EGT3
ENGINEERING TRIPOS PART IIB

## Module 4C15

MEMS DESIGN

Answer not more than three questions.

All questions carry the same number of marks.

The approximate percentage of marks allocated to each part of a question is indicated in the right margin.

Write your candidate number not your name on the cover sheet.

STATIONERY REQUIREMENTS
Single-sided script paper

## SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM <br> CUED approved calculator allowed <br> Attachment: 4C15 MEMS Design data sheet, 2018 (4 pages). <br> Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 An electrostatically actuated silicon MEMS switch is implemented in the form of a cantilever beam shown in top view in Fig. 1. The out-of-plane structural thickness is $10 \mu \mathrm{~m}$. The cantilever is $200 \mu \mathrm{~m}$ long and $4 \mu \mathrm{~m}$ wide as shown. The beam is actuated using two parallel-plate electrodes that extend inward from the tip of the cantilever as shown in Fig. 1. The length of each electrode is $20 \mu \mathrm{~m}$ and the gap spacing between each electrode and the adjacent cantilever surface is 200 nm .
(a) The net van der Waals force of attraction $F(h)$ per unit area between two flat planar surfaces can be written as

$$
F(h)=\frac{8 w}{3 h_{0}}\left\{\left(\frac{h}{h_{0}}\right)^{-3}-\left(\frac{h}{h_{0}}\right)^{-9}\right\}
$$

where $h$ is the gap between the surfaces, $h_{0}$ is the equilibrium gap spacing, and $w$ is the work of adhesion of the materials concerned. Sketch the form of this relationship and obtain a value for the maximum force of attraction between the two surfaces.
(b) Calculate the pull-in voltage for this switch, assuming the electrode surfaces remain parallel during the actuation process.
(c) The work of adhesion between the two surfaces is $100 \mathrm{~mJ} \mathrm{~m}^{-2}$ and the value of $h_{0}$ is 0.5 nm . Estimate the force required to pull the cantilever from the surface and express this as a fraction of the electrostatic force required to bring the surfaces into contact.
(d) Through surface treatment, the work of adhesion between the two surfaces is reduced to $1 \mathrm{~mJ} \mathrm{~m}^{-2}$. A second electrode is symmetrically placed on the opposite side of the cantilever to actuate the beam back into motion while the voltage on the first electrode is reduced to 0 V . Estimate the pull-off voltage that must be applied to the second electrode in this case. You may assume that the electrostatic force can be approximated by a parallel-plate model for pull-off.
(e) Determine the value for the maximum work of adhesion such that the cantilever pulls off as soon as the voltage on the first electrode is reduced to 0 V . Sketch the current-voltage transfer characteristics of the switch for the values provided in this case assuming that the maximum current is restricted to $10 \mu \mathrm{~A}$ when the switch is closed.

moveable

fixed

Fig. 1

2 A square plate MEMS resonator of side length 2.0 mm , anchored at its corners, is designed in a silicon micromachining process as shown in Fig. 2. The structural thickness of the device layer is $25 \mu \mathrm{~m}$. The resonator is excited in the square extensional mode with mechanical parameters of the equivalent single degree-of-freedom resonator specified as effective mass $m=233 \times 10^{-9} \mathrm{~kg}$, effective stiffness $k=44.4 \times 10^{6} \mathrm{~N} \mathrm{~m}^{-1}$ and a Quality factor $Q=10000$ for motion in air. The nominal overlap length $L$ of the electrodes is 1.8 mm and the gap $g$ between the electrodes is $3 \mu \mathrm{~m}$. The resonator is driven using a combination of an AC voltage $v_{A C}$ and a DC bias voltage $V_{P}$ applied to the 4 electrodes with respect to the plate. The motion is transduced by measuring the total capacitive current generated due to the motion of the square plate.
(a) Obtain an expression for the motional current as a function of the displacement of the resonator assuming that all 4 sides of the mass extend and contract uniformly in the square extensional mode.
(b) Estimate the amplitude of the motional current at resonance for applied voltages $v_{A C}=100 \mathrm{mV}$ and $V_{P}=50 \mathrm{~V}$.
(c) Derive expressions for the electromechanical motional parameters of this device and calculate specific values of these parameters for the drive conditions in (b).
(d) Sketch graphs showing the dependence of the motional resistance as a function of the bias voltage, transduction gap, and structural thickness. Hence, discuss approaches to minimise motional resistance subject to processing constraints.


Fig. 2

## Version AAS/4

3 A schematic of a capacitive accelerometer is shown in Fig. 3. The mechanical assembly consists of a suspended spring-supported mass. The response of the mass is transduced using a single-ended parallel-plate electrode arrangement, implemented as a parallel array of capacitive unit cells. The mechanical parameters are specified as mass $m=10^{-6} \mathrm{~kg}$, stiffness $k=100 \mathrm{~N} \mathrm{~m}^{-1}$ and damping constant, $c=10^{-4} \mathrm{~N} \mathrm{~m}^{-1} \mathrm{~s}$. The geometrical parameters for the device include a structural thickness $t=10 \mu \mathrm{~m}$, a nominal electrode gap $g=1 \mu \mathrm{~m}$, and electrode overlap length $L=100 \mu \mathrm{~m}$. The number of capacitive unit cells is $N=100$ and the gap spacing between successive unit cells is $10 \mu \mathrm{~m}$. A DC voltage is applied between the fixed electrode and the moving mass to sense the displacement response, and all the fixed electrodes are tied to the same potential.
(a) Estimate upper and lower bound values for the pull-in voltage of this device.
(b) Calculate the fractional change in capacitance in response to an external acceleration of $1 \mathrm{~m} \mathrm{~s}^{-2}$ along the sensitive axis.
(c) Estimate the thermo-mechanical noise equivalent acceleration response at 300 K .
(d) The maximum displacement of the mass is limited to $10 \%$ of the gap spacing. Estimate the dynamic range of the accelerometer and the deviation in response from linear behaviour at the top end of the measurement range, assuming that the nonlinearity is dominated by the transducer only.


Fig. 3

## Version AAS/4

4 A glass-based microfluidic device consists of 3 fluidic ports and an asymmetric Tshaped channel geometry shown in Fig. 4. The channels have a square cross-section of dimensions $100 \mu \mathrm{~m} \times 100 \mu \mathrm{~m}$. A buffer solution consisting of two molecular species with differing electrophoretic mobilities is pumped from Port 1 to Port 2 initially under a uniform electric field using electroosmosis. The glass wall potential is estimated to be 100 mV and the relative permittivity of the medium is 80 . You may assume that the dynamic viscosity of water is $10^{-3} \mathrm{~Pa}$.
(a) Estimate the volumetric flow rate for an applied voltage of 100 V between ports 1 and 2.
(b) Estimate the equivalent pressure difference that must be applied across ports 1 and 2 to achieve an identical flow rate to (a) above.
(c) At a certain instant, the electric field between ports 1 and 2 is switched off while the voltage at port 3 is set at 200 V relative to the junction potential so that a plug of fluid is transported between the channel junction towards port 3. Determine the flow velocity of the plug.
(d) Estimate the separation distance between the two molecular species when the plug has been transported a further distance of 4 mm down the separation channel. The difference in the electrophoretic mobilities is $10^{-8} \mathrm{~m}^{2} \mathrm{~V}^{-1} \mathrm{~s}^{-1}$.
(e) Comment on the optimisation of device design and geometry to obtain a good separation between the two molecular species.


Fig. 4

Version AAS/4

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## ENGINEERING TRIPOS Part IIB

## Module 4C15 Data Sheet

## Elastic Hertzian point contact under load $P$

Reduced radius $R$ given by $\frac{1}{R}=\frac{1}{R_{1}}+\frac{1}{R_{2}}$ (Suffixes 1, 2 refer to the two bodies in contact)
Contact modulus $E^{*}$ given by $\frac{1}{E^{*}}=\frac{1-v_{1}^{2}}{E_{1}}+\frac{1-v_{2}^{2}}{E_{2}}$
Radius of contact circle $a=\left\{\frac{3 P R}{4 E^{*}}\right\}^{1 / 3}$
Maximum contact pressure $p_{0}=\frac{3 P}{2 \pi a^{2}}=\left\{\frac{6 P E^{* 2}}{\pi^{3} R^{2}}\right\}^{1 / 3}$
Mean contact pressure $\bar{p}=\frac{2}{3} p_{0}$
Approach of distant points $\delta=\frac{a^{2}}{R}=\left\{\frac{9 P^{2}}{16 R E *^{2}}\right\}^{1 / 3}$

Maximum shear stress is of magnitude $0.31 p_{0}$ and at depth $0.48 a$.

Lennard-Jones potential between point atoms

$$
U(r)=-\frac{\mathrm{C}}{r^{6}}+\frac{\mathrm{D}}{r^{12}}=-4 U_{0}\left\{\left(\frac{1.12 r}{r_{0}}\right)^{-6}-\left(\frac{1.12 r}{r_{0}}\right)^{-12}\right\}
$$

where $U_{0}$ is bond energy and $r_{0}$ is bond length, i.e. spacing at which $U(r)$ is minimum.

Smooth surface adhesion $p(h)=\frac{8 w}{3 h_{0}}\left\{\left(\frac{h}{h_{0}}\right)^{-3}-\left(\frac{h}{h_{0}}\right)^{-9}\right\}$
$w$ is the work of adhesion, in principle $w=\gamma_{1}+\gamma_{2}-\gamma_{12}$
Elastic spherical contact with adhesion, JKR $\frac{4 E * a^{3}}{3 R}=P+2 \sqrt{2 \pi w E^{*} a^{3}}$
Pressure drop across meniscus $\Delta p=\frac{\gamma}{r}$ for each liquid/vapour interface
Yield stress in shear $k \approx H / 6$

Archard wear, $\quad$ dimensional wear rate $\propto \frac{\text { pressure } \times \text { sliding speed }}{\text { hardness } H}$

## SURFACE ENERGIES AT ROOM TEMPERATURE*

High energy solids

| Material | Surface energy $\mathrm{mJ} \mathrm{m}^{-2}$ |
| :---: | :---: |
| NaCl | 160 |
| $\mathrm{Al}_{2} \mathrm{O}_{3}$ | 641 |
| Si | 1280 |
| Al | 1120 |
| Ag | 1440 |
| Fe | 2400 |
| W | 4490 |

Low energy solids

| Material | Surface energy $\mathrm{mJ} \mathrm{m}^{-2}$ |
| :---: | :---: |
| nylon | 46.5 |
| polyvinyl chloride | 38.9 |
| polystyrene | 33.0 |
| polyethylene | 30.4 |
| paraffin wax | 25.0 |
| PTFE | 18.3 |
| Diamond-Like-Carbon | $25-40$ |

## Liquids

| Material | Surface energy $\mathrm{mJ} \mathrm{m}^{-2}$ |
| :---: | :---: |
| water | 73.1 |
| benzene | 28.8 |
| n-pentane | 16.0 |
| n-octane | 21.6 |
| n-dodecane $\left(\mathrm{C}_{12} \mathrm{H}_{26}\right)$ | 25.5 |
| n-hexadecane $\left(\mathrm{C}_{16} \mathrm{H}_{34}\right)$ | 27.6 |
| n-octadecane $\left(\mathrm{C}_{18} \mathrm{H}_{38}\right)$ | 28.0 |
| Fomblin Zdol | $20 \sim 25$ |

* from: Adamson, A. W., Physical Chemistry of Surfaces, Wiley (1990) and Israelachvili, J., Intermolecular and Surface Forces, Academic Press (1992)


## Electrostatic forces

$F_{P P}=\frac{\varepsilon A V^{2}}{2 g^{2}}$; magnitude of the electrostatic force for a gap-closing parallel-plate actuator where $\varepsilon$ is the permittivity for the medium between the plates, $A$ is the area of overlap, $g$ is the gap spacing between the electrodes, $V$ is the voltage applied.
$V_{P I}=\sqrt{\frac{8 k g_{0}^{3}}{27 \varepsilon A}}$; pull-in voltage for a gap-closing parallel-plate actuator where $k$ is the effective spring constant, $g_{0}$ is the initial gap spacing with 0 V applied, $\varepsilon$ is the permittivity for the medium between the plates, and $A$ is the area of overlap.
$F_{\text {COMB }}=\frac{\varepsilon t V^{2}}{2 g}$; magnitude of the electrostatic force generated between a pair of electrodes arranged in the form of a comb drive where $\varepsilon$ is the permittivity for the medium between the plates, $t$ is the structural thickness, $g$ is the gap spacing between the electrodes, $V$ is the voltage applied.

## Thermo-mechanical Noise

$\bar{F}_{n}=\sqrt{4 k_{B} T b}$ in units of $N / \sqrt{H z}$; analytical expression for the thermo-mechanical force noise spectral density where $k_{\mathrm{B}}$ is the Boltzmann constant, $T$ is the temperature, and $b$ is the damping constant.

## Damping constants

$b=\frac{\eta A}{h}$; analytical expression for damping constant determined by Couette flow where $\eta$ is the dynamic viscosity of the fluid, $A$ is the area of overlap of the two surfaces and $h$ is the constant gap spacing between the surfaces.
$b=\frac{96 \eta L W^{3}}{\pi^{4} h^{3}}$; analytical expression for damping constant determined by squeeze film effects where $\eta$ is the dynamic viscosity of the fluid, $L$ is the overlap length (long dimension), $W$ is the width (short dimension), and $h$ is the nominal gap spacing between the surfaces.

## Equivalent Circuit parameters for resonators

$L_{m}=m / \eta^{2}$; where $m$ is the effective mass and $\eta$ is the transduction parameter.
$C_{m}=\eta^{2} / k$; where $k$ is the effective stiffness and $\eta$ is the transduction parameter.
$R_{m}=b / \eta^{2}$; where $b$ is the damping constant and $\eta$ is the transduction parameter.
$\eta=\frac{V_{P} \varepsilon A}{g^{2}}$; transduction parameter for a parallel-plate electrode geometry with a symmetric drive and sense configuration, where $\varepsilon$ is the permittivity for the medium between the plates, $A$ is the electrode overlap area, $g$ is the gap spacing between the electrodes, $V_{\mathrm{P}}$ is the DC polarization voltage applied across the electrode(s) and the resonator.

## Microfluidics

$\Delta P=\frac{12 \eta L}{W h^{3}} Q$; relationship between pressure drop $(\Delta P)$ and volumetric flow rate $(Q)$ for Poiseuille Flow in a microchannel with rectangular cross-section, where $\eta$ is the dynamic viscosity of the fluid, $L$ is the channel length, $W$ is the channel width and $h$ is the channel height.
$Q=\frac{\pi a^{4}}{32 \eta} \frac{\Delta P}{L}$; relationship between volumetric flow rate $(Q)$ and pressure drop $(\Delta P)$ for Poiseuille Flow in a microchannel with circular cross-section, where $\eta$ is the dynamic viscosity of the fluid, $a$ is the channel radius and $L$ is the channel length.
$U_{0}=-\frac{\sigma_{\mathrm{w}} \mathrm{E}_{x} L_{D}}{\eta}=-\frac{\varepsilon \zeta}{\eta} \mathrm{E}_{x}$; Electroosmotic plug flow velocity $\left(U_{0}\right)$ as a function of the glass wall charge $\left(\sigma_{\mathrm{w}}\right)$, the zeta potential ( $\zeta$ ), magnitude of the Electric field ( $\mathrm{E}_{x}$ ) for electroosmotic transport, dynamic viscosity $(\eta)$, Debye Length $\left(L_{\mathrm{D}}\right)$, permittivity of the fluid medium ( $\varepsilon$ ).
$v_{e p}=\mu_{e p} \mathrm{E}_{x}$; electrophoretic velocity $\left(v_{e p}\right)$ as a function of the electrophoretic mobility $\left(\mu_{e p}\right)$ and the magnitude of the Electric field $\left(\mathrm{E}_{x}\right)$.

