Q1 (a) The three influences on permeability: (i) viscosity of the fluid, (ii) size and continuity of pore spaces, which are a function of: size and shape of the soil particles, density, structure, (iii) presence of discontinuities. Hazen's equation accounts for only the particle size and is $K = 0.01 D_{10}^2$.   [10%]

(b) Soil is rarely isotropic, quit elikely to be stratified, during the sedimentation process. Usually Horizontal K is much higher than vertical K, as horizontal flow will occur along more permeable layers, while vertical flow has to go through all layers.   [10%]

(c)

$$Q = A.v = 2\pi.r.b.K\frac{d\overline{h}}{dr}$$

$$\overline{h}_\infty - \overline{h}_w = \frac{Q}{2.\pi.b.K}\ln\frac{R}{r_w}$$
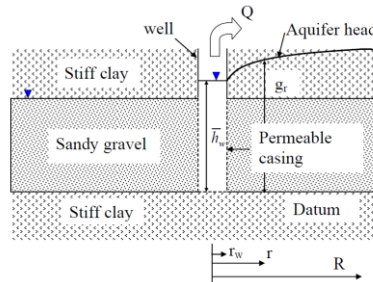
B = 7m, r_w = 0.1m, K = 2 x 10⁻³m/s, Q = 1700m³/day = 0.02m³/s, R = 35m, potential head at far field = groundwater level = 9m. Substitution in above equation will give potential head in well = 7.67m. Hence drawdown is 1.33m.   [25%]

(d)   (i) See annotated flow net.   [25%]

(ii) Total potential drop = 4m with 12 potential head intervals, 4 flow channels on each side, hence total is 8 channels. Q = K Δh N_f/N_h = 10⁻⁵ x 4 x 8/12 = 1.33 x 10⁻⁵ m³/s/m length.   [10%]

(iii) Take datum at base of excavation. Pwp at A: potential head = 0.66m, y = 2m below datum, pressure head 2.66m, pwp = 26.6kPa.   [10%]
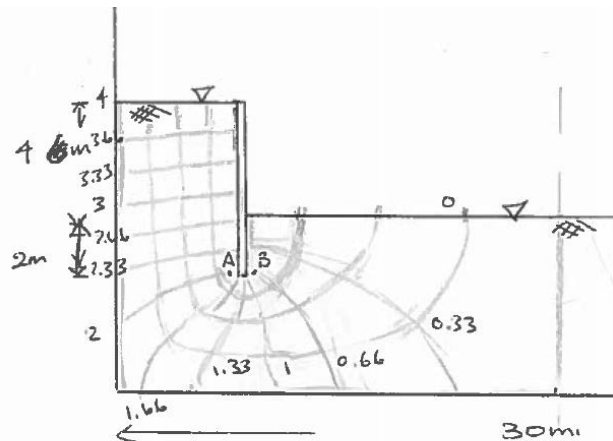
(iv) The soil in the excavation could liquefy if the effective stress reduces to zero. Calculate the total vertical stress acting downwards (γz) and the pore water pressure acting up ($h_{crit} \gamma_w$). If effective stress is zero then the two terms are equal and the critical hydraulic gradient is calculated from $(\gamma - \gamma_w)/ \gamma_w$, which is around 0.8-1.0.   [10%]

2. (a) Three main sources of groundwater contamination: (i) Direct disposal of hazardous materials, examples: landfill, surface impoundment, underground injection, (ii)indirect disposal of hazardous materials, examples: leaks from underground storage tanks, from process facilities, from treatment facilities, (iii) non-point source, examples: agricultural operations, mixing activities, oil & gas explorations. [10%]

(b) Molecular diffusion: contaminant transport mechanism due to differences in concentration of the contaminant and follows Fick's law. In porous media, diffusion occurs along concentration gradients but the coefficient is modified to take into account the tortuosity of the paths the contaminant has to take through the porous media.
Mechanical dispersion: is the mixing that occurs in porous media as a consequence of local variations in velocity around some mean flow velocity, due to (i) different pore sizes, (ii) different paths taken and (iii) boundary friction, hence the contaminants become mixed and transported due to these changes in flow velocities. [10%]

(c) Peclet number is $v_f.d/D_d$, where v_f is flow velocity, d is average particle size and D_d coefficient of diffusion.

On log-log axes this plot indicates that:
$P_e < 1$ D_l is independent of v_f and diffusion is the dominant dispersive mechanism, so $D_l \approx D_d^*$
$P_e > 10$ slope is approximately equal to one, diffusion is negligible in comparison with mechanical dispersion, so $D_l \approx \alpha.v_f$ [10%]
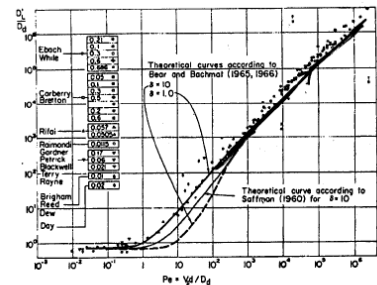
(d) (i) Length of time it will take for the contaminant to break through the base liner of the landfill.

c/co = 0.00001, hence β = 3.0, hence Z/β = $\sqrt{4 D_d^* t}$, therefore 4/3 = $\sqrt{4 \times 0.9 \times 10^{-9} \times 0.35 \times t}$,

1.777 = 1.26 x 10^{-9} t, giving t = 1.41 x 10^{-9} sec = 44.72 years. [20%]

(ii) The depth within the liner will c/co = 0.5.

erfc (β) = 0.4795 for β = 0.5 and erfc (β) = 0.5245 for β = 0.45, Interpolating the values for erfc (β) = 0.5, gives β = 0.47722.

Therefore, z / [4 x 0.9 x 10^{-9} x 0.35 x 1.41 x 10^{-9}] = 0.47722, giving z = 0.636m. [20%]

(iii) Sketch of concentration profile with depth, curved profile, with salient points. [10%]
(iv) The steady state concentration profile will be linear between the initial concentration and zero concentration at depth of 4m, due to the imposed boundary conditions and pure chemical gradient effect, sketch shown. [10%]
(v)   When advection is not negligible, the steady state concentration profile would be parabolic, the other way round to that of the diffusion. Advection dominates, leading to co throughout, but with 4m boundary condition at 0, leads to the profile shown (approximated by power series), sketch shown. [10%]
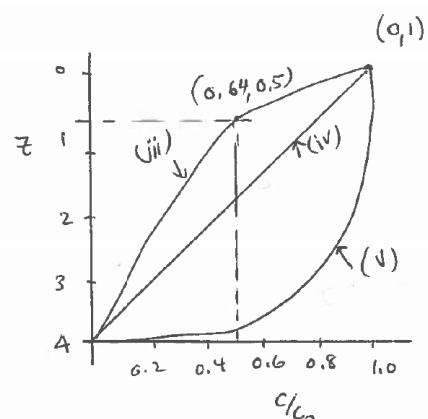
II (a)

Operative temperature is the mean radiant temperature & ambient air temperature weighted by their respective heat transfer coefficients. It is used as a measure of thermal comfort experienced by a person within a room.

III (b) (i)

$$(T_i^4 - T_j^4) - 4\bar{T}_{ij}^3(T_i - T_j) \quad\text{——} \quad ①$$

Rewriting ①

$$\Rightarrow (T_i^2 + T_j^2)(T_i + T_j)(T_i - T_j) - 4\bar{T}_{ij}^3(T_i - T_j)$$

Remove / Ignore $(T_i - T_j)$

$$\Rightarrow (T_i^2 + T_j^2)(T_i + T_j) - 4\bar{T}_{ij}^3 \quad\text{——} \quad ②$$

Given

$$\bar{T}_{ij} = \frac{T_i + T_j}{2} \quad\&\quad \Delta = \frac{T_i - T_j}{2}$$

$$\therefore \quad T_i = \bar{T}_{ij} + \Delta \quad\text{and}\quad T_j = \bar{T}_{ij} - \Delta$$

Substituting with $\bar{T}_{ij}$ & Δ in ② gives:

$$\Rightarrow [(\bar{T}_{ij} + \Delta)^2 + (\bar{T}_{ij} - \Delta)^2][(\bar{T}_{ij} + \Delta) + (\bar{T}_{ij} - \Delta)] - 4\bar{T}_{ij}^3$$

$$\Rightarrow [\bar{T}_{ij}^2 + \Delta^2 + 2\bar{T}_{ij}\Delta + \bar{T}_{ij}^2 + \Delta^2 - 2\bar{T}_{ij}\Delta][2\bar{T}_{ij}] - 4\bar{T}_{ij}^3$$

$$\Rightarrow [2\bar{T}_{ij}^2 + 2\Delta^2][2\bar{T}_{ij}] - 4\bar{T}_{ij}^3$$

$$\Rightarrow 4\bar{T}_{ij}^3\left[1 + \frac{\Delta^2}{\bar{T}_{ij}^2}\right] - 4\bar{T}_{ij}^3$$

$$\underline{\underline{\qquad\qquad}}$$
error
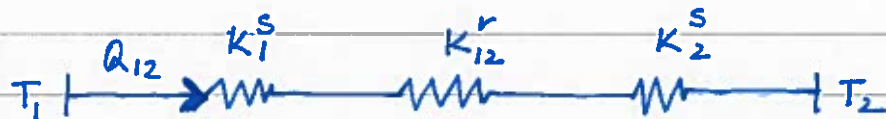
III (b) (ii)

$$T_i - T_j = 10 + 273 \, K$$

$$\overline{T_{ij}} = 273 \, K$$

$$\Delta = \frac{T_i - T_j}{2} = \frac{283}{2} = 141.5$$

$$\frac{\Delta^2}{\overline{T_{ij}}^2} = \frac{141.5^2}{273^2} = 0.27$$

$$\approx 3\% \text{ error}$$

III (b) (iii)

$$K_1^S = \frac{\epsilon_1 A_1}{1 - \epsilon_1} 4\sigma \overline{T_1}^3 \quad —① \qquad K_2^S = \frac{\epsilon_2 A_2}{1 - \epsilon_2} 4\sigma T_2^3 \quad —②$$

$$K_{12}^r = A_1 F_{12} 4\sigma \overline{T_{12}}^3 \quad —③$$

Conductances in series can be reduced:

$$K_{total}^r = \frac{1}{1/K_1^S + 1/K_{12}^r + 1/K_2^S} \quad —④$$

$$\overline{T_{1,2}} = \frac{T_1 + T_2}{2} \quad \approx \quad \frac{2T_1}{2} = T_1 \qquad \left.\begin{array}{c} \\ \\ \end{array}\right\} \text{ since } T_1 \approx T_2$$

$$\approx \quad \frac{2T_2}{2} = T_2$$

$$\therefore \quad T_1^3 \approx \overline{T_{12}}^3 \approx T_2^3$$

Accounting for surface radiative conductance:

$$Q_{12} = K^r_{total} (T_1 - T_2)$$

$$\Rightarrow \quad \frac{4\sigma (T_1 - T_2)}{\frac{1-\epsilon_1}{\epsilon_1 A_1 T_{12}^3} + \frac{1}{A_1 F_{12} T_{12}^3} + \frac{1-\epsilon_2}{\epsilon_2 A_2 T_{12}^3}}$$

$$\ast$$

$$\Rightarrow \quad \frac{4\sigma A_1 \overline{T_{12}}^3 (T_1 - T_2)}{\frac{1-\epsilon_1}{\epsilon_1} + \frac{1}{F_{12}} + \frac{1-\epsilon_2}{\epsilon_2} \frac{A_1}{A_2}}$$

III b) (iv) $\epsilon_1 = \epsilon_2 = 0.9$

$$T_1 = 10 + 273 \text{ K} \qquad T_2 = 20 + 273 \text{ K}$$

$$Q_{12} = \frac{4 \times 5.67 \times 10^{-8} \times (293 - 283) \times \overline{T_{12}}^3}{\frac{1-0.9}{0.9} + 1 + \frac{1-0.9}{0.9} \times 1}$$

$$\overline{T_{12}}^3 = \left(\frac{293 + 283}{2} \right)^3$$

$$Q_{12} = 44.3 \text{ W}/\text{m}^2 \text{ of glass}$$

Inserting there we get

$$I_{ij} = h_r (T_l - T_2) = \frac{4\sigma(T_1 - T_2)}{\frac{1-\epsilon_1}{\epsilon_1 A_1 T_1^3} + \frac{1}{A_1 F_{12} \bar{T}_{12}^3} + \frac{1-\epsilon_2}{\epsilon_2 A_2 T_2^3}}$$
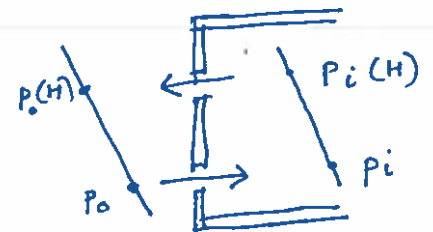
For $T_1/T_2 \approx 1$, we get:

$$I_{ij} = \frac{4\sigma A_1 \bar{T}_{12}^3 (T_1 - T_2)}{\frac{1-\epsilon_1}{\epsilon_1} + \frac{1}{F_{12}} + \frac{1-\epsilon_2}{\epsilon_2}\frac{A_1}{A_2}}$$

IV (a)

ext: $P_o(H) = P_o - \rho_o g H$

int: $P_i(H) = P_i - \rho_i g H$

pressure difference across bottom & top:

$$\Delta P_b = P_o - P_i$$

$$\Delta P_t = (P_i - \rho_i g H) - (P_o - \rho_o g H) = (P_i - P_o) + (\rho_o - \rho_i) g H$$

Total pressure difference between the two openings:

$$\Delta P_s = \Delta P_b + \Delta P_t = (\rho_o - \rho_i) g H \quad\quad —— ①$$

Assuming $\rho \propto \frac{1}{T}$ with $273K$ as benchmark

$$\frac{\rho}{\rho_{273}} = \frac{273}{T} \Rightarrow \rho = \rho_{273}\frac{273}{T}$$

Rewriting ①

$$\Delta P_s = \rho_{273} g H \left(\frac{273}{T_o} - \frac{273}{T_i}\right)$$

IV **(b)**

$T_{ext} = 0°C$ $RH_{ext} = 65\%$

$T_{int} = 21°C$ $RH_{int} = 40\%$

Attic :

$T_{attic} = T_{ext}$ $Area_{attic} = 90 \, m^2$

$n = 0.3 \, ACH$ $Z_{floor} = 3 \times 10^6 \, s \, m^{-1}$

$Volume = 120 \, m^3$

Air gap :

$b = 0.003 \, m$

$L = 0.2 \, m$

$$Area \; of \; gap = 2 \times (0.6 + 0.8) \times b$$

$$\mu_{air} = 18 \times 10^{-6} \, kg/ms \quad (data \; book)$$

(i) Air Pressure Difference

$h = 4 \, m$

$$\Delta P_s = \rho_{273} \, g \, H \left(\frac{273}{T_o} - \frac{273}{T_i} \right)$$

$\rho_{273} = 1.2 \, kg/m^3 \quad (data \; book)$

$g = 9.8 \, m/s^2 \quad (data \; book)$

$$\Delta P_s = 1.2 \times 9.8 \times 4 \left(\frac{273}{273+0} - \frac{273}{273+21} \right)$$

$$= \underline{\underline{3.36}}$$

(ii)   Air   leakage

Calculate S'_e $= \dfrac{1.8 \, P}{2 \cdot A^2 gap}$ (data book)

$$= \dfrac{1.8 \times 1.2}{2 \times [2(0.6 + 0.8) \times 0.003]^2}$$

$\neq \quad \underline{15306}$

Calculate Sg $= \dfrac{12 \, \mu L}{b^2 \, A \, gap}$ (databook)

$$= \dfrac{12 \times 18 \times 10^{-6} \times 0.2}{(0.003)^2 \times \left(2 \times (0.6 + 0.8) \times 0.003\right)}$$

$$= \quad \underline{571 \cdot 43}$$

$$R_a = \dfrac{1}{2 S'_e} \left[\sqrt{S^2g + 4 \Delta P \cdot S'_e} - Sg \right] \quad \left(\text{data book}\right)$$

$$\Rightarrow \quad \underline{\underline{5 \times 10^{-3} \quad m^3/s}}$$

(iii)   Moisture   Balance   under   Steady   State

$$V_e \;\; \underset{nV}{-\!\!\!\bigwedge\!\!\!\bigwedge\!\!\!-} \;\; \overset{}{\underset{V_{attic}}{\bullet}} \;\; \underset{R_a}{-\!\!\!\bigwedge\!\!\!\bigwedge\!\!\!-} \;\; V_i$$

$$V_{attic} = \dfrac{R_a \, V_s (T_i) \, RHi + {}^n/_{3600} V \, V_s (T_{ext}) \, RH_{ex}}{R_a + {}^n/_{3600} V}$$

$$\left. \begin{array}{l} V_s (T_i) = \quad 18.32 \; g/m^3 \\ V_s (T_0) = \quad 4.85 \; g/m^3 \end{array} \right] \quad \text{table in data book}$$

$$V_{attic} = 4.57$$

Relative humidity in attic

$$\frac{V_{attic}}{V_s(0)} \times 100 = 94\%$$

Very damp attic !

(iv) Including Moisture Diffusion

$$\Rightarrow \frac{\left(R_a + \frac{A}{z_{floor}}\right) V_s(T_i) RH_i + \frac{n}{3600} V \cdot V_s(T_{ext}) RH_{ext}}{R_a + \frac{A}{z_{floor}} + \frac{n}{3600} V}$$

$$\Rightarrow \underline{4.57}$$

very little effect compared to moisture transfer by convection.