EGT2
ENGINEERING TRIPOS PART IIA

Fri 27 Apr 2018   9.30 to 11.10

Module 3A6

HEAT AND MASS TRANSFER

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 A lamp is composed of a linear filament and focused using a parabolic reflector, as shown in Fig. 1. The filament has an area per unit length $A_1 = 0.01\ \mathrm{m}^2$ with an emissivity of $\varepsilon_1 = 0.8$ and it has a temperature of $T_1 = 1800$ K at steady state. The heater is partially enclosed by a long, thin parabolic reflector with area per unit length $A_2 = 0.5\ \mathrm{m}^2$, whose inner and outer surface emissivities are $\varepsilon_{2i} = 0.1$ and $\varepsilon_{2o} = 0.8$, respectively. The system may be assumed to be at steady state within an infinite medium with temperature $T_3 = 300$ K. The filament is held by an optically transparent support, which has a conductive thermal resistance of $R_\mathrm{c} = 10\ \mathrm{K\ m\ W^{-1}}$ and is only in thermal contact with the filament and the environment at temperature T_3. Convective losses can be neglected.

(a) Sketch the appropriate radiation circuit, and write expressions for each of the network resistances. [20 %]

(b) Calculate the appropriate view factors for each surface with the others. You may assume that the filament is small relative to the surroundings such that the view factor of the surroundings to the inner surface of the reflector is $F_{32i} = 1$. [15 %]

(c) Calculate the effective resistance $R_\mathrm{eff,rad}$ for the total radiation circuit from the filament to the environment. [25 %]

(d) Calculate the radiative heat loss, $\dot{q}_{1,\,\mathrm{rad}}$, and conductive heat loss, $\dot{q}_{1,\,\mathrm{c}}$, from the filament. [15 %]

(e) For a fixed filament power $\dot{q}_1 = \dot{q}_{1,\,\mathrm{rad}} + \dot{q}_{1,\,\mathrm{c}}$, determine the limits of filament temperature T_1 for high and low conduction, for which $R_\mathrm{c} \to 0$ and $R_\mathrm{c} \to \infty$, respectively. Sketch the variation of the filament temperature T_1 versus R_c. [25 %]

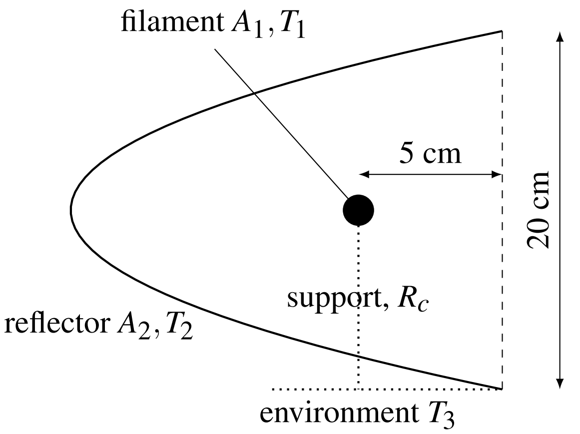

Fig. 1

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2 A metal sphere of diameter D, initially at a uniform temperature T_0 is cooled by a flow of fluid at a free stream velocity U_∞ and temperature T_∞. A cylindrical wake is established at some distance z downstream of the sphere, where the local mean axial velocity u and temperature T are found to fit the following functions

$$\frac{U_\infty - u}{\Delta u} = \frac{T - T_\infty}{\Delta T} = \exp(-\beta r^2)$$

where Δu and ΔT are the absolute velocity and temperature differences between the centreline and outer flow, respectively, r is the radial coordinate, and β is a constant. The properties of the metal are density ρ_m, specific heat capacity c_m, and conductivity λ_m. The fluid has constant density ρ_∞, specific heat capacity at constant pressure c_∞ and conductivity λ_∞. The mean Nusselt number for heat transfer with the sphere is $Nu_D = \dfrac{hD}{\lambda_\infty}$, where h is the convective heat transfer coefficient.

(a) Sketch the physical layout of the flow and object, including the wake and corresponding velocity and temperature profiles. Label your diagram clearly. [10%]

(b) Obtain a criterion for which the temperature of the sphere T_s can be assumed to be uniform. [15%]

(c) Assuming uniform temperature for the sphere, show that the temperature difference $T_s - T_\infty$ between the sphere and the surrounding fluid is given by: [20%]

$$T_s - T_\infty = (T_0 - T_\infty)\exp(-t/\tau)$$

where $\tau = \dfrac{\rho_m c_m D^2}{6\lambda_\infty Nu_D}$

(d) Show that the enthalpy flow difference between a cross section downstream and upstream of the sphere, $\Delta\dot{H}$ is: [30%]

$$\Delta\dot{H} = \rho_\infty U_\infty c_\infty T_\infty \frac{\pi}{\beta}\left[\frac{\Delta T}{T_\infty} - \frac{\Delta u}{U_\infty} - \frac{1}{2}\frac{\Delta u}{U_\infty}\frac{\Delta T}{T_\infty}\right]$$

You may use the fact that $\int_0^\infty \exp(-Kr^2)\, 2\pi r\, dr = \frac{\pi}{K}$.

(e) Assuming that the flow conditions remain quasi-steady, with negligible accumulation in the wake, obtain an expression for $\frac{\Delta T}{T_s - T_\infty}$ as a function of $\frac{\Delta u}{U_\infty}$ and Nu_D, including any other non-dimensional parameters. [25%]

3 Consider the diffusion of nutrients and wastes through membrane barriers such as those in sea urchins. Each spike can be considered as an extended surface for mass transfer of a target substance. We can analyse a single protrusion as a cylindrical surface, with a length L and radius R, shown as a cross section in Fig. 2. The inside of the organism contains a uniform mass fraction Y_0, which can diffuse through the fluid of density ρ with a mass diffusion coefficient D. The interface of the membrane is porous to the target substance only, and not the diluent water. It offers a mass diffusion resistance R_m, and is subject to an outer convection coefficient h_m for surroundings with mass fraction Y_∞. Convection terms can be neglected for the transport inside the cylinder, and the overall flow of the permeable species is steady and outwards.

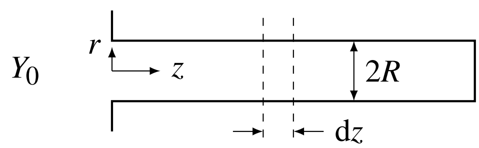

Fig. 2

(a) Assume first that gradients of species in the radial direction within the spike are negligible. Using a species balance, write a differential equation for the species mass fraction Y across an element $\mathrm{d}z$ as a function of the fluid properties and the flux, j_m, which is assumed positive in the outward direction. [20%]

(b) Solve the equation for the mass fraction, for $Y = Y_0$ at $z = 0$, and the assumption of a very long cylinder. Sketch the mass fraction and its flux across the membrane as a function of z. [20%]

(c) Determine a condition involving the fluid properties, geometry and flow resistances for which the assumption of one-dimensional diffusion in part (a) is reasonable. [20%]

(d) Derive the general species conservation equation *including* one-dimensional convection in the z direction with a constant velocity u_z, and diffusion in r and z. [20%]

(e) Write detailed boundary conditions for the equation in part (d), for a given concentration at the inlet, radial fluxes controlled subject to membrane resistance and outer convection, and a very long cylinder. Do not solve it. [20%]

4 A sphere of radius R with initial temperature of T_o is plunged into a liquid bath at time $t = 0$ with temperature T_∞, where $T_o > T_\infty$. The heat exchange between the bath and the sphere is fast such that the surface temperature of the sphere is constant.

(a) Sketch the temperature T versus the radius r within the sphere from 0 to R for increasing times. [10%]

(b) The governing conservation of energy equation for spherical coordinates is

$$\frac{\partial T}{\partial t} = \frac{\alpha}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial T}{\partial r}\right)$$

where thermal diffusivity is given by $\alpha = \lambda/(\rho c)$, and λ is the thermal conductivity, ρ is the density and c is the specific heat capacity of the sphere material. Give the equations for the appropriate initial and boundary conditions. *Hint*: Use symmetry at $r = 0$ to define one boundary condition. [10%]

(c) Using the results from (b) derive a non-dimensional governing energy conservation equation for 1-D spherical conduction, as well as non-dimensional boundary conditions. Use $\theta = \dfrac{T - T_\infty}{T_o - T_\infty}$ as the non-dimensional temperature. [25%]

(d) Show that you may reduce the problem to an equivalent cartesian 1-D heat transfer problem with the use of $\theta(\hat{r}, \hat{\tau}) = \psi(\hat{r}, \hat{\tau})/\hat{r}$ resulting in

$$\frac{\partial \psi}{\partial \hat{\tau}} = \frac{\partial^2 \psi}{\partial \hat{r}^2},$$

where $\hat{r}$ and $\hat{\tau}$ are the non-dimensional radial dimension and time, respectively. Provide the equivalent boundary and initial conditions in terms of ψ. [25%]

(e) Use separation of variables to solve the non-dimensional differential energy equation. [30%]

END OF PAPER

Answers

1.
(a) -
(b) $F_{13} = 0.35$, $F_{12i} = 0.65$, $F_{2i1} = 0.013$, $F_{2i3} = 0.40$, $F_{2i2i} = 0.286$
(c) 126.6 kW m^{-1}
(d) $q_{1,rad}$ = 4695 W m^{-1}, $q_{1,c}$ = 150 W m^{-1}
(e) -

2.
(a) -
(b) $hd/\lambda_m << 1$
(c) -
(d) -
(e) $\frac{\Delta T}{T_s - T_\infty} = Nu_D \dfrac{\alpha_\infty \beta D}{U_\infty} \left(1 - \frac{1}{2}\frac{\Delta U}{U_\infty}\right)^{-1}$

3.
(a) $\frac{d^2T}{dz^2} + \frac{2j_m}{\rho DR} = 0$
(b) $\frac{j_m}{\rho D} = \frac{Y - Y_\infty}{R_m + 1/h_m}$
(c) $\frac{R/D}{R_m + 1/h_m} << 1$
(d) $\frac{\partial^2 Y}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial Y}{\partial r}\right) - \frac{u_z}{D}\frac{\partial Y}{\partial z} = 0$
(e) $Y(0) = Y_0$, $\rho D \left[\dfrac{\partial Y}{\partial r}\right]_{r=R} = \rho \frac{Y(R) - Y_\infty}{R_m + 1/h_m}$, $Y(L \to \infty) = Y_\infty$

4.
(a) -
(b) $T(r,0) = T_0$, $\left[\frac{\partial T}{\partial r}\right]_{r=0} = 0$, $T(R) = T_\infty$
(c) $\frac{\partial \theta}{\partial \hat{\tau}} = \frac{1}{\hat{r}^2}\frac{\partial}{\partial \hat{r}}\left(\hat{r}^2 \frac{\partial \theta}{\partial \hat{\tau}}\right)$
B.C.: $\left[\frac{\partial \theta}{\partial \hat{r}}\right]_{\hat{r}=0} = 0$, $\theta(1, \hat{\tau}) = 0$, I.C.: $\theta(\hat{r}, 0) = 1$
(d) B.C.: $\psi(1, \hat{\tau}) = 0$, $\left[\frac{\partial \psi}{\partial r}\right]_{\hat{r}=0} = 0$, B.C.: $\psi(\hat{r}, 0) = \hat{r}$
(e) $\theta = 2\sum_{n=1}^{\infty}(-1)^{n+1} e^{-\omega^2 \hat{\tau}} \left(\frac{\sin(\omega_n \hat{r})}{\omega \hat{r}}\right)$, $\omega_n = \sqrt{n\pi}$