4 c (cont'd)

$$\frac{\partial T}{\partial t} = \frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)$$

$T = \Theta(T_\infty - T_0) + T_0 \qquad t = \frac{R_0^2}{\alpha}\hat{\tau} \qquad r = \hat{r}R_0 \qquad \hat{\tau} = \frac{\alpha}{R_0^2}t$

$$\frac{\partial\hat{\tau}}{\partial t}\frac{\partial T}{\partial\hat{\tau}} = \frac{\alpha}{r^2}\frac{\partial\hat{r}}{\partial r}\frac{\partial}{\partial\hat{r}}\left(r^2\frac{\partial\hat{r}}{\partial r}\frac{\partial T}{\partial\hat{r}}\right) \qquad \frac{\partial\hat{\tau}}{\partial t} = \frac{\alpha}{R_0^2} \qquad \frac{\partial\hat{r}}{\partial r} = \frac{1}{R_0}$$

$$\frac{\cancel{\alpha}}{\cancel{R_0^2}}\frac{\partial(\Theta(T_0 - T_\infty) + T_\infty)}{\partial\hat{\tau}} = \frac{\cancel{\alpha}}{\hat{r}^2\cancel{R_0^2}}\frac{1}{\cancel{R_0}}\frac{\partial}{\partial\hat{r}}\left(\cancel{R_0^2}\hat{r}^2\frac{1}{\cancel{R_0}}\frac{\partial(\Theta(T_\infty - T_\infty) + T_\infty)}{\partial\hat{\tau}}\right)$$

$$\boxed{\frac{\partial\Theta}{\partial\tau} = \frac{1}{\hat{r}^2}\frac{\partial}{\partial\hat{r}}\left(\hat{r}^2\frac{\partial\Theta}{\partial\hat{\tau}}\right)}$$

B.C.'s $\left.\frac{\partial T}{\partial r}\right|_{r=0} = 0 = \frac{1}{R_0}\left.\frac{\partial(\Theta(T_0 - T_\infty) + T_\infty)}{\partial\hat{r}}\right|_{\hat{r}=0} = 0$

$$\left.\frac{\partial\Theta}{\partial\hat{r}}\right|_{\hat{r}=0} = 0$$

$\boxed{\left.\frac{\partial\Theta}{\partial\hat{r}}\right|_{\hat{r}=0} = 0}$ B.C. 1

B.C. 2 $T_{r=R} = T_\infty \qquad \Theta_R = \frac{T_\infty - T_\infty}{T_0 - T_\infty}$

$\boxed{\Theta|_{\hat{r}=1} = 0}$ B.C. 2

I.C. $T|_{t=0} = T_0 \qquad \Theta_0 = \frac{T_0 - T_\infty}{T_0 - T_\infty}$

$\boxed{\Theta_{\tau=0} = 1}$ I.C.

d) $\Theta(\hat{r},\tau) = \psi(\hat{r},\tau)/\hat{r}$

$$\frac{\partial(\psi/\hat{r})}{\partial\hat{\tau}} = \frac{1}{\hat{r}^2}\frac{\partial}{\partial\hat{r}}\left(\hat{r}^2\frac{\partial(\psi/\hat{r})}{\partial\hat{r}}\right) \Rightarrow \frac{1}{\hat{r}}\frac{\partial\psi}{\partial\tau} = \frac{1}{\hat{r}^2}\frac{\partial}{\partial\hat{r}}\left(\hat{r}^2\left(-\frac{\psi}{\hat{r}^2} + \frac{1}{\hat{r}}\frac{\partial\psi}{\partial\hat{r}}\right)\right)$$

$$\frac{\partial\psi}{\partial\hat{\tau}} = \frac{1}{\hat{r}}\frac{\partial}{\partial\hat{r}}\left(-\psi + \hat{r}\frac{\partial\psi}{\partial r}\right) = \frac{1}{\hat{r}}\left(-\cancel{\frac{\partial\psi}{\partial r}} + \cancel{\frac{\partial\psi}{\partial r}} + \hat{r}\frac{\partial^2\psi}{\partial r^2}\right)$$

$$\boxed{\frac{\partial\psi}{\partial\hat{\tau}} = \frac{\partial^2\psi}{\partial r^2}}$$

w/ B.C.'s

$\boxed{\left.\frac{\partial\psi}{\partial\hat{r}}\right|_{\hat{r}=0} = 0}$ B.C. 1

$\boxed{\psi|_{\hat{r}=1} = 0}$ B.C. 2

$\boxed{\psi|_{\tau=0} = \hat{r}}$ I.C. 1

4 e) $$\frac{\partial\psi}{\partial\hat{\tau}} = \frac{\partial^2\psi}{\partial\hat{r}^2}$$

B.C. 1 $\quad \psi'|_{\hat{r}=0} = 0$

B.C. 2 $\quad \psi|_{\hat{r}=1} = \underline{1}\,\hat{r}$

I.C. $\quad \psi|_{\hat{\tau}=0} = 0$

$\psi = T\,R$ – seperation of variables

$$R\frac{dT}{\partial\hat{\tau}} = T\frac{d^2R}{d\hat{r}^2} \Rightarrow \frac{1}{T}\frac{dT}{\partial\hat{\tau}} = \frac{1}{R}\frac{d^2R}{d\hat{r}^2} = -\omega^2$$

$R^2 + \omega^2 R = 0, \quad R_0 = 0, \quad R'|_{\hat{r}=1} + R|_{\hat{r}=1}(B_i - 1) = B_i$

$R(\hat{r}) = A_1 \sin\omega^2\hat{r} + B_1\cos\omega^2\hat{r}$ $\quad$ w/ $R(0) = 0$, $R(1) = 0$

Applying B.C.'s $\quad B_1 = 0$

$R(\hat{r}=1) = A_1 \sin\omega^2 1 = 0$

$\omega^* = \sqrt{n\pi}$, $n = 1, 2, 3, \ldots$

$T' + \omega^2 T = 0$

$T = C_1 e^{-\omega^2\tau}$

$$\psi = RT = \sum_{n=1}^{\infty} \overbrace{D_1}^{A_1C_1} e^{-\omega^2\tau}\sin\omega^2\hat{r}$$

I.C. $\quad \psi(0) = \hat{r} = \sum_{n=1}^{\infty} D_1 \cancelto{1}{e^{-\omega^2 0}} \sin\omega^2\hat{r}$

$D_1 = (-1)^{n+1}\frac{2}{\omega}$

$$\therefore \boxed{\Theta = 2\sum_{n=1}^{\infty}(-1)^{n+1} e^{-\omega^2\tau}\frac{\sin(\omega_n\hat{r})}{\omega_n\hat{r}}, \quad \omega_n = \sqrt{n\pi}}$$