EGT2: IIA
ENGINEERING TRIPOS PART IIA

Friday 4 May 2018 9.30-11.10

Module 3E3

MODELLING RISK

Answer not more than ***two*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed.
Attachment: 3E3 Modelling Risk data sheet (3 pages).

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) A queuing system with 4 servers is observed for a long period of time and data are collected on the proportion of time the system is in each of the states. Capacity is limited, so whenever there is an arrival when 6 customers are present in the system, the arriving customers balks and goes elsewhere for service. Assume an infinite calling population.

Each state, denoted by n, represents the number of customers present in the system. In the following table, estimates of steady-state probabilities are given.

State, n	Probability, π_n
0	0.05
1	0.1
2	0.2
3	0.3
4	0.25
5	0.05
6	0.05

(i) What is the probability that all servers are idle? [5%]

(ii) What is the probability that a customer will not have to wait? [5%]

(iii) What is the probability that a customer will have to wait in the queue? [5%]

(iv) What is the probability that an arriving customer will balk? [5%]

(v) What is the expected number of customers in the queue? [2.5%]

(vi) What is the expected number of customers in servers? [2.5%]

(vii) If the expected arrival rate is 6 customers per hour, then determine the expected time customers spend in the system. [5%]

(viii) What is the utilisation of the servers? [5%]

(b) Briefly explain in what sense Capital Asset Pricing Model (CAPM) is a pricing model. [10%]

(c) Briefly explain what the omitted variable bias is, and how it can be addressed. [10%]

(d) What is a ‘stochastic matrix’? Is the matrix M below stochastic? [10%]

 (cont.

$$M = \begin{pmatrix} 5/12 & 1/4 & 1/3 \\ 5/12 & 1/4 & 1/2 \\ 1/6 & 1/2 & 1/3 \end{pmatrix}$$

(e) Eli plans on selling boxes of donuts in front of the football stadium on the day of the championship match. From last year's sales history, Eli estimated that the demand for boxes of donuts during the day would be normally distributed with mean 300 and standard deviation 29.8. Eli buys the donuts from his supplier for £12 per box and decided to sell them for £25 per box. Since Eli does not know how to calculate the optimal order quantity that will maximise his expected profit, he asked Dr. Erhun for help. After solving the problem using a newsvendor model, Dr. Erhun told Eli "You should order 320 boxes of donuts considering that you can sell every unsold donut box to the nearby grocery store after the game."

(i) When solving the newsvendor problem, Dr. Erhun assumed that the nearby grocery store would buy entire left over boxes of donuts at £s per box, where $s < 12$. What is the value of s? [10%]

(ii) Suppose the salvage value for unsold donuts is such that the underage and overage costs are equal to each other. In this case, Dr. Erhun claims that it is optimal to order more than average demand. Is Dr. Erhun's claim correct? Briefly explain your answer. [10%]

(iii) Now assume that $s = 6$. The weather forecast predicts that there is a high chance of heavy snow on the day of the championship match. Based on this report, Eli comes up with the following discrete demand distribution:

Q	240	245	250	255	260	265	270
$P(x = Q)$	0.07	0.12	0.23	0.17	0.16	0.20	0.05

What is the optimal order quantity under this new demand forecast assuming that all unsold donut boxes can be salvaged at the nearby grocery store at £6 per box? [15%]

2 (a) Butterfly Inc. manufactures and sells wind turbines to worldwide customers. You are working in the operations strategy group in Butterfly Inc., and you are asked to find out the optimal production plan for a specific type of product, WT260, for the next four months: November, December, January and February. The customer demands for the four months are summarised in the table below:

November	December	January	February
70	80	90	60

The fixed production set-up cost for WT260 is £10,000 per month if you plan to produce the product in the month and £0 otherwise. At the end of each month, the unsold products should be stored in warehouse. The inventory holding cost is £100 per unit per month. Assume that no stock-out is allowed.

(i) Formulate this problem using dynamic programming by identifying the:

A. stage (t) and state (s_t); [5%]

B. decision (x_t) and constraint on x_t as a function of s_t; [5%]

C. state transformation equation ($s_{t+1} = g(s_t, x_t)$); [5%]

D. cost function $c(s_t, x_t)$. [5%]

(ii) Solve this problem numerically, using dynamic programming. What is the optimal production plan? [25%]

(iii) How many units of the November demand can be postponed to December without any changes in the production plan? [15%]

(b) VARMAX Realty collected the following data on twenty houses:

- the selling price of each house (in £000s);
- the total area of the house, measured in ft^2;
- a rating of the quality of the neighbourhood in which the house is located, rated on a scale of 1 through 5 with 1 being the lowest ranking; and
- a rating of the general condition contains a rating of the general condition of the house, also rated as a number between 1 and 5 with 1 being the lowest ranking.

VARMAX used the data to develop a linear regression model to predict the price of houses as a function of the area of the house, the neighbourhood rating, and the general condition rating of the house:

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.90
R Square	0.81
Adjusted R Square	0.77
Standard Error	49.07
Observations	20

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	163167.7802	54389.3	22.59	5.39018E-06
Residual	16	38525.96977	2407.9		
Total	19	201693.75			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	-166.69	65.24	-2.56	0.02	-305.00	-28.39
Area	0.09	0.01	7.16	0.00	0.06	0.11
Neighbourhood rating	39.92	8.59	4.65	0.00	21.72	58.12
General condition rating	42.70	9.74	4.39	0.00	22.06	63.33

Suppose that the standard linear regression assumptions hold.

(i) Evaluate the regression output of your regression model. Do you recommend using this regression model? [10%]

(ii) What is the regression equation produced by the linear regression model? [10%]

(iii) What is the predicted price of a house whose area is 3,000ft^2 with a neighbourhood ranking of 5 and a general condition ranking of 4? [10%]

(iv) Based on the regression analysis, estimate how much 5,000ft^2 of additional living space is worth as far as the selling price is concerned. In addition, compute a 99% confidence interval for your estimate. [10%]

3 (a) I have a farm, and I have already invested £50 per acre in seed, water, fertiliser, and labour. I will need to invest another £15 per acre to produce and harvest a marketable crop. If the weather stays favourable, I estimate my crop will fetch a market price of £26 per acre. However, if the weather becomes unfavourable, I estimate my crop will fetch a market price of £12 per acre. Currently, the local forecasters are predicting favourable weather conditions with a probability of 0.7. The owner of the farm next to mine, Farmer Joe, is growing the same product and has made the same £50 per acre investment. Farmer Joe has just decided not to invest the additional £15 per acre to produce and harvest a marketable crop as this would just be "throwing good money after bad".

(i) Build a decision tree for my decision. [5%]

(ii) Should I bring the crop to market or not? [5%]

(iii) By how much would the probability of favourable weather have to change before my answer to the question in part (ii) would change? [10%]

(iv) By how much would the £15 per acre cost of bringing the crop to market have to change before my answer to the question in part (ii) would change? [10%]

(v) In the past the local forecasters made mistakes in predicting the weather conditions. I can secure additional information from the national weather department. The national weather department's assessment of 'favourable' and 'unfavourable' is described by the following conditional probabilities: $P(a_1|s_1) = 0.8$ and $P(a_2|s_2) = 0.9$, where a_1 and a_2 represent 'favourable assessment' and 'unfavourable assessment', respectively, by the national weather department, and s_1 and s_2 represent 'favourable weather' and 'unfavourable weather', respectively. Assuming the national weather department's assessment is favourable, what is the maximum amount I should be willing to pay to Farmer Joe for the rights to harvest his farm as well? [10%]

(vi) What other factors should I consider before making my decision? [10%]

(b) Aaron finds a bill on his desk. He either puts it on his wife's desk to be dealt with the next day or he leaves it on his own desk for the next day, or he pays it immediately with probabilities 0.5, 0.2 and 0.3, respectively. Similarly, his wife Ada can keep it until the next day, put it on Aaron's desk, or pay it immediately with probabilities 0.3, 0.6, and 0.1, respectively.

(i) Express the situation as a Markov Chain and specify the corresponding transition matrix. [10%]

(ii) Find the probability a bill now on Aaron's desk will be paid exactly on the third day. [15%]

(iii) If the bill is on Aaron's desk on a given day, what is the probability it will be paid within five days? [15%]

(c) Define the utilisation factor and the term 'steady state' of a queuing system. Does every queuing system eventually reach a steady state? Explain. [10%]

END OF PAPER

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ENGINEERING TRIPOS PART IIA
Friday 4 May 2018, Module 3E3, Questions 1-3.

SPECIAL DATA SHEET

Standard errors

$$STEM = \frac{\sigma}{\sqrt{n}} \approx \frac{s}{\sqrt{n}}, \quad STEP = \sqrt{\frac{p(1-p)}{n}} \approx \sqrt{\frac{q(1-q)}{n}},$$

$$STEDM = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2}} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}.$$

Covariance, Correlation and Regression

Consider data pairs (X_1, Y_1), (X_2, Y_2), …, (X_n, Y_n).

Let m_X and m_Y denote the respective means of the X and Y data.

Let s_X and s_Y denote the respective standard deviations of the X and Y data.

Covariance between X and Y is given by

$$\mathrm{cov}(\mathrm{X},\mathrm{Y}) = \frac{\sum_{i=1}^{n}(\mathrm{X}_i - m_\mathrm{X})(\mathrm{Y}_i - m_\mathrm{Y})}{n} = \frac{\sum_{i=1}^{n}\mathrm{X}_i\mathrm{Y}_i}{n} - m_\mathrm{X} m_\mathrm{Y}$$

The correlation coefficient between X and Y is given by

$$\mathrm{correl}(\mathrm{X},\mathrm{Y}) = r = \frac{\mathrm{cov}(\mathrm{X},\mathrm{Y})}{s_\mathrm{X} s_\mathrm{Y}}.$$

The line of best fit is given by

$$\mathrm{Y} - m_\mathrm{Y} = \frac{r s_\mathrm{Y}}{s_\mathrm{X}}(\mathrm{X} - m_\mathrm{X}).$$

Variance of a portfolio

Consider three random variables x, y and z with means m_x, m_y, and m_z, respectively; variances Var(x), Var(y), and Var(z), respectively; and covariance between x and y, for example, given by the formula above. Given any numbers α_x, α_y, α_z, let $v = \alpha_x x + \alpha_y y + \alpha_z z$. Then the variance of v is given by

$$\begin{aligned}\mathrm{Var}(v) &= \alpha_x{}^2\mathrm{Var}(x) + \alpha_y{}^2\mathrm{Var}(y) + \alpha_z{}^2\mathrm{Var}(z) \\ &\quad + 2\big(\alpha_x\alpha_y\mathrm{cov}(x,y) + \alpha_y\alpha_z\mathrm{cov}(y,z) + \alpha_x\alpha_z\mathrm{cov}(x,z)\big)\end{aligned}$$

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Time Series Forecasting (Winters' multiplicative smoothing method)

$$E_t = \alpha \frac{X_t}{S_{t-c}} + (1-\alpha)(E_{t-1} + T_{t-1})$$
$$T_t = \beta(E_t - E_{t-1}) + (1-\beta)T_{t-1}$$
$$S_t = \gamma \frac{X_t}{E_t} + (1-\gamma)S_{t-c}$$

$$F_{t+k} = (E_t + kT_t)S_{t+k-c}$$

Markov Chains (calculate probabilities for first passage time and expected first passage times)

$$f_{ij}(1) = P_{ij}$$
$$\vdots$$
$$f_{ij}(n) = P_{ij}^{(n)} - f_{ij}(1)P_{jj}^{(n-1)} - \ldots - f_{ij}(n-1)P_{jj}^{(1)}.$$

$$E(H_{ij}) = 1 + \sum_{k \neq j} E(H_{kj})P_{ik}, \forall i.$$

Queueing Theory (Poisson distribution, exponential distribution, performance metrics for the M/M/s queue, the M/M/1 queue is a special case of the M/M/s queue)

$$P(X = k) = \frac{(\lambda t)^k e^{-\lambda t}}{k!}, \quad k = 0,1,\ldots$$

$$P(X \le t) = 1 - e^{-\mu t}, \quad \forall t \ge 0.$$

$$p_0 = \frac{1}{\sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!}\left(\frac{s\mu}{s\mu - \lambda}\right)}$$

$$p_n = \begin{cases} \frac{(\lambda/\mu)^n}{n!} p_0 & \text{if } 0 \le n \le s \\ \frac{(\lambda/\mu)^n}{s! s^{n-s}} p_0 & \text{if } n \ge s \end{cases}$$

$$L_q = \left(\frac{(\lambda/\mu)^{s+1}}{(s-1)!(s-\lambda/\mu)^2}\right) p_0.$$

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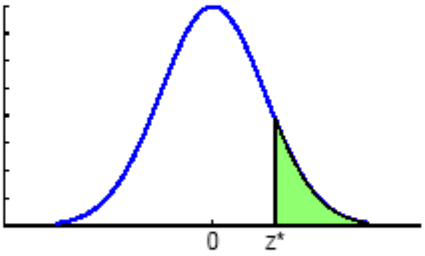
Standard Normal Distribution Table

(Areas under the standard normal curve beyond z*, i.e., shaded area)



z*	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010