1) a) i)

The work input is given by $w = \int_{1\,bar}^{100\,bar} v\,dp$, where v can be taken as constant

$w = v((100 - 1)\times 10^5)$, with v = 1/997.0 m^3 for water at 25 °C and 1 bar.

$w = 0.001003\times 99\times 10^5 = 9.93\ kJ/kg$

ii) Heater (stations [2] → [3]):

$$Q = h_3 - h_2$$

At [3] the fluid must be saturated as it is a two-phase mixture, so using the saturated enthalpies at 100 bar,

$$h_3 = 1407.9(1 - x) + (x)2725.5 = 2066.7\ kJ/kg$$

$$h_2 = 104.9 + 9.93 = 114.8$$

(i.e. the value before the pump + the work done by the pump)

$$Q = 1952\ kJ/kg$$

iii) The throttle (stations [3] → [4]) is adiabatic and there is no work, so the enthalpy remains constant.

$$h_4 = 2066.7\ kJ/kg$$

At 1 bar, this enthalpy is between the values for the saturated liquid and vapour, so temperature is 99.61 °C and state is two-phase. Dryness fraction from enthalpy:

$$h_4 = 2066.7 = 417.5(1 - x) + (x)2674.9$$

$$x = 0.731$$

The entropy is then

$$s_4 = 1.303(1 - x) + (x)7.359 = 5.73\ \text{kJ/kg/K}$$

b)

i) Partial pressure of water:

$$Pw = Psat\times .5 = 0.12352\times 0.5 = 0.06176$$

Hence wet air has 15.19 moles of air per mole of water.

$$\frac{m_{water}}{m_{air}} = \frac{18}{15.19\times 29} = 0.0409$$

ii)

The partial pressure of the water vapour is 0.06176 bar. From the saturation data in the thermofluids data book

Psat	Tsat
0.06	36.16
0.08	41.51

Interpolating

$$T_{dew} = 36.16 + (41.51 - 36.16) \times \frac{(0.06176 - 0.06)}{(0.08 - 0.06)} = 36.6\,°C$$

c) i) Enthalpy of a perfect gas is a function of temperature only. Hence enthalpy of water vapour at 0.06176 bar and 50 °C is equal to that of saturated vapour at 50 °C:

$$h_{wout} = 2591.3 \text{ kJ/kg}$$

ii)

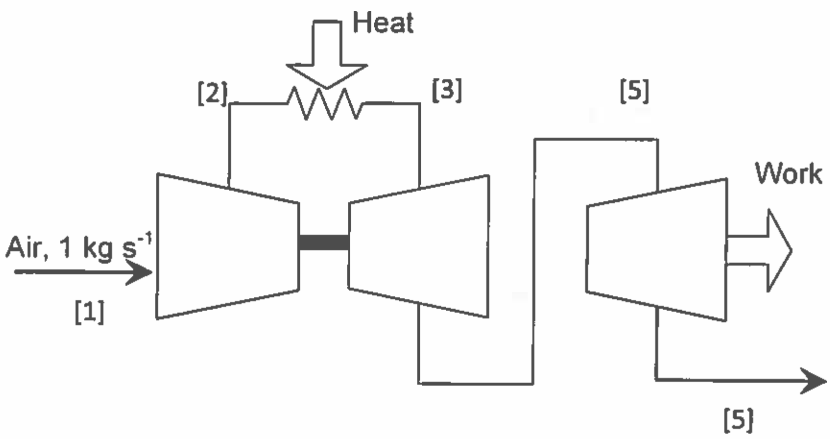
$$q = h_{air_{out}} m_{air}/m_w + h_{wout} - h_4 - h_{air_{in}} m_{air}/m_w$$

There is no change in air temperature, so there is no difference in its enthalpy. Hence

$$q = 2591.3 - 2066.7 = 525 \text{ kJ/kg}$$

Positive, hence heat must be added.

2)



a i)

$$T_2^{isen} = T_1(r)^{\frac{\gamma-1}{\gamma}}$$

$$\frac{T_2^{isen} - T_1}{\eta_{isen}} = T_2 - T_1$$

$$T_2 = T_1 + \frac{T_2^{isen} - T_1}{\eta_{comp}}$$

$$= 298.15 + \frac{298.15\times(10)^{\frac{0.4}{1.4}} - 298.15}{0.8} = 645\ K$$

ii) The compressor work must equal the work from the first turbine

$$W_{comp} = c_p(T_2 - T_1) \ = c_p(T_3 \ - T_4^{isen})\eta_{turbine}$$

$$(T_2 - T_1) = (T_3 \ - T_3 \ r_{t1}^{\ \frac{(\gamma-1)}{\gamma}})\eta_{turbine}$$

$$r_{t1} = \left(1 - \frac{(T_2 - T_1)}{T_3 \ \eta_{turbine}}\right)^{\left\{\frac{(\gamma)}{\gamma-1}\right\}} = 3.88$$

iii) The work out from the second turbine is

$$W_{comp} = c_p(T_4 \ - T_5^{isen})\eta_{turbine}$$

$$T_4 = T_3 + \ (T_4^{isen} - T_3)\eta_{turbine}$$

$$= \ 1200 + \left(1200\times(1/3.88)^{\frac{0.4}{1.4}} - 1200\right) * 0.9 = 853\ K$$

Similarly the pressure ratio across the second turbine is 10/3.88, allowing T5 to be calculated

$$T_5 = T_4 + (T_5^{isen} - T_4)\eta_{turbine}$$

$$= 843 + \left(843\times(10/3.88)^{\frac{0.4}{1.4}} - 843\right) * 0.9 = 671\ K$$

$$W = c_p(T_4 - T_5) = 183\ kW$$

b) i) The power potential added in the heater is

$$E_Q = \Delta h - T_o\Delta S$$

$$= c_p(T_3 - T_2) - T_o\left(c_p \ln\left(\frac{T_3}{T_2}\right)\right)$$

$$= 371.7kW$$

ii) The total loss in work potential is the sum of that from the turbines and the compressors. They are adiabatic so

$$E_{loss} = T_o\Delta S_{irrev} = T_o\Delta S$$

Where

$$\Delta S = \Delta S_{comp} + \Delta S_{turbine1} + \Delta S_{turbine2}$$

$$\Delta S = \left(c_p\, ln\left(\frac{T_2}{T_1}\right) - Rln\left(\frac{P_2}{P_1}\right) + c_p\, ln\left(\frac{T_4}{T_3}\right) - Rln\left(\frac{P_4}{P_3}\right) + c_p\, ln\left(\frac{T_5}{T_4}\right) - Rln\left(\frac{P_5}{P_4}\right)\right)$$

$$\Delta S = c_p\, ln\left(\frac{T_2}{T_1}\frac{T_4}{T_3}\frac{T_5}{T_4}\right) - Rln\left(\frac{P_2}{P_1}\frac{P_4}{P_3}\frac{P_5}{P_4}\right)$$

$$\Delta S = c_p\, ln\left(\frac{T_2}{T_1}\frac{T_5}{T_3}\right) - Rln\left(\frac{10}{3.88}\frac{3.88}{10}\right)$$

$$\Delta S = c_p\, ln\left(\frac{T_2}{T_1}\frac{T_5}{T_3}\right) = 191.6\, J\, kg^{-1}\, K^{-1}$$

$$E_{loss} = 298.15 * 191.6 = 57.1\ kW$$

iii) Overall we expect the change in the availability of the air between the inlet and outlet to be

$$\Delta B = E_q - W - E_{loss}$$

(note the minus signs are from taking the work and the loss as positive numbers despite work leaving the system)

Thus, the difference between $W + E_{loss}$ and the availability supplied in the heat is used to give the air leaving the system a positive availability. Another way to express this is to say that the air leaving the second turbine is hot and thus itself has the potential to do work.

c) The heat capacity of the combustion gases has the same Cp and mass flow as the air, this means that in the heat exchange process, the temperature difference between the hot gases and the cold air remains constant in a counter current heat exchanger.

The irreversibility is the difference between the loss of availability of the combustion gases and the gain in availability of the air (calculated previously). The temperature difference is constant so the combustion gases cool from 1500 to 645+300.

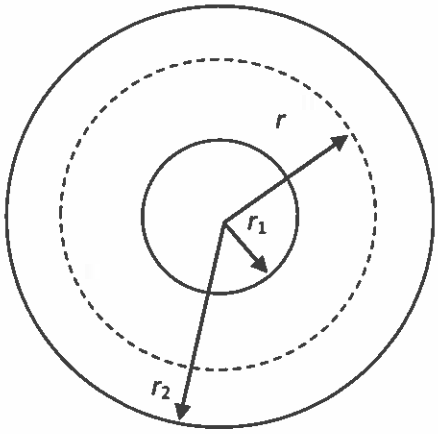
$E\_Q_{\ combustiongas} \ = c_p(945 - 1500) - T_o\left(c_p \ln\left(\frac{945}{1500}\right)\right)$ = -419.3 kW (i.e. a loss)

Therefore the amount lost due to heat transfer across a finite temperature difference is 419.3-371.

7 = 47.6 kW

3)

a)



Consider a circular cylinder as shown, of length *l*. With boundary conditions:- at r = r_1, T = T_1, and at r = r_2, T = T_2. The fluids is at Tf

$$\dot{Q} = -2\pi r l \lambda \frac{dT}{dr}$$

$$\dot{Q} \int_{r_1}^{r_2} \frac{1}{r} dr = -2\pi l \lambda \int_{T_1}^{T_2} dT$$

$$\dot{Q} \ln\left(\frac{r_2}{r_1}\right) = 2\pi l \lambda (T_1 - T_2)$$

$$\dot{Q} = \frac{2\pi\lambda}{\ln\left(\frac{r_2}{r_1}\right)} (T_1 - T_2)$$

So the thermal resistance per unit length is

$$R = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi\lambda}$$

Or

$$R_1 = \frac{\ln\left(\frac{r + t_1}{r}\right)}{2\pi\lambda_1}$$

A similar resistance can be written for the outer material

$$R_1 = \frac{\ln\left(\frac{r + t_1 + t_2}{r \quad + t_1}\right)}{2\pi\lambda}$$

The total resistance is then

$$R_2 + R_1 = R_{tube} = \frac{1}{2\pi\lambda_2} \ln\left(\frac{r_1 + t_1 + t_2}{r_1 + t_1}\right) + \frac{1}{2\pi\lambda_1} \ln\left(\frac{r_1 + t_1}{r_1}\right) = 0.012257K\, W^{-1}$$

b)i) In the Reynolds analogy, the heat and momentum diffusivity are equal (Pr =1). The same random movement of either particles in laminar flow or packets of fluid in turbulent flow results in the same diffusion of heat and momentum. If the flow is turbulent in the pipe, the thermal and momentum boundary layers will also be equivalent. This means the equations governing heat transfer and momentum transfer end up being the same when non-dimensionalised and will have the same boundary conditions and solutions. The temperature gradient (heat flux) and velocity gradient (shear stress) at the wall are linked. When non dimensionalised this leads to the Stanton number being half the friction factor (as in the data book). The arguments made in the lectures about flow over a plate apply equally well to flow in a pipe, and the analogy did not depend on the actual shape of the velocity profile. For this particular case this analogy is easier to see as the Reynolds number is high enough for the flow to be turbulent, so a thin boundary layer must exist at the wall. If the boundary layer is thin, then a round tube wall can be analysed as flat plate. In this case the flow is fully developed and the boundary layer thickness is constant, however as noted above the arguments in the Reynolds analogy do not depend on shape or size of the velocity profile. There is no form drag.

ii)The heat flow through the wall is the same as that into the fluid per unit length

$$\dot{Q} = \frac{1}{R_{tube}}(T_1 - T_2) = h2\pi r_1(T_f - T_1)$$

$$\frac{1}{2\pi r_1 R_{tube}}\frac{(T_1 - T_2)}{(T_f - T_1)} = h$$

$$h = \frac{1}{2\times\pi\times10^{-2}\times0.012257}\frac{(10)}{(52)} = 249.7\ w\ m^{-1}K^{-1}$$

Now calculate Stanton number and friction factor.

$$St = \frac{h}{\rho V Cp} = \frac{246.6}{1\times100\times1000} = 0.002497$$

From the Reynolds analogy

$$St = \frac{1}{2}Cf$$

$$Cf = 0.004994$$

The pressure drop per unit length is then

$$\frac{dP}{dz} = \frac{2\rho Cf V^2}{d} = \frac{2\rho Cf V^2}{d} = 4994\ Pa\ m^{-1}$$

[Note the data book gives Cf in terms of wall shear stress, so students may have to convert from shear stress to pressure drop per unit length]

c) The Nusselt Number scales with $Re^{.8}$. Since the physical properties of the fluid and dimensions remain unchanged.

$$\frac{Nu_{new}}{Nu_{old}} = \frac{Re_{new}^{0.8}}{Re_{old}^{0.8}}$$

$$\frac{h_{new}}{h_{old}} = \frac{V_{new}^{0.8}}{V_{old}^{0.8}}$$

$$h_{new} = \left(\frac{20}{100}\right)^{0.8} \times 249.7 = 68.9 w\, m^{-1} K^{-1}$$

The pressure drop can be found from Stanton number, since we know how Nusselt number scales with Reynolds number.

$$St = \frac{Nu}{RePr} = \frac{1}{2} Cf$$

$$\frac{Nu_{new} Re_{old}}{Re_{new} Nu_{old}} = \frac{\frac{1}{2} Cf_{new}}{\frac{1}{2} Cf_{old}}$$

$$\frac{Re_{new}^{0.8} Re_{old}}{Re_{new} Re_{old}^{0.8}} = \frac{Cf_{new}}{Cf_{old}}$$

$$Cf_{new} = Cf_{old} \frac{Re_{old}^{0.2}}{Re_{new}^{0.2}} = Cf_{old} \left(\frac{100}{20}\right)^{0.2} = 0.0688$$

$$\Delta P = \frac{2\rho Cf V^2 L}{d} = \frac{2\times 1\times 0.0683\times 20^2\times 1}{2\times 10^{-2}} = 275 \;\; Pa\, m^{-1}$$

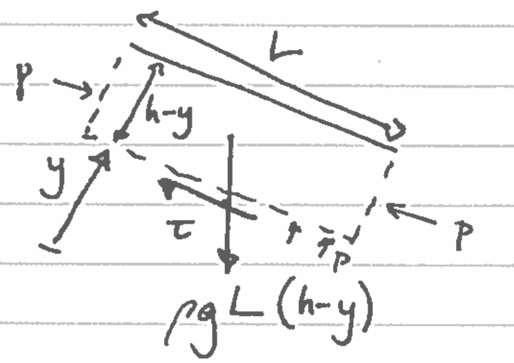
a) (i) Pressure (gauge) on surface is zero.

Parallel streamlines $\Rightarrow$ pressure variation is hydrostatic

$$p = \rho g (h-y)\cos\theta$$

(and independent of x)

(ii) Apply momentum to control vol; no change in momentum flux, so reduces to force balance



N.B. a(i) follows from y-direction equilib^{m}.

(End) Pressures cancel; hence (resolving in x-direction)

$$\rho g L (h-y)\sin\theta = \tau L \qquad (1)$$

$$\rho g (h-y)\sin\theta = \mu \frac{dV}{dy}$$

$$V = \frac{\rho g \sin\theta}{\mu}\left[hy - \frac{y^2}{2} + c\right]$$

$V = 0$ at $y = 0$ (no-slip)

$\Rightarrow c = 0$

$$V = \frac{\rho g \sin\theta}{2\mu}\, y\,(2h-y)$$

N.B. Alternatively, use elemental CV for $\mu d^2V/dy^2 = -\rho g \sin\theta$ and integrate twice, with $dV/dy|_{y=h} = 0$.

b) Force balance is modified in that weight of plate is added to that of fluid:

$$\left[MgL + \rho gL(h-y)\right]\sin\theta = \tau L \qquad (\text{cf. } ①)$$

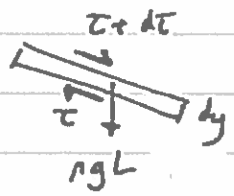
$$\left[M+\rho(h-y)\right]g\sin\theta = \mu\frac{dV}{dy}$$

From $\mu\frac{d^2V}{dy^2} = -\rho g\sin\theta$ change $y=h$ BC to $\tau = Mg\sin\theta$

$$V = \frac{g\sin\theta}{\mu}\left[(M+\rho h)y - \frac{\rho y^2}{2}\right] \qquad \text{(const. of integration again zero)}$$

$$V = \frac{g\sin\theta}{2\mu}\, y\left[2(M+\rho h) - \rho y\right]$$

c) Easiest now to start from force balance on fluid lamina:



$$\Rightarrow L\,d\tau + \rho gL\,dy\sin\theta = 0$$

$$\text{i.e. } \mu\frac{d^2V}{dy^2} = -\rho g\sin\theta$$

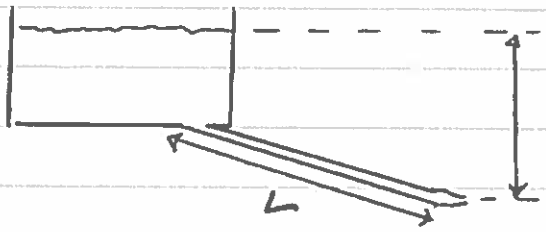
Integrate twice and apply no-slip condition at $y=0 \Rightarrow \frac{\mu V}{\rho g\sin\theta} = \left(Ay - \frac{y^2}{2}\right)$

At $y=h$, $V=-U$

$$-\frac{\mu U}{\rho g h^2\sin\theta} = \frac{A}{h} - \frac{1}{2} \Rightarrow A = h\left[\frac{1}{2} - \frac{\mu U}{\rho g h^2\sin\theta}\right] \quad ②$$

Vol. flow rate $\frac{\mu Q}{\rho g\sin\theta} = \int_0^h \left(Ay - \frac{y^2}{2}\right)dy = h^2\left[\frac{A}{2} - \frac{h}{6}\right] = 0 \quad ③ \quad A = \frac{h}{3}$

Compare ②, ③ : $\frac{\mu U}{\rho g h^2} = \frac{\sin\theta}{6}$ $\quad U = \frac{\rho g h^2}{6\mu}\sin\theta$



In reservoir, ~~stagnation~~ total pressure is $\rho g H + p_a$ ← atmospheric pressure

At nozzle exit, ∥ s/lines $\Rightarrow p = p_a$;
hence total pressure is $\frac{1}{2}\rho (Q/A_n)^2 + p_a$

Total / Stagnation pressure loss in pipe is $\frac{4c_f L}{d} \cdot \frac{1}{2}\rho \left(\frac{4Q}{\pi d^2}\right)^2$

↑ velocity in pipe

So $\rho g H - \frac{1}{2}\rho \left(Q/A_n\right)^2 = \frac{32 c_f L \rho}{\pi^2 d^5} Q^2$

$$H = \frac{Q^2}{g}\left[\frac{1}{2A_n^2} + \frac{32 c_f L}{\pi^2 d^5}\right]$$

Henceforth work with total head h (= gauge total pressure / ρg).

b) (i) No flow from main when $H_1 = 10\text{m} \Rightarrow p_{0J} = \rho g H_1 + p_a$; $h_J = 10\text{m}$

Nozzle flow has constant head $\Rightarrow h_J = \frac{1}{2g}(Q/A_n)^2$; $Q = 0.1\text{m}^3/\text{s}$

Hence $A_n = \underline{7.14} \times 10^{-3}\ \text{m}^2$

Reservoir pipe flow: $H_2 - h_J = \frac{32 c_f L_2 Q^2}{\pi^2 d_2^5 g} = k_2 Q^2$, where ...

b)(i) cont'd

$$k_2 = \frac{\overset{H_2}{1000} - \overset{h_J}{10}}{0.1^2_{\leftarrow Q}} = 99\times10^3$$

$$\Rightarrow \quad d_2 = 0.176\,m$$

b)(ii) Denote flow rates as Q_1 and Q_2; we have ....

$$\left.\begin{aligned} &① \quad h_J = \tfrac{1}{2g}\left(\frac{Q_1+Q_2}{A_m}\right)^2 = 1000\,(Q_1+Q_2)^2 \\ &② \quad H_2 - h_J = 99\times10^3\,Q_2^2 \end{aligned}\right\}\ \text{cf. (i)}$$

$$③ \quad H_1 - h_J = 1.6525\,Q_1^2 \qquad \left\{ H_1 - h_J = \frac{32 c_f L_1}{\pi^2 d_1^5 g}\,Q_1^2 \right\}$$

If $Q_1 = Q_2$, then ①, ② give $Q_2 = 0.09853\ m^3/s$

$Q_1 = Q_2$ and ③ $\Rightarrow H_1 - h_J = 0.016\,m$

① $\Rightarrow h_J = 38.835\,m$; hence $H_1 = 38.8(5)\,m$

(iii) From values conveniently pre-calculated in (ii) ....

difference $H_1 - h_J = 0.016\,m$ cf. $H_1 = 38.85\,m$, ie. $h_J \simeq H_1$.

Losses in main supply pipe are negligible.

(iv) From (iii) $h_J \simeq H_1 = 100\,m$, so ① $\Rightarrow Q_1+Q_2 = 0.316\ m^3/s$

N.B. $Q_2 \neq 0$, nor is it $0.1\,m^3/s$ (value when $H_1 = 10\,m$). Post-calc (not required) gives, from ②: $Q_2 = 0.0953\ m^3/s$.

a) Q depends on ρ μ h D Ω

ML^2T^{-2} ML^{-3} $ML^{-1}T^{-1}$ L L T^{-1}

Q/ρ — " — μ/ρ h D Ω

L^5T^{-2} L^2T^{-1} L L T^{-1}

$Q/\rho\Omega^2$ — " — $\mu/\rho\Omega$ h D

$$\underline{\underline{\frac{Q}{\rho\Omega^2 D^5} \text{ depends on } \frac{\mu}{\rho\Omega D^2}, \ h/D}}$$

$\leftarrow$ inverse Reynolds no.

b) – h/D is maintained unchanged by scaling
– to maintain Reynolds no., require $\Omega D^2 = \text{const}$

$$\Rightarrow \quad \underline{\underline{\Omega_{model} = 16\,\Omega_{full\text{-}scale}}}$$

c) m: model
f: full-scale

$$\frac{Q_m}{\rho\Omega_m^2 D_m^5} = \frac{Q_f}{\rho\Omega_f^2 D_f^5}$$ when dynamical similarity achieved

$$\frac{Q_m}{Q_f} = \left(\frac{\Omega_m}{\Omega_f}\right)^2 \left(\frac{D_m}{D_f}\right)^5 = \underline{\underline{\frac{1}{4}}}$$

d) (i) Mechanical power $= Q\Omega = 8\text{ kW}$, all goes to heat.

So rate of heat loss $= 8 - 7.8 = \underline{\underline{0.2\text{ kW}}}$

(ii) Power to be dissipated in the full-scale tank will be

$$\left(\cancel{4} \times 1000\right) \times \left(\tfrac{1}{16} \times 8\right) = 2\text{ kW}$$

Rate of heat loss from surface (assuming no change in heat-transfer coefficient, so scales on area) is

$$4^2 \times 0.2 = 3.2\text{ kW}$$

Hence we can expect full-scale tank to cool naturally; cooling system unlikely to be required, but insulation may be! In practice, reliability of model heat-loss estimate could be uncertain. Even if heat-transfer coefficient remains constant, we're assuming the same environmental temperature at full-scale. In reality, heat-transfer coefficient will have some dependence on size, via (Prandtl and) Grashof number(s).