CRIB for 3C9

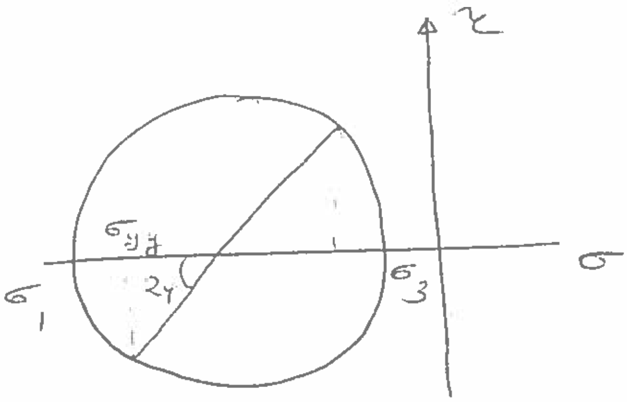
EGT2, ENGINEERING TRIPOS
PART IIA

Paper 3C9 : Fracture Mechanics of Materials and Structures.

(2017 - 2018)

CRIB

1. (a)



$$\sigma_{xy} = \frac{\sigma_3 - \sigma_1}{2} \sin 2\psi$$

(b) $\sigma_{\theta\theta}$ due to crack tip field is solely due to σ_{xy} as the normal stress component σ_{yy} is compressive & shuts the crack.

$\sigma_{\theta\theta}$ from K_{II} field in data sheet is

$$\sigma_{\theta\theta} = \frac{-\sigma_{xy}\sqrt{\pi a}}{\sqrt{2\pi r}} \frac{3}{4}\left(\sin\frac{\theta}{2} + \sin 3\frac{\theta}{2}\right)$$

Given $K_I \equiv \sigma_{\theta\theta}\sqrt{2\pi r}$

$$K_I = -(\sigma_3 - \sigma_1)\sin 2\psi \sqrt{\pi a}\,\frac{3}{4}\left[\sin\frac{\theta}{2} + \sin 3\frac{\theta}{2}\right]$$

Maximise w.r.t. θ $\Rightarrow$ maximise $\sin\frac{\theta}{2} + \sin\frac{3\theta}{2}$

$$\frac{d}{d\theta}\left(\sin\frac{\theta}{2} + \sin 3\frac{\theta}{2}\right) = \frac{1}{2}\cos\frac{\theta}{2} + \frac{3}{2}\cos\frac{3\theta}{2}$$

$$= \frac{3}{2}\left[\cos\frac{3\theta}{2} + \cos\frac{\theta}{2}\right] - \cos\frac{\theta}{2}$$

$$= 3\cos\theta\cos\frac{\theta}{2} - \cos\frac{\theta}{2}$$

$$\cos\frac{\theta}{2}\left(3\cos\theta - 1\right) = 0$$

$\Rightarrow$ K_I max at ~~θ~~ $\cos\theta = \frac{1}{3}$

$$\sin\theta = \frac{2\sqrt{2}}{3}$$

$$\Rightarrow \cos^2\frac{\theta}{2} = \frac{1}{2}\left(1+\cos\theta\right)$$

$$\cos\frac{\theta}{2} = \sqrt{\frac{2}{3}}$$

$$\Rightarrow K_I^{max} = -\frac{3}{2}\left(\sigma_3 - \sigma_1\right)\sin 2\psi\,\sqrt{\pi a}\,\left(\sin\theta\cos\frac{\theta}{2}\right)$$

where $\cos\theta = \frac{1}{3}$

$$\Rightarrow K_I^{max} = -\frac{2}{\sqrt{3}}\left(\sigma_3 - \sigma_1\right)\sin 2\psi\,\sqrt{\pi a}$$

(c) K_I^{max} maximised at max $(\sin 2\psi)$

$\Rightarrow$ $\psi = 45^\circ$ for critical cracks

(d)

$$-\frac{2}{\sqrt{3}}\left(\sigma_3 - \sigma_1\right)\sqrt{\pi a}\,\sin 2\underset{}{\overset{\psi=45^\circ}{\psi}} = K_{IC}$$

$$\Rightarrow \sigma_1 - \sigma_3 = \frac{\sqrt{3}\,K_{IC}}{2\sqrt{\pi a}}$$

(e) If friction co-eff μ between crack flanks,

max. frictional stress $= \mu \sigma_{yy}$

$$\sigma_f = \mu \left[\frac{\sigma_1 + \sigma_3}{2} - \frac{(\sigma_3 - \sigma_1)}{2} \sin 2\psi \right]$$

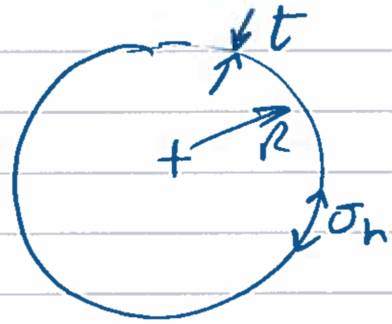
$\Rightarrow$ ~~[illegible]~~

$\Rightarrow$ No. singular stress field for $\sigma_{xy} \leq \sigma_f$

& for $\sigma_{xy} > \sigma_f$ repeat above analysis

with σ_{xy} replaced by $\sigma_{xy}' = \sigma_{xy} - \sigma_f$

Q2. (a)



$$p\pi R^2 = 2\pi R t \sigma_h$$

$$\Rightarrow \sigma_h = \frac{R}{2t} p$$

Semi-circular flaw $a = c$

$$\Rightarrow K = \frac{1.12}{\Phi} \sigma \sqrt{\pi a} \qquad \Phi = 1.56$$

$$\Rightarrow K = \frac{1.12\sqrt{\pi}}{\Phi} \sigma \sqrt{a} = 1.273\,\sigma\sqrt{a}$$

write $\alpha = 1.273$, then $K = \alpha \sigma \sqrt{a}$

$$K = K_{IC} \Rightarrow \alpha \sigma_p \sqrt{a_0} = \alpha p_p \frac{R}{2t} \sqrt{a_0} = K_{IC}$$

$$\Rightarrow a_0 = \left(\frac{2t}{\alpha R} \frac{K_{IC}}{p_p} \right)^2$$

(b) (i) If $\Delta K \leq \Delta K_{th}$, then the fatigue life is infinite.

$$\Delta K = \alpha p_{max} \frac{R}{2t} \sqrt{a_0} \leq \Delta K_{th}$$

$$\Rightarrow \frac{p_{max}}{p_p} \leq \frac{\Delta K_{th}}{K_{IC}} \quad \text{for infinite life.}$$

$$\frac{da}{dN} = C \Delta K^m = C\left(\alpha p_{max} \frac{R}{2t} \sqrt{a}\right)^m$$

$$\Rightarrow \int_{a_0}^{a_f} \frac{da}{a^{m/2}} = N_f\, C \left(\alpha p_{max} \frac{R}{2t}\right)^m$$

$$\Rightarrow \frac{2}{m-2} a_0^{\frac{2-m}{2}} - \frac{2}{m-2} a_f^{\frac{2-m}{2}} = N_f\, C \left(\alpha \frac{p_{max} R}{2t}\right)^m$$

for $m \neq 2$

Q2. (b) (i) contd.

If $m = 2$: $\quad \ln \dfrac{a_f}{a_0} = N_f \, C \left(\alpha \dfrac{p_{max} R}{2t} \right)^m$

$a_f = ?$ $\quad K_{max} = K_{Ic} \Rightarrow a_f = \left(\dfrac{2t}{\alpha R} \dfrac{K_{Ic}}{p_{max}} \right)^2$

(ii)

Here, $a_f \gg a_0$ and so the fatigue life is dictated by the value of a_0.

A large K_{Ic} value $\Rightarrow$ a large safe a_0 value in the proof test $\Rightarrow$ a shorter fatigue life N_f. This is counter-intuitive.

A better way of measuring a_0 is to perform non-destructive evaluation tests (NDE) e.g. by X-ray.

(iii) For leak-before-break ensure that a_f exceeds t.

At $a_f = t$ we have

$$K = \alpha \, p_{max} \frac{R}{2t} \sqrt{t} = K_{Ic}$$

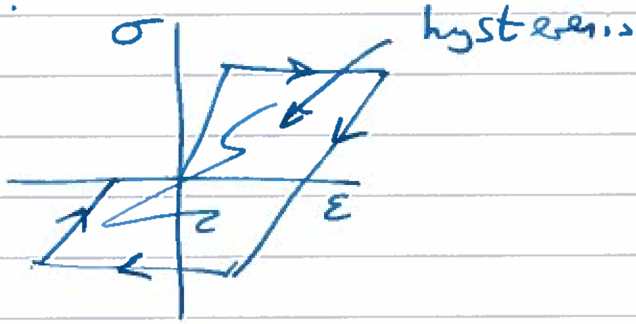
So ensure that $K_{Ic} > \alpha \, p_{max} \dfrac{R}{2\sqrt{t}}$

Q2. (c)
- use a higher pressure p_p in proof test
- replace ~~the~~ proof test by NDE methods to actually measure a_0
- reduce a_0 by grinding the surface
- shot peen the surface to confer compressive residual stress

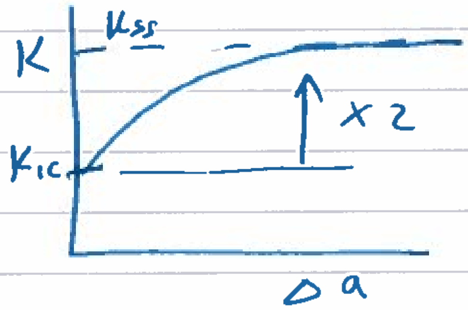
Q2.(d) Make sure the steel has a low volume fraction of small inclusions, to give a large spacing of voids at the crack tip.

Q3. (a) The increasing resistance to crack advance is due to irreversible plastic work.

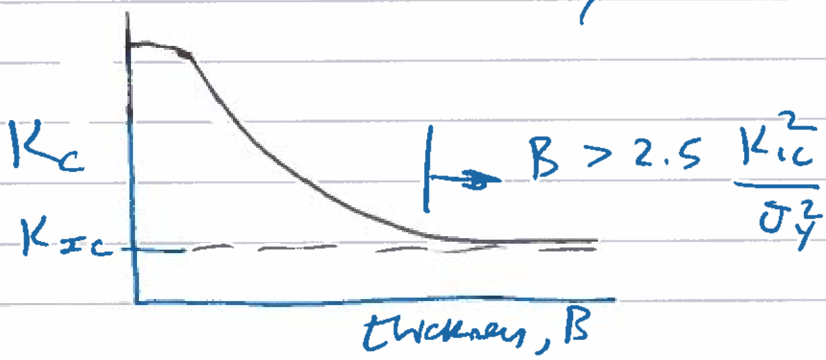
As the crack advances, material is sheared in one direction and then back in the opposite direction, and plastic hysteresis occurs.



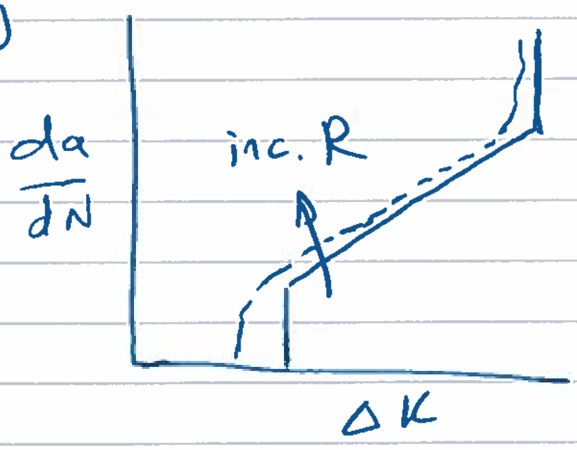
Hence :



The thickness of the plate dictates the level of hydrostatic stress ∴ plastic constraint raises the hydrostatic stress and lowers the R-curve for a thick specimen.

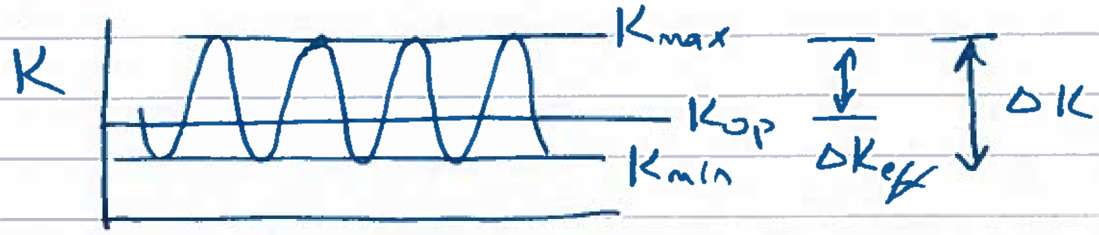


Q3. (b)



$$R = \frac{K_{min}}{K_{max}}$$

In mid-regime ('Paris regime') of the da/dN versus ΔK plot, plasticity-induced crack closure occurs with a value of K for crack tip opening, $K_{op} > 0$.



Fatigue crack growth is driven by $\Delta K_{eff} = K_{max} - K_{op}$

$$\frac{da}{dN} = C' \Delta K_{eff}^{m'} \quad \text{in mid-regime.}$$

As R increases, ΔK_{eff} increases at fixed ΔK.

Near threshold and near $K_{max} = K_{Ic}$, ~~environ~~ microstructural effects ~~become~~ become more important and there can be a switch in crack advance mechanism.

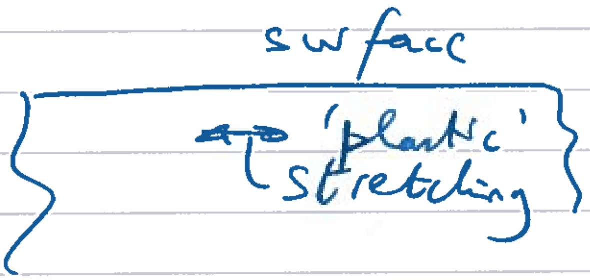
Near threshold, oxide and roughness induced crack closure can also become important.

Near $K_{max} = K_{Ic}$, microvoid coalescence can occur driven by K_{max} rather than ΔK.

Q4(b) contd.

Overloads lead to plastic stretching ahead of the crack tip & to a transient of enhanced plasticity-induced crack closure.
Overloads can also promote crack branching or to microvoid coalescence.

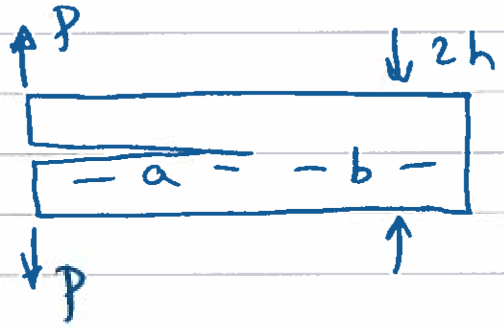
Q4(c)



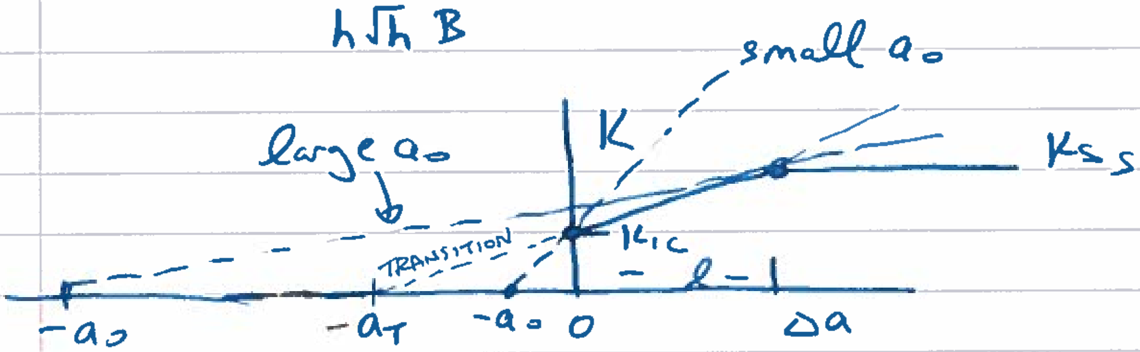
$\Rightarrow$ compressive residual stress

Compressive stresses delay crack initiation (recall the Goodman correction) and lead to a lower R-value, hence retarded da/dN.

Q3 (b) (i)



$$K = \frac{2\sqrt{3}\,a}{h\sqrt{h}\,B} \cdot P$$



At small a_0 : $K = K_{IC} = \frac{2\sqrt{3}\,a_0}{h\sqrt{h}\,B} P_{max}$

So P_{max} is set by $K = K_{IC}$

At large a_0 : $K = K_{SS} = \frac{2\sqrt{3}\,a_0}{h\sqrt{h}\,B} P_{max}$

At transition value for $a_0 = a_T$
we have :

$$\frac{K_{IC}}{a_T} = \frac{K_{SS} - K_{IC}}{l}$$

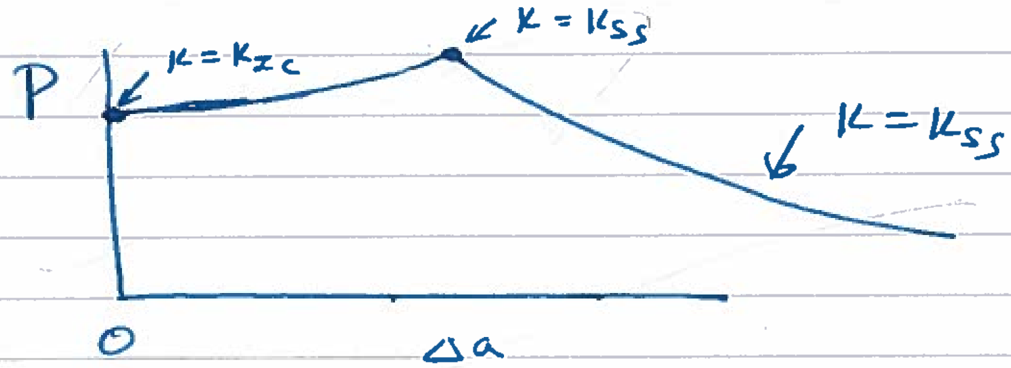
Hence a_T.

For $a_0 \gg a_T$, get initial crack advance at $K = K_{IC}$

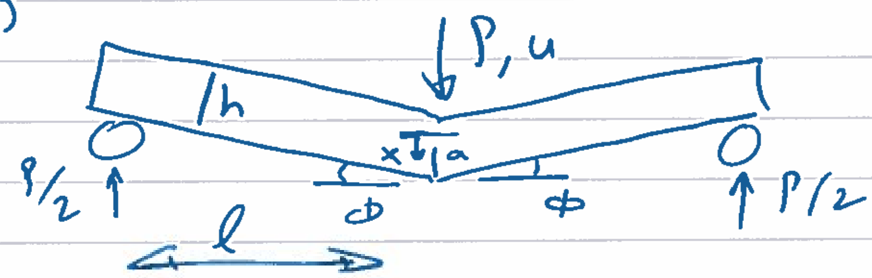
3(d)(ii)

$$\frac{2\sqrt{3}}{h\sqrt{h}B}(a_0+\Delta a)P = K_{Ic} + \left(\frac{K_{ss}-K_{Ic}}{\ell}\right)\Delta a$$

$$\Rightarrow P = \frac{h\sqrt{h}B}{2\sqrt{3}}\left[K_{Ic} + \left(\frac{K_{ss}-K_{Ic}}{\ell}\right)\Delta a\right](a_0+\Delta a)^{-1}$$

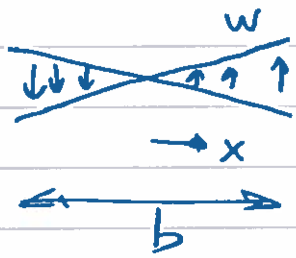


Q4. (a)

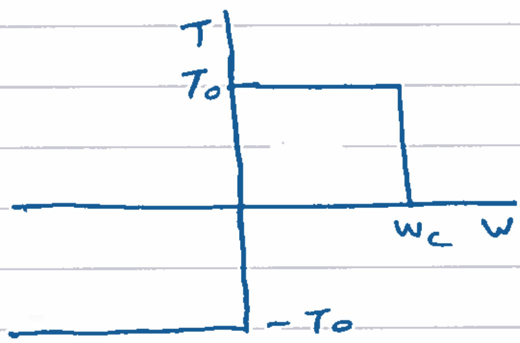


$\phi = \dfrac{u}{\ell}$

At ligament



$w = 2\phi x$



Maximum moment at ligament, $M = \dfrac{P\ell}{2}$

$M_p = T_0 \dfrac{b^2 h}{4} \Rightarrow \dfrac{P\ell}{2} = M_p$

$\Rightarrow P = \dfrac{T_0 b^2 h}{2\ell}$ independent of u.

(b) $\Psi = \int P \, du = \dfrac{T_0 b^2 h u}{2\ell}$

(c) $J = -\left.\dfrac{\partial \Psi}{\partial a}\right|_u \dfrac{1}{h} = \left.\dfrac{\partial \Psi}{\partial b}\right|_u \dfrac{1}{h} = \dfrac{T_0 \cancel{k} b u}{\ell \cancel{k}}$

Now, crack advance occurs at $J = J_{Ic}$

$\Rightarrow \dfrac{T_0 b u_c}{\ell} = J_{Ic} \Rightarrow u_c = \dfrac{\ell J_{Ic}}{T_0 b}$

Q4. d)

$$\phi = \frac{u}{l} \quad , \quad \text{and} \quad w = 2\phi . x$$

where x is the distance from the neutral section of the ligament of height b

$$w_c = 2\phi . \frac{b}{2} = \phi b \quad \text{and} \quad \phi = \frac{u}{l}$$

$$\Rightarrow \quad w_c = u_c \frac{b}{l}$$

Now $\quad J_{Ic} = \frac{\tau_0 b u_c}{l} = \underline{\tau_0 w_c}.$

e) The joint may be in a different metallurgical state from that of the bulk solid (different grain size, contain porosity, contain segregation of contaminants)!

Interfacial failure may occur for the joint, or failure within the brazed layer.

The degree of plastic constraint within the joint can be different (higher) than ~~that~~ in the bulk for the case of a thin brazed joint.