Version RS/2

EGT3
ENGINEERING TRIPOS PART IIB

Wednesday 26 April 2017 9:30 to 11

Module 4F2

ROBUST & NONLINEAR CONTROL

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 Consider the closed loop system in Figure 1 where

$$G_0(s) = \frac{1}{s-1}$$

is the transfer function of an unstable plant, $K(s)$ represents the controller, and $\Delta(s)$ represents multiplicative uncertainties.

(a) The controller

$$K(s) = \frac{s-1}{s+1}$$

guarantees stable transfer functions $T_{r\to y}$ and $T_{r\to e}$, respectively from r to y and from r to e. However, it is known that zero/pole cancellations of unstable poles should be avoided.

(i) Define the notion of internal stability. [15%]

(ii) Show that the nominal closed loop system is not internally stable. [15%]

(b) The simpler proportional controller

$$K(s) = k$$

guarantees nominal closed loop stability for any $k > 1$.

(i) Compute the nominal performance $\|T_{d\to y}\|_\infty$ achieved by the controller. [15%]

(ii) Using Parseval's theorem $\frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{f}(j\omega)^* \bar{g}(j\omega) d\omega = \int_{-\infty}^{\infty} f(t)^* g(t) dt$ show that $\sup_{d\neq 0} \frac{\|y\|_2}{\|d\|_2} \leq \|T_{d\to y}\|_\infty$. [20%]

(c) Assuming $K(s) = k > 1$, use the small gain theorem to answer the following:

(i) Find the largest bound on the unstructured uncertainty $\|\Delta(s)\|_\infty$ for which the proportional controller guarantees robust stability. [15%]

(ii) Suppose that $|\Delta(j\omega)| \leq \frac{\omega+2k}{4k}$. For which $k > 1$, if any, is the closed loop system stable? Motivate your answer. [20%]

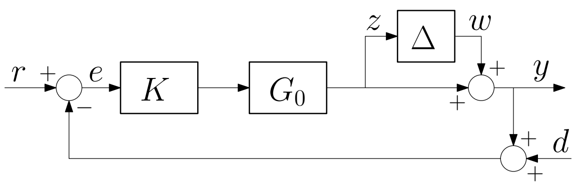

Fig. 1

2 Consider the transfer function

$$G_\alpha(s) = \frac{1}{s+\alpha} \quad \text{where} \quad -\frac{1}{2} \le \alpha \le \frac{1}{2} \,. \tag{1}$$

(a) We want to find a controller K that stabilizes $G_\alpha(s)$ for any admissible value of α. We approach the design by perturbation to coprime factors.

(i) Find a normalized left coprime factorization for $G_0(s) = \frac{\tilde{N}(s)}{\tilde{M}(s)}$. [10%]

(ii) Write $G_\alpha(s) = \frac{\tilde{N}(s)+\Delta_N(s)}{\tilde{M}(s)+\Delta_M(s)}$ as a perturbation of $G_0(s)$. Evaluate $\|[\Delta_N, \Delta_M]\|_\infty$. [20%]

(b) Recall the definition of the "stability margin"

$$b(G_0, K) = \| \left[\begin{smallmatrix} K \\ I \end{smallmatrix}\right] [I - G_0 K]^{-1} \tilde{M}^{-1} \|_\infty^{-1} \,.$$

Using the small gain theorem, derive the condition on $b(G_0, K)$ that guarantees stability of the closed loop system in Figure 2. Compute the value of $b(G_0, K)$ that guarantees stability for any $G_\alpha(s)$ defined in (1). [50%]

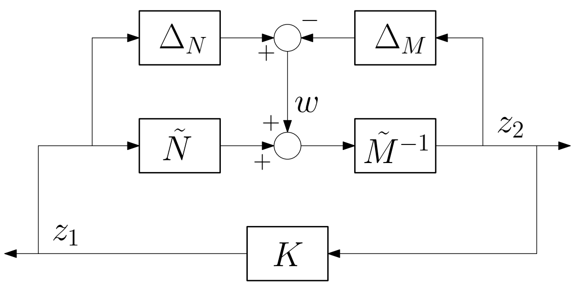

Fig. 2

(c) For the particular controller $K(s) = -k$, find the range of gains $k > 0$ that guarantee closed loop stability. [20%]

3 Consider the (negative) feedback interconnection of the linear system of transfer function $H(s) = \frac{1}{s(s+1)(s+2)}$, with a differentiable sector nonlinearity $y = \varphi(u)u$ described by the nonlinear gain

$$0 < \varphi(u) \leq k_2$$

(a) Sketch the Nyquist plot of $H(s)$ and use the Nyquist criterion to determine the maximal value of k_2 such that the zero equilibrium of the *linearised* system is asymptotically stable. Is the equilibrium of the *nonlinear* system also asymptotically stable in this case ? Justify your answer. [20%]

(b) Use the circle criterion to provide a lower bound on k_2 that guarantees *global* asymptotic stability of the zero equilibrium. [20%]

(c) Applying the circle criterion is equivalent to applying the passivity theorem after a suitable loop transformation. In your answer to question (b), what is the underlying loop transformation ? Show that the transformed nonlinearity is strictly passive and that the transformed transfer function is strictly positive real. [20%]

(d) Could the lower bound provided by the circle criterion in question (b) be improved by applying the Popov criterion ? Justify your answer. [20%]

(e) Consider now a sign nonlinearity

$$y = \text{sign}(u)$$

In simulation, the negative feedback interconnection with $H(s)$ results in a limit cycle oscillation of frequency 1.3 rad s^{-1} and amplitude 0.2 . Compare this simulation result with the prediction of the describing function method. [20%]

4 The Hopfield neural network

$$\dot{x}_i = -x_i + S(u_i - \frac{1}{n-1}\sum_{j\neq i} x_j), \quad i = 1,\ldots,n \tag{2}$$

is sometimes called a *winner-take-all* network. For the analysis below, assume that S is differentiable but well approximated by the piecewise linear function

$$S(y) = \begin{cases} 0, & y \leq 0, \\ 2y & 0 \leq y \leq 1, \\ 2, & y \geq 1 \end{cases}$$

(a) Consider $n = 2$ and an input vector with entries $u_1 = u_2 = 1$. Determine the equilibria of the system and sketch the phase portrait of the linearised system around each of them. [30 %]

(b) Sketch a phase portrait of this nonlinear network for the two following situations: (i) $1 \approx u_1 \approx 2u_2$, and (ii) $u_1 \approx u_2 \approx 1$. Explain the behaviour of the network from the phase portraits. [40 %]

(c) For the two situations considered in (b), sketch the time-trajectory of $x_2(t)$ as a function of time for an initial condition $(x_1(0), x_2(0)) = (\varepsilon, 0)$ with $\varepsilon > 0$ and small. [20%]

(d) From your previous analysis, how would you describe the properties of a winner-take-all model for any n? [10%]

END OF PAPER

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