Version GV/3

EGT2
ENGINEERING TRIPOS PART IIA

Friday 4 May 2018 2.00 to 3.40

Module 3F2

SYSTEMS AND CONTROL

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 Consider the double pendulum of Fig. 1. The individual bobs have masses m_1 and m_2 and the carriage to which they are pinned has mass M. All other components are massless and rigid. An accelerometer measures α, the acceleration of the carriage. The linearised equations of motion are given by

$$
\begin{aligned}
M\alpha - m_1 g\theta_1 - m_2 g\theta_2 &= F \\
m_1 g\theta_1 + m_1(\alpha - l_1\ddot{\theta}_1) &= 0 \\
m_2 g\theta_2 + m_2(\alpha - l_2\ddot{\theta}_2) &= 0
\end{aligned}
$$

(a) Put these equations into state-space form, with input the force F and output α. [30%]

(b) Under what conditions on the parameters is the system observable? [40%]

(c) Describe the structure of an observer to estimate the values of θ_1 and θ_2 from measurements of α. What design choices would have to made for a practical design? [30%]

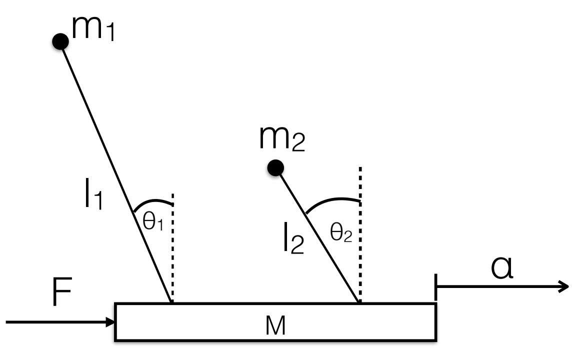

Fig. 1

2 (a) Consider the system

$$\ddot{x} + x = 0$$

Put this into state-space form, with states x and $\dot{x}$, and show that the state trajectories take the form of circles. [30%]

(b) Now consider the system

$$\ddot{x} + (x^2 + \dot{x}^2 - 1)\dot{x} + x = 0$$

(i) Find tangents to the trajectories of this system along the axes $x = 0$ and $\dot{x} = 0$, and hence sketch the state trajectories of this system starting at the points

$$\dot{x} = 0\,,\ x = -2, -1, 0.5, 1, 2$$

and

$$x = 0\,,\ \dot{x} = -2, -1, 0.5, 1, 2\ .$$ [50%]

(ii) For what initial conditions, if any, are the trajectories of this system identical to those of the system of part a)? [20%]

3 Consider the system

$$\underline{\dot{x}} = \begin{bmatrix} -1 & 4 & 6 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \underline{u}$$

(a) Find $\Phi(t)$, the state transition matrix relating $\underline{x}(t)$ to $\underline{x}(0)$ when $\underline{u} = \underline{0}$ [30%]

(b) Characterize the set of states which can be held constant by an appropriate choice of constant u. [20%]

(c) For $\underline{u} = 0$, characterize the set of initial states $\underline{x}(0)$ for which $\underline{x}(t)$ converges to 0 as t goes to infinity. [20%]

(d) If

$$\underline{y} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \underline{x}$$

Calculate the transfer function matrix from $\underline{\bar{u}}(s)$ to $\underline{\bar{y}}(s)$. [30%]

4 Consider the system with transfer function

$$G(s) = \frac{s+1}{s^3+s^2+s},$$

which is to be controlled in a negative feedback configuration by a controller $K(s)$.

(a) Draw the root locus diagram for the system when static controllers of the form $K(s) = k$ are used. Find the approximate location of the closed-loop poles for large k, and characterise the resultant form of the response of the system in this case. [50%]

(b) Now consider a controller of the form

$$K(s) = k\frac{s+2}{s+a}$$

(i) How large must a be to ensure that oscillations decay faster than $\exp(-4t)$ for large k?

(ii) Sketch the root locus diagram for a value of a satisfying this condition.

(iii) Choose locations for the oscillatory poles which would guarantee that oscillations decay faster than $\exp(-4t)$.

(iv) Estimate the value of k which would give these closed-loop poles. [50%]

END OF PAPER