Version FC/3

EGT2
ENGINEERING TRIPOS PART IIA

Monday 23 April 2018   2 to 3.40

Module 3D4

STRUCTURAL ANALYSIS AND STABILITY

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper
Graph paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: Data Sheet for Question 2: Stiffness Matrices (1 page)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Figure 1(a) shows a thin-walled unsymmetric channel-like cross-section with constant thickness t.

(i) Compute the St. Venant torsion constant of the cross-section. [10%]

(ii) Compute directions of the principal axes and the principal second moments of area of the cross-section. [40%]

(b) Figure 1(b) shows a thin-walled channel-like cross section with a single symmetry axis and a constant thickness t. Compute the shear centre of the cross-section. [35%]

(c) Describe how you would compute the shear centre of the cross-section shown in Fig. 1(a). No numerical computations are necessary. [15%]

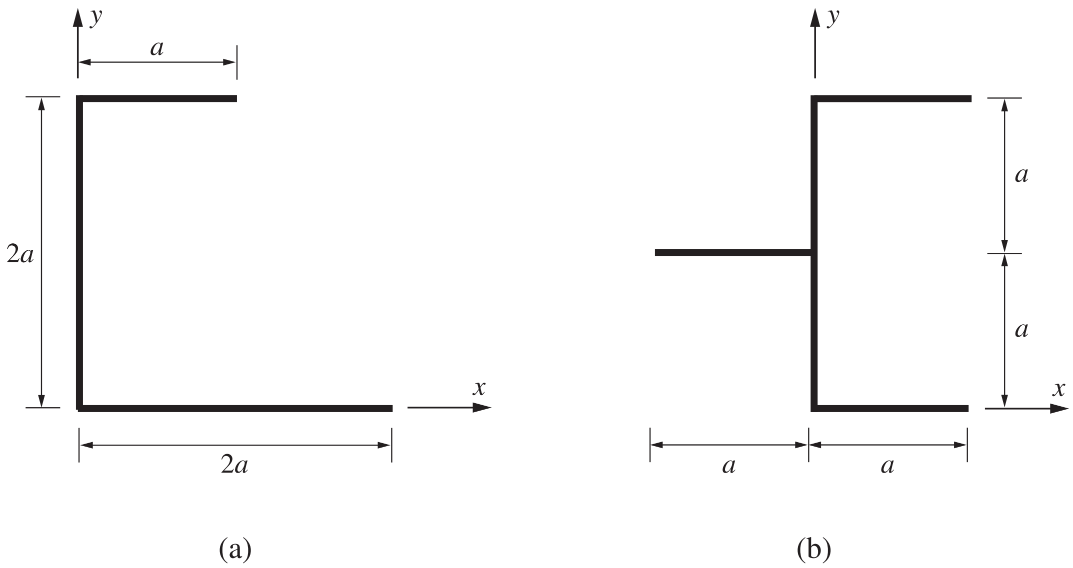


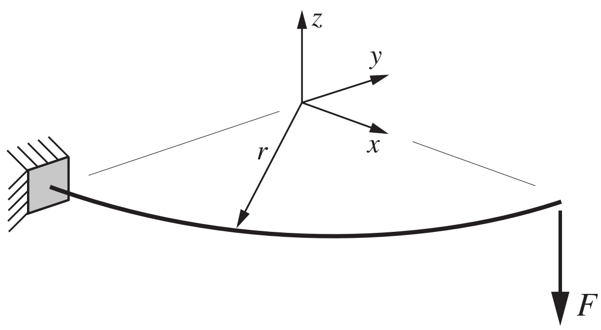
Fig. 1

2 (a) Figure 2(a) shows a circular arc cantilever with radius r which lies in the $x-y$ plane. The flexural stiffness of the cantilever is EI and its torsional stiffness is $GJ = 3EI$. It is loaded at its free end by a force F acting in the negative z-direction. Compute the tip displacement at the cantilever's free end. [35%]

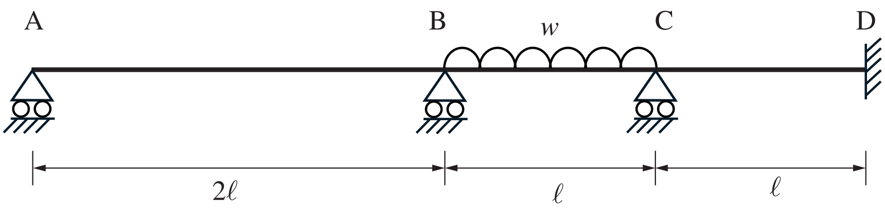
(b) Figure 2(b) shows the side view of a continuous beam over three spans with uniform flexural rigidity EI. The beam is simply supported at its left end, clamped at its right end and continuous over the two mid-span supports. The centre span caries a uniformly distributed load w.

(i) Sketch the bending moment diagram over the entire beam length, marking its salient features. No numerical computations are necessary. [20%]

(ii) Compute the hogging moment over support B using the displacement method. [45%]



(a)



(b)

Fig. 2

3 Figure 3(a) shows a pin-ended strut with bending stiffness EI carrying a compressive load P. Consider the deformation of the strut shown in Fig. 3(b), where the strut takes up a shape $y = \psi(x)$.

(a) Show that the bending energy U stored in the strut is given by

$$U = \frac{1}{2}EI\int_{-L}^{L}\left(\frac{d^2\psi}{dx^2}\right)^2 dx$$

and that the work V done by the load P is given by

$$V = \frac{1}{2}P\int_{-L}^{L}\left(\frac{d\psi}{dx}\right)^2 dx\,.$$

[30%]

(b) Estimate the buckling load of the strut using a mode shape

$$\psi_1 = a\left(1-\left(\frac{x}{L}\right)^2\right)\,.$$

[20%]

(c) The buckling load of the strut will be estimated using a mode shape

$$\psi_2 = a\left(1-\left(\frac{x}{L}\right)^2\right) + b\left(1-\left(\frac{x}{L}\right)^4\right)\,.$$

The total potential energy maybe expressed as

$$U - V = \frac{1}{2}\begin{bmatrix} a & b \end{bmatrix}\begin{bmatrix} \boldsymbol{K} \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix}\,.$$

Determine the matrix $\boldsymbol{K}$ and without further calculation explain how this may be used to compute the buckling load. [50%]

Fig. 3

4 Figure 4 shows three frame structures that are all braced against out-of-plane displacements. Each member has flexural rigidity EI for bending deformations in the plane of the frame.

(a) The central point A of the beam shown in Fig. 4(a) has a support that prevents displacement but not rotation. Show that the stiffness of the structure against rotation at A is $4EI/L$. [10%]

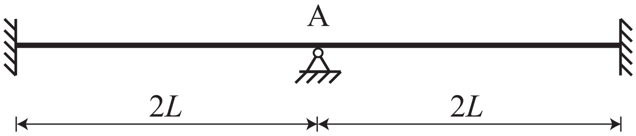
(b) A vertical load F is applied at A to the frame shown in Fig. 4(b). Using the s&c functions given in Table 1, where the stiffness factor s and carry-over factor c are given for a column with Euler buckling load P_E carrying a compressive load P, estimate the critical load to cause buckling. [40%]

(c) A vertical load F is applied at A to the frame shown in Fig. 4(c). Again using s&c functions, estimate the critical load to cause buckling, and describe the mode of bucking. [50%]

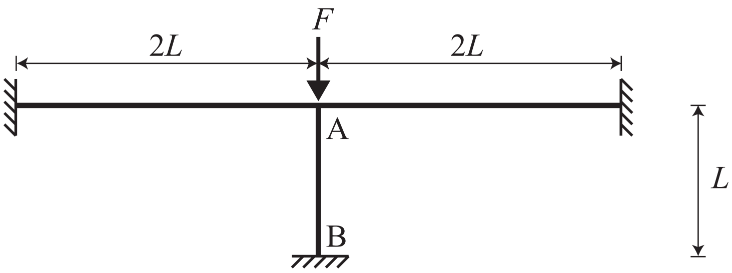
P/P_E	s	c
0.0	4.0000	0.5000
0.2	3.7297	0.5550
0.4	3.4439	0.6242
0.6	3.1403	0.7136
0.8	2.8159	0.8330
1.0	2.4674	1.0000
1.2	2.0901	1.2487
1.4	1.6782	1.6557

P/P_E	s	c
1.6	1.2240	2.4348
1.8	0.7170	4.4969
2.0	0.1428	24.6841
2.2	−0.5194	−7.5107
2.4	−1.3006	−3.3703
2.6	−2.2490	−2.2312
2.8	−3.4449	−1.7081
3.0	−5.0320	−1.4157

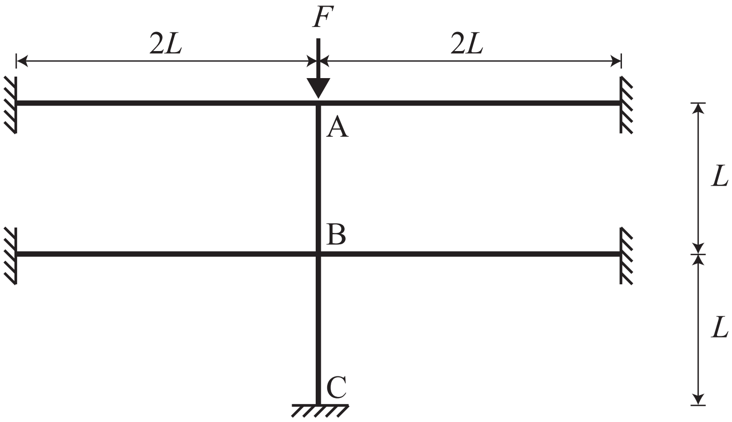
Table 1



(a)



(b)



(c)

Fig. 4

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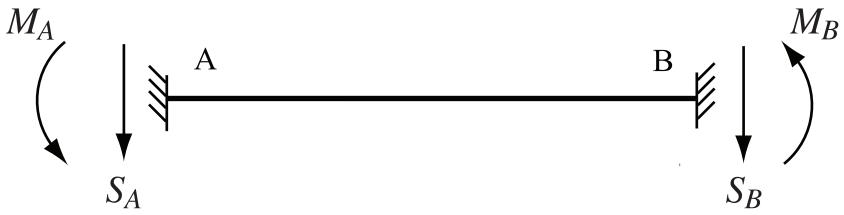
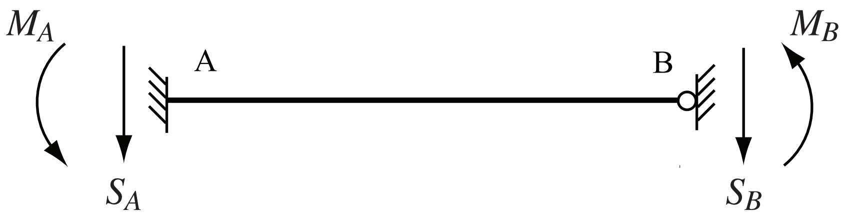
Data Sheet for Question 2: Stiffness Matrices.

Beam type I



$$\begin{bmatrix} S_A \\ M_A \\ S_B \\ M_B \end{bmatrix} = \begin{bmatrix} \frac{12EI}{L^3} & -\frac{6EI}{L^2} & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ -\frac{6EI}{L^2} & \frac{4EI}{L} & \frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & \frac{6EI}{L^2} & \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ -\frac{6EI}{L^2} & \frac{2EI}{L} & \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{bmatrix} w_A \\ \phi_A \\ w_B \\ \phi_B \end{bmatrix}$$

Beam type II



$$\begin{bmatrix} S_A \\ M_A \\ S_B \\ M_B \end{bmatrix} = \begin{bmatrix} \frac{3EI}{L^3} & -\frac{3EI}{L^2} & -\frac{3EI}{L^3} & 0 \\ -\frac{3EI}{L^2} & \frac{3EI}{L} & \frac{3EI}{L^2} & 0 \\ -\frac{3EI}{L^3} & \frac{3EI}{L^2} & \frac{3EI}{L^3} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w_A \\ \phi_A \\ w_B \\ \phi_B \end{bmatrix}$$