EGT2
ENGINEERING TRIPOS PART IIA

Friday 4 May 2018 2 to 3.40

Module 3C9

FRACTURE MECHANICS OF MATERIALS AND STRUCTURES

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 3C9 formulae sheet (8 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 A simple model for the compressive failure of a brittle solid can be developed by considering an infinite elastic plate with a crack of length $2a$ at an angle ψ with respect to the X_1 axis as shown in Fig. 1. The plate is subjected to in-plane compressive principal stresses σ_1 and σ_3 ($|\sigma_1| > |\sigma_3|$) as shown in Fig. 1 and you may assume that there is no friction between the crack surfaces. A local co-ordinate system $x-y$ aligned with the crack is also indicated in Fig. 1 with r the distance from the crack tip and θ an angle measured with respect to the crack plane.

(a) Calculate the shear stress σ_{xy} remote from the crack. [10%]

(b) Calculate the stress $\sigma_{\theta\theta}$ in the asymtoptic limit $r \to 0$ and hence determine the mode I stress intensity factor defined as $K_I = \sigma_{\theta\theta}\sqrt{2\pi r}$. At what value of θ does K_I attain its maximum value $K_I^{\max}$? [40%]

(c) The most dangerous crack in the brittle solid is that lying at an angle ψ which maximises $K_I^{\max}$. Determine the angle ψ of this critical crack. [10%]

(d) Given that the mode I fracture toughness of the solid is K_{IC}, derive an expression relating σ_1 and σ_3 at the instant crack growth initiates. [20%]

(e) Briefly outline (without performing calculations) how the above analysis could be modified to account for a Coloumb friction coefficient μ between the crack surfaces. [20%]

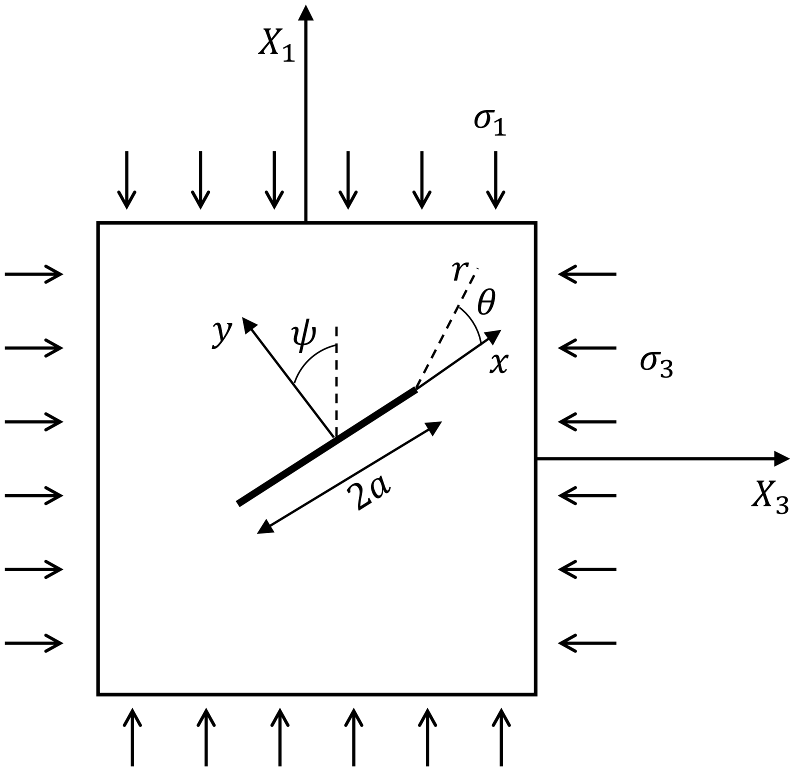

Fig. 1

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2 A thin-walled spherical pressure vessel of radius R, and wall thickness t, is subjected to service loading in the form of an internal cyclic pressure p from a minimum value of zero to a maximum value p_{max}. It is assumed that the internal wall of the pressure vessel contains surface-breaking, semi-circular thumbnail cracks of radius $a \ll t$.

(a) In order to ensure an adequate fatigue life, the pressure vessel is subjected to a proof test by a pressure p_p such that the worst flaw does not lead to fast fracture. Obtain an expression for this flaw size a_0 assuming that the pressure vessel has a fracture toughness K_{IC} and a yield strength σ_Y. [15%]

(b) Assume that fatigue cracks grow in a self-similar manner, at a growth rate $\mathrm{d}a/\mathrm{d}N$ that depends upon the stress intensity range ΔK according to $\mathrm{d}a/\mathrm{d}N = C\Delta K^m$, where C and m are material constants, provided that ΔK exceeds the threshold value ΔK_{th}.

(i) Determine an expression for the fatigue life of the pressure vessel under the service loading. [20%]

(ii) Comment on the sensitivity of fatigue life to the fracture toughness. [10%]

(iii) Determine an expression for the minimum value of fracture toughness that guarantees leak-before-break rather than fast fracture. [15%]

(c) What practical steps could be taken in order to increase the fatigue life of the pressure vessel for a given steel? [20%]

(d) In the selection of steels for the pressure vessel, a high toughness is chosen. Briefly explain the features of a steel microstructure that endow it with a high toughness. [20%]

3 (a) Explain the physical basis of the R-curve in the fracture of a sheet made from a metallic alloy, and the dependence of the R-curve upon thickness. [25%]

(b) 'Crack closure effects can account for the effect of mean stress and overloads on fatigue crack growth in metallic alloys'. Explain what is meant by this claim, and give any qualifications on its veracity. [25%]

(c) Explain why compressive residual stresses can be generated at the surface of a steel component by shot peening or by carburisation. What is the effect of such stresses upon fatigue crack initiation and growth? [25%]

(d) A double cantilever beam (DCB), of height $2h$ and thickness B, contains a crack of initial length a_0 and ligament dimension b, with $h \ll a_0$ and $h \ll b$ (geometry and K calibration are given in the datasheet). The DCB is made from a high strength steel, and the ends of each arm are subjected to a load P. Assume that the crack growth resistance curve, K versus Δa, for this steel is piecewise linear, such that crack advance initiates when K attains the value $K = K_{IC}$. With ensuing crack advance Δa, K rises to the steady state value $K = K_{SS}$ in a linear fashion over a material length scale ℓ such that

$$K = K_{IC} + (K_{SS} - K_{IC})\frac{\Delta a}{\ell}, \qquad 0 < \Delta a \leq \ell$$

$$K = K_{SS}, \qquad \Delta a > \ell.$$

(i) Show that the value of crack advance at peak load depends upon a_0/ℓ. [10%]

(ii) Sketch the load versus crack advance at large a_0/ℓ, and label salient points. [15%]

4 Two steel bars of square cross-section $h \times h$ are brazed together. The toughness of the brazed joint is determined by introducing a crack of length a across the width h of the joint, and then loading the brazed bars in 3-point bending, as sketched in Fig. 2. The central roller displaces by u under a central load P. In order to model the deformation of the brazed layer in the 3-point bend test, it is assumed that the layer behaves as a cohesive zone of constant tensile strength T_0 for an opening displacement w between zero and a critical value w_c. The cohesive zone carries zero tensile traction when w exceeds w_c. In compression, the brazed layer has a constant compressive strength $-T_0$, regardless of the magnitude of closing displacement.

(a) Calculate the load P versus mid-roller displacement u response of the bar in the 3-point bend test, assuming no crack extension. [30%]

(b) Obtain an expression for the potential energy ψ as a function of the prescribed displacement u, and of the crack ligament $b = h - a$. [15%]

(c) Determine J as a function of u up to the point of crack extension, and thereby obtain an expression for the critical displacement u_c of the beam in terms of J_{IC}. [20%]

(d) By considering the geometric relationship between the roller displacement u and the cohesive zone opening w at the crack tip, obtain J_{IC} as a function of T_0 and w_c. [20%]

(e) Explain why the measured toughness of the brazed joint might be different from that measured for a large volume of brazed material. [15%]

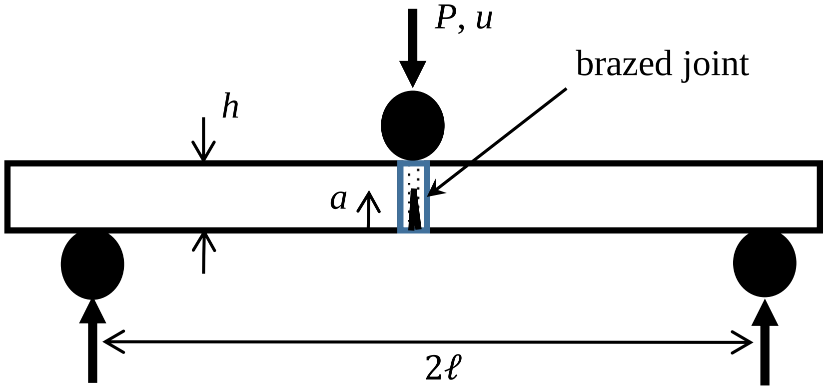

Fig. 2

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ENGINEERING TRIPOS PART IIA

Module 3C9 – FRACTURE MECHANICS OF MATERIALS AND STRUCTURES

DATASHEET

Crack tip plastic zone sizes

$$\text{diameter, } d_p = \begin{cases} \dfrac{1}{\pi}\left(\dfrac{K_I}{\sigma_y}\right)^2 & \text{Plane stress} \\ \dfrac{1}{3\pi}\left(\dfrac{K_I}{\sigma_y}\right)^2 & \text{Plane strain} \end{cases}$$

Crack opening displacement

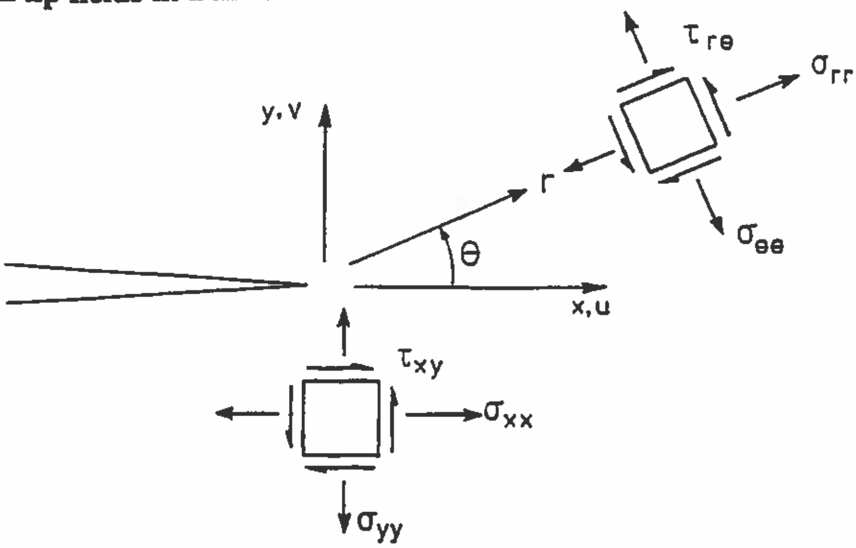
$$\delta = \begin{cases} \dfrac{K_I^2}{\sigma_y E} & \text{Plane stress} \\ \dfrac{1}{2}\dfrac{K_I^2}{\sigma_y E} & \text{Plane strain} \end{cases}$$

Energy release rate

$$G = \begin{cases} \dfrac{1}{E}K_I^2 & \text{Plane stress} \\ \dfrac{1-\nu^2}{E}K_I^2 & \text{Plane strain} \end{cases}$$

Related to compliance C: $G = \dfrac{1}{2}\dfrac{P^2}{B}\dfrac{dC}{da}$

Asymptotic crack tip fields in a linear elastic solid



Mode I

$$\sigma_{yy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\sigma_{xx} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\tau_{xy} = \frac{K_I}{\sqrt{2\pi r}} \cos\frac{\theta}{2}\ \sin\frac{\theta}{2}\ \cos\frac{3\theta}{2}$$

$$\sigma_{rr} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{5}{4}\cos\frac{\theta}{2}-\frac{1}{4}\cos\frac{3\theta}{2}\right)$$

$$\sigma_{\theta\theta} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{3}{4}\cos\frac{\theta}{2}+\frac{1}{4}\cos\frac{3\theta}{2}\right)$$

$$\tau_{r\theta} = \frac{K_I}{\sqrt{2\pi r}}\left(\frac{1}{4}\sin\frac{\theta}{2}+\frac{1}{4}\sin\frac{3\theta}{2}\right)$$

$$u = \begin{cases} \dfrac{K_I}{G}\sqrt{\dfrac{r}{2\pi}}\left(\dfrac{1-\nu}{1+\nu}+\sin^2\dfrac{\theta}{2}\right)\cos\dfrac{\theta}{2} & \text{Plane stress} \\ \dfrac{K_I}{G}\sqrt{\dfrac{r}{2\pi}}\left(1-2\nu+\sin^2\dfrac{\theta}{2}\right)\cos\dfrac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$v = \begin{cases} \dfrac{K_I}{G}\sqrt{\dfrac{r}{2\pi}}\left(\dfrac{2}{1+\nu}-\cos^2\dfrac{\theta}{2}\right)\sin\dfrac{\theta}{2} & \text{Plane stress} \\ \dfrac{K_I}{G}\sqrt{\dfrac{r}{2\pi}}\left(2-2\nu-\cos^2\dfrac{\theta}{2}\right)\sin\dfrac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$w = 0$$

Crack tip stress fields (cont'd)

Mode II

$$\sigma_{yy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \sin\frac{\theta}{2} \cos\frac{3\theta}{2}$$

$$\sigma_{xx} = -\frac{K_{II}}{\sqrt{2\pi r}} \sin\frac{\theta}{2} \left(2 + \cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)$$

$$\tau_{xy} = \frac{K_{II}}{\sqrt{2\pi r}} \cos\frac{\theta}{2} \left(1 - \sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

$$\sigma_{rr} = \frac{K_{II}}{\sqrt{2\pi r}} \left(-\frac{5}{4}\sin\frac{\theta}{2} + \frac{3}{4}\sin\frac{3\theta}{2}\right)$$

$$\sigma_{\theta\theta} = -\frac{K_{II}}{\sqrt{2\pi r}} \left(\frac{3}{4}\sin\frac{\theta}{2} + \frac{3}{4}\sin\frac{3\theta}{2}\right)$$

$$\tau_{r\theta} = \frac{K_{II}}{\sqrt{2\pi r}} \left(\frac{1}{4}\cos\frac{\theta}{2} + \frac{3}{4}\cos\frac{3\theta}{2}\right)$$

$$u = \begin{cases} \dfrac{K_{II}}{G}\sqrt{\dfrac{r}{2\pi}}\left(\dfrac{2}{1+\nu} + \cos^2\dfrac{\theta}{2}\right)\sin\dfrac{\theta}{2} & \text{Plane stress} \\ \dfrac{K_{II}}{G}\sqrt{\dfrac{r}{2\pi}}\left(2 - 2\nu + \cos^2\dfrac{\theta}{2}\right)\sin\dfrac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$v = \begin{cases} \dfrac{K_{II}}{G}\sqrt{\dfrac{r}{2\pi}}\left(\dfrac{\nu-1}{1+\nu} + \sin^2\dfrac{\theta}{2}\right)\cos\dfrac{\theta}{2} & \text{Plane stress} \\ \dfrac{K_{II}}{G}\sqrt{\dfrac{r}{2\pi}}\left(-1 + 2\nu + \sin^2\dfrac{\theta}{2}\right)\cos\dfrac{\theta}{2} & \text{Plane strain} \end{cases}$$

$$w = 0$$

Mode III

$$\tau_{zx} = -\frac{K_{III}}{\sqrt{2\pi r}} \sin\frac{\theta}{2}$$

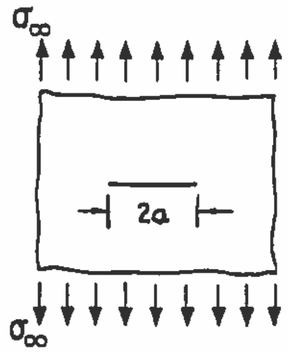
$$\tau_{yz} = \frac{K_{III}}{\sqrt{2\pi r}} \cos\frac{\theta}{2}$$

$$\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \tau_{xy} = 0$$

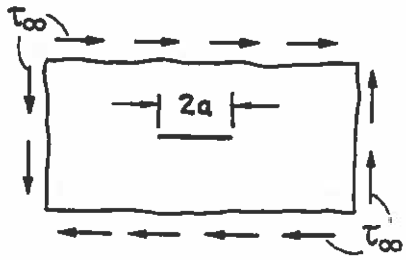
$$w = \frac{K_{III}}{G}\sqrt{\frac{2r}{\pi}} \sin\frac{\theta}{2}$$

$$u = v = 0$$

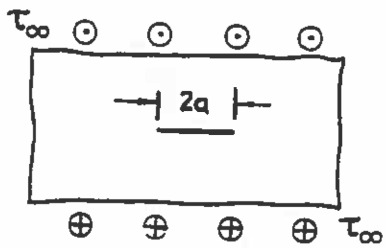
Tables of stress intensity factors



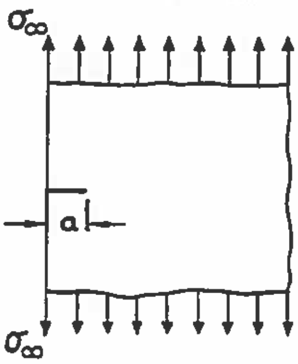
$$K_I = \sigma_\infty \sqrt{\pi a}$$



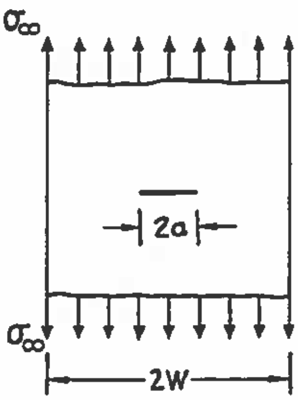
$$K_{II} = \tau_\infty \sqrt{\pi a}$$



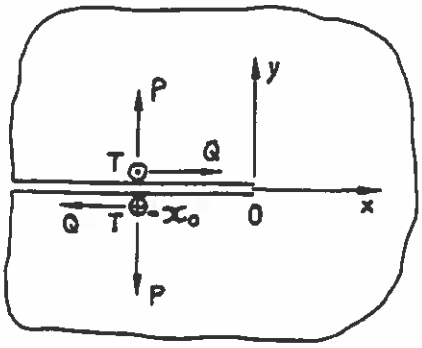
$$K_{III} = \tau_\infty \sqrt{\pi a}$$



$$K_I = 1.12\,\sigma_\infty \sqrt{\pi a}$$



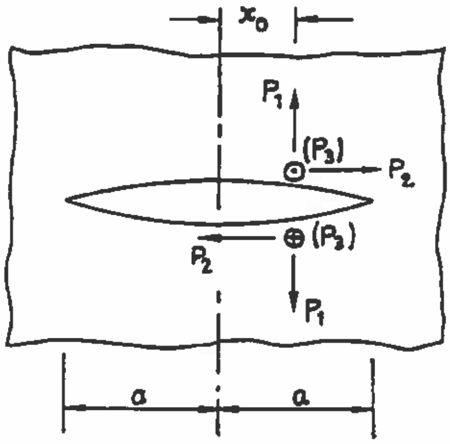
$$K_I = \sigma_\infty \sqrt{\pi a} \left(\frac{1 - a/2W + 0.326a^2/W^2}{\sqrt{1 - a/W}} \right)$$



$$K_I = \frac{2P}{\sqrt{2\pi x_o}}$$

$$K_{II} = \frac{2Q}{\sqrt{2\pi x_o}}$$

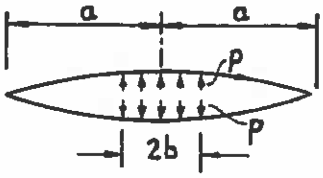
$$K_{III} = \frac{2T}{\sqrt{2\pi x_o}}$$



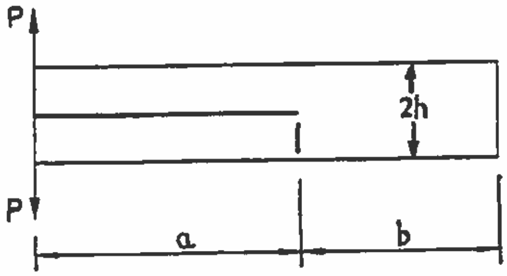
$$K_I = \frac{P_1}{\sqrt{\pi a}}\sqrt{\frac{a+x_o}{a-x_o}}$$

$$K_{II} = \frac{P_2}{\sqrt{\pi a}}\sqrt{\frac{a+x_o}{a-x_o}}$$

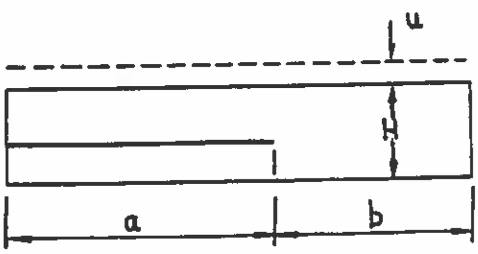
$$K_{III} = \frac{P_3}{\sqrt{\pi a}}\sqrt{\frac{a+x_o}{a-x_o}}$$



$$K_I = \frac{2pb}{\sqrt{\pi a}}\frac{a}{b}\arcsin\frac{b}{a}$$

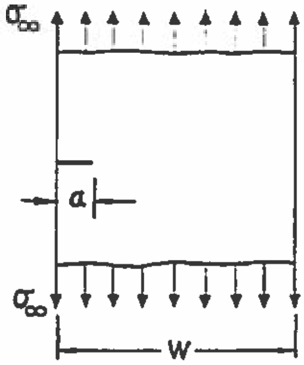


$$K_I = \frac{2\sqrt{3}}{h\sqrt{h}}\frac{Pa}{B} \qquad h \ll a \text{ and } h \ll b$$



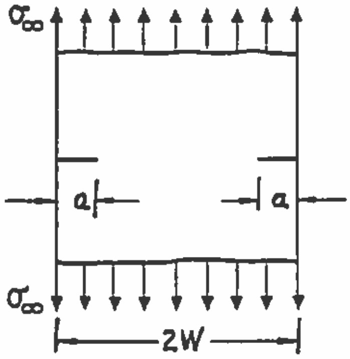
$$K_I = \sqrt{\frac{1}{2\alpha H}}Eu \qquad H \ll a \text{ and } H \ll b$$

$$\alpha = \begin{cases} 1-\nu^2 & \text{Plane stress} \\ 1-3\nu^2-2\nu^3 & \text{Plane strain} \end{cases}$$

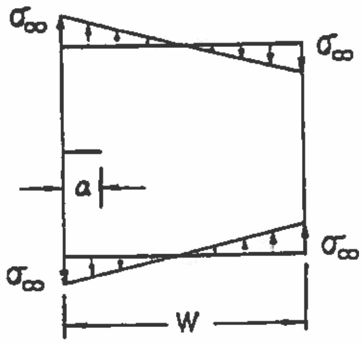


$a/W < 0.7$

$$K_I = \sigma_\infty \sqrt{\pi a}\left(1.12 - 0.23\frac{a}{W} + 10.6\frac{a^2}{W^2} - 21.7\frac{a^3}{W^3} + 30.4\frac{a^4}{W^4}\right)$$

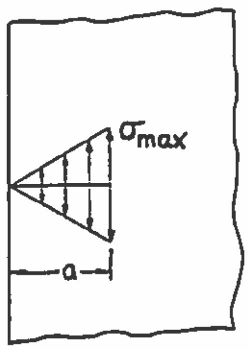


$$K_I = \sigma_\infty \sqrt{\pi a}\left(\frac{1.12 - 0.61a/W + 0.13a^3/W^3}{\sqrt{1 - a/W}}\right)$$

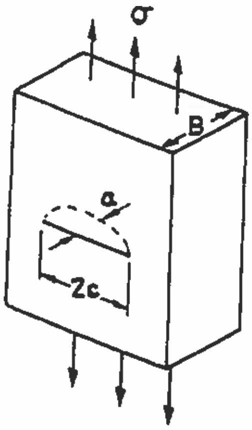


$a/W < 0.7$

$$K_I = \sigma_\infty \sqrt{\pi a}\left(1.12 - 1.39\frac{a}{W} + 7.3\frac{a^2}{W^2} - 13\frac{a^3}{W^3} + 14\frac{a^4}{W^4}\right)$$

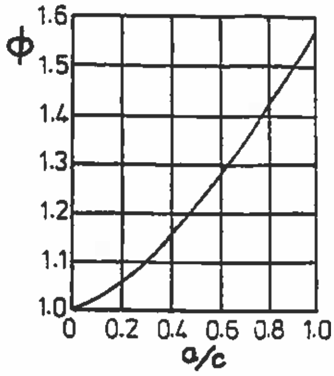


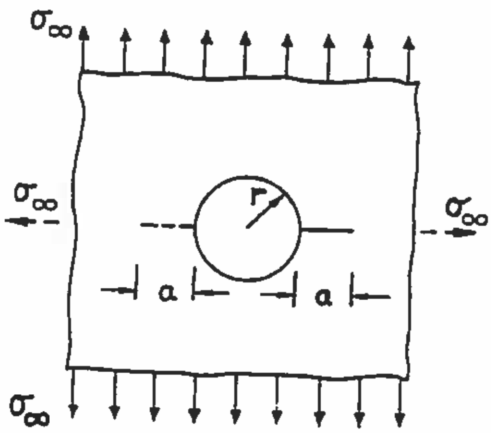
$$K_I = 0.683\,\sigma_{max}\sqrt{\pi a}$$



$$K_I = \frac{1.12}{\Phi}\sigma\sqrt{\pi a}$$

$$\Phi = \int_0^{\pi/2}\left(1 - \frac{c^2 - a^2}{c^2}\sin^2\theta\right)^{\frac{1}{2}} d\theta$$



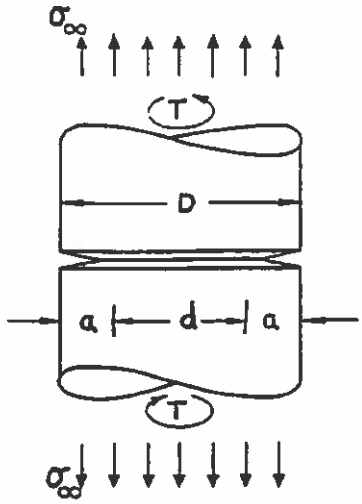


$$K_I = \sigma_\infty\sqrt{\pi a}\; F\left(\frac{a}{r}\right)$$

value of $F\,(a\,/\,r\,)^\dagger$

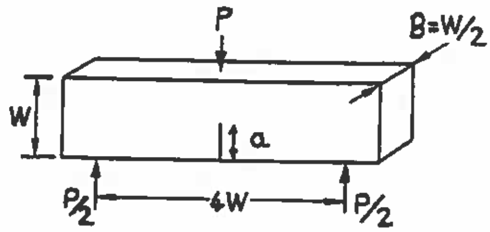
	One crack		Two cracks	
$\frac{a}{r}$	U	B	U	B
0.00	3.36	2.24	3.36	2.24
0.10	2.73	1.98	2.73	1.98
0.20	2.30	1.82	2.41	1.83
0.30	2.04	1.67	2.15	1.70
0.40	1.86	1.58	1.96	1.61
0.50	1.73	1.49	1.83	1.57
0.60	1.64	1.42	1.71	1.52
0.80	1.47	1.32	1.58	1.43
1.0	1.37	1.22	1.45	1.38
1.5	1.18	1.06	1.29	1.26
2.0	1.06	1.01	1.21	1.20
3.0	0.94	0.93	1.14	1.13
5.0	0.81	0.81	1.07	1.06
10.0	0.75	0.75	1.03	1.03
∞	0.707	0.707	1.00	1.00

†U = uniaxial σ_∞ B = biaxial σ_∞.

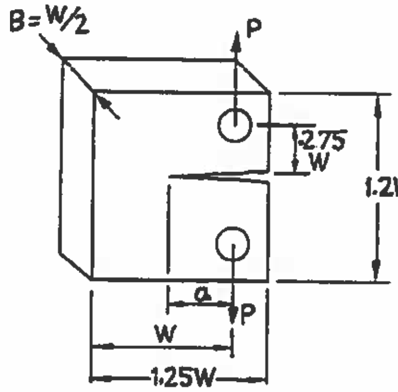


$$K_I = \sigma_\infty \sqrt{\pi a}\left(\frac{D}{d}+\frac{1}{2}+\frac{3}{8}\frac{d}{D}-0.36\frac{d^2}{D^2}+0.73\frac{d^3}{D^3}\right)\frac{1}{2}\sqrt{\frac{D}{d}}$$

$$K_{III} = \frac{16T}{\pi D^3}\sqrt{\pi a}\left(\frac{D^2}{d^2}+\frac{1}{2}\frac{D}{d}+\frac{3}{8}+\frac{5}{16}\frac{d}{D}+\frac{35}{128}\frac{d^2}{D^2}+0.21\frac{d^3}{D^3}\right)\frac{3}{8}\sqrt{\frac{D}{d}}$$



$$K_I = \frac{4P}{B}\sqrt{\frac{\pi}{W}}\left\{1.6\left(\frac{a}{W}\right)^{1/2}-2.6\left(\frac{a}{W}\right)^{3/2}+12.3\left(\frac{a}{W}\right)^{5/2}-21.2\left(\frac{a}{W}\right)^{7/2}+21.8\left(\frac{a}{W}\right)^{9/2}\right\}$$



$$K_I = \frac{P}{B}\sqrt{\frac{\pi}{W}}\left\{16.7\left(\frac{a}{W}\right)^{1/2}-104.7\left(\frac{a}{W}\right)^{3/2}+369.9\left(\frac{a}{W}\right)^{5/2}-573.8\left(\frac{a}{W}\right)^{7/2}+360.5\left(\frac{a}{W}\right)^{9/2}\right\}$$

NAF
March 2010