Engineering Tripos Part IB 2017

Paper 8

CRIB

IB Paper 8 Section A 6 pages

Introductory Business Economics

1. (a) Consider the market for Butter. Use the supply and demand curves to represent and explain changes in the market equilibrium following.

   (i) A decrease in the price of margarine

   (ii) An decrease in the price of milk

   (iii) An increase in consumer income

   (iv) The introduction of a new and efficient technology in butter production.

[5]

The supply curve is upward sloping, and the demand curve is downward sloping. It is important to make a distinction between the movement along the demand or supply curve and the shifts of the demand or supply curve.

i- ***An increase in the demand for margarine and a DECREASE in the demand for butter leading the demand curve for butter to shift left.***

ii- ***This has an impact the cost of producing butter which might lead to an increase in demand. But also this would increase the supply of milk. Both supply and demand curves shift to the right.***

iii- ***Increases the demand – demand curve shifts right.***

iv- ***Increases the supply of butter.***

(b) With reference to market structure

(i) Define a monopolist firm and the market characteristics

(ii) Explain the different types of entry barriers and show and analyse diagrammatically the market equilibrium.

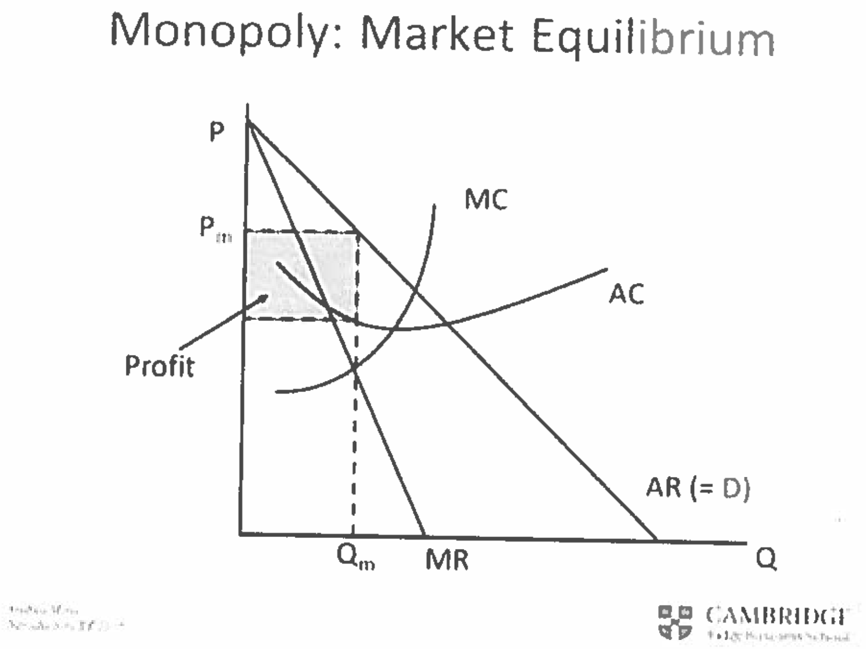
[10]

- A monopoly is the sole supplier (and potential supplier) of a particular good or service

→ the firm is the market (since there is only one firm) and the SR and the LR are the same (since no entry into the industry is possible…)

- Characteristics:
  - No close substitutes
  - Barriers to entry into the market
- **Structural** (or Innocent Barriers) – due to differences in costs

- **Statutory barriers** – entry barriers given force of law (National Lottery or television and radio broadcasting licences)
- **Strategic barriers**
  - designed to **block potential entrants** from entering a market (making the market 'less contestable')
  - to **protect the monopoly** power of incumbent and therefore maintain supernormal profits



Monopoly: Market Equilibrium

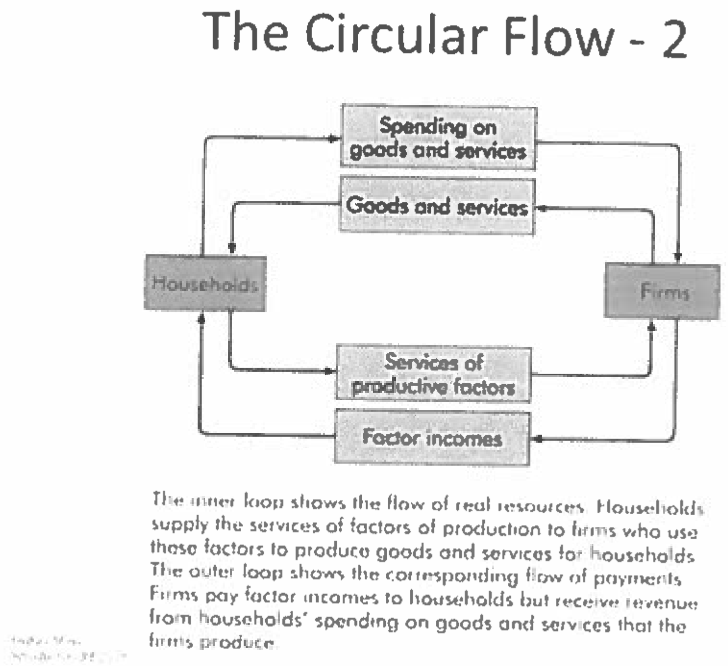
- Patents
- Vertical Integration
- Predatory Pricing
- Advertising and Marketing
- Brand Development

(c) What is the Circular flow of income in macroeconomic analysis? Explain and analyse the impact of the leakages and injections on the flow.

[10]

- Ignoring government spending and transactions with foreigners, the transactions between firms and households may be summarised as follows:
- *Households* own and supply factors of production to firms, receive income in return for these factors, and spend income on goods and services.

- *Firms* use factors of production supplied by households to produce goods and services, pay households for these factors, and sell goods and services to households.



The Circular Flow - 2

The inner loop shows the flow of real resources. Households supply the services of factors of production to firms who use those factors to produce goods and services for households. The outer loop shows the corresponding flow of payments. Firms pay factor incomes to households but receive revenue from households' spending on goods and services that the firms produce.

2. (a) Consider a market structure where there is an oligopoly.

   (i) Explain the different models of oligopoly.

   (ii) Show diagrammatically and explain the profit maximization output.

[5]

Imperfectly competitive markets fall between the extremes of perfect competition and pure monopoly. Imperfect competition is a more realistic market structure but the analysis is more complicated. There is no single model of an imperfectly competitive market. A major source of complication is the fact that imperfectly competitive markets often involve strategic or interdependent decision-making by firms, especially when there are relatively few firms in a market…

OLIGOPOLY is:

- ***A market dominated by a few large firms i.e. "Competition amongst the few"***
- ***High level of market concentration***
  - ***Concentration ratio is the market share of the leading firms***
- ***Entry barriers exist – long run supernormal profits***
- ***Mutual interdependence between competing firms***
- ***Intensive non-price competition is a common feature of oligopoly***

- *Periodic aggressive price wars (fights for market share /dominance)*
- *Strong tendency for many market structures to tend towards oligopoly in the long run*
- *Can be collusive or non-co-operative*
- *Several models of Oligopoly*
    - *Sweezy (Kinked-Demand Curve) Model*
    - *Cournot Model*
        - *Simultaneous Output Decisions*
    - *Bertrand Model*
        - *Simultaneous Price Decisions*
    - *Stackelberg Model*
        - *Sequential Output Decisions (Not analysed here)*
- *Collusion*
- *Other features of Oligopoly*
    - *Price wars*
    - *Non-price competition*

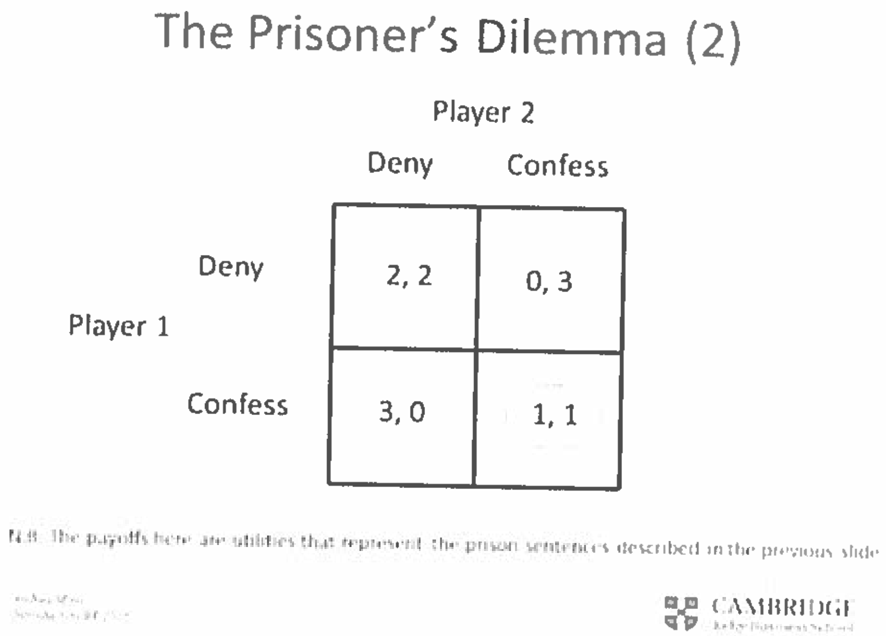
(b) Discuss the importance of the concept of game theory in making economic decisions by firms, and explain the concept of the prisoner's dilemma.

[10]

- Game Theory uses the idea of a Game in order to describe strategic situations
- The key components of a game are
    - A set of **players** (individuals, firms, governments etc.)
    - A set of **strategies** for each player
        - A strategy is essentially a plan of action
    - A complete specification of the possible **payoffs** to each player
        - Payoffs are generally shown as **utilities**
        - Payoffs include **everything** that a player cares about

An example of Game theory is Prisoner's dilemma:

- Two ex-convicts are arrested by police on suspicion of carrying out a spate of burglaries in a particular neighbourhood. The police lack the evidence to charge the pair with the burglaries immediately, so instead hold them in separate cells and offer each the chance to confess to the crimes in return for a reduced sentence.
- Each is told that if they both continue to deny the crimes, they will be charged with the lesser crime of possessing stolen goods, which carries a **1**-year prison sentence. If both confess, each will receive a **5**-year sentence for the multiple burglaries. However if one confesses to the crimes while the other denies, then the one who confesses will be released without charge while the other will be sentenced to the maximum term of **10** years.



The Prisoner's Dilemma (2)

		Player 2	
		Deny	Confess
Player 1	Deny	2, 2	0, 3
	Confess	3, 0	1, 1

N.B. The payoffs here are utilities that represent the prison sentences described in the previous slide

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(c) Illustrate the fundamental principles of Keynesian consumption theory.

[10]

Components of aggregate demand:

C = consumer demand for g & s

I = demand for investment goods

G = government demand for g & s

(closed economy: no NX)

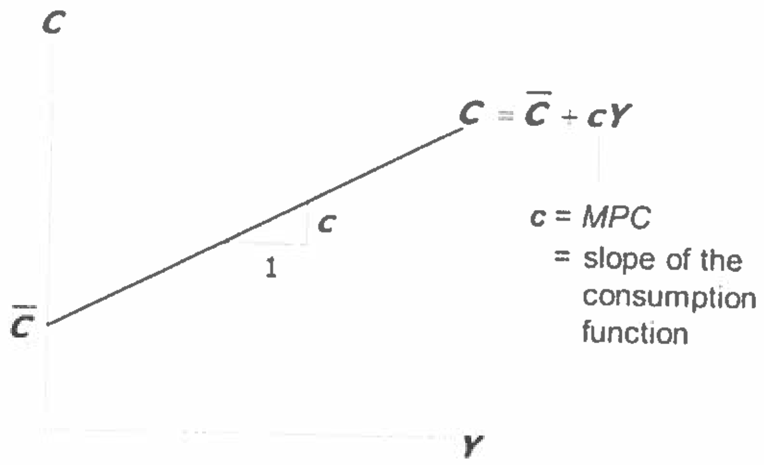
- **Disposable income** is total income minus total taxes: $Y - T$.

- Consumption function: $C = C\,(Y - T\,)$

  Shows that $\uparrow(Y - T\,) \Rightarrow \uparrow C$

- **Marginal propensity to consume (MPC)** is the change in C when disposable income increases by one pound.

The Keynesian consumption function - 1



$c = MPC$
= slope of the consumption function

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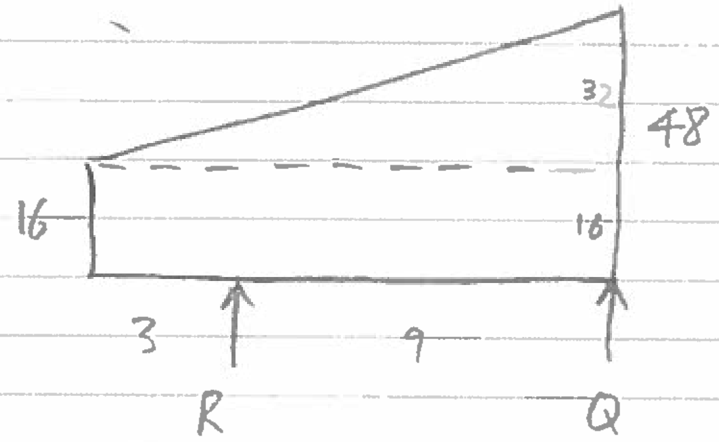
3. (NB. loads 4x values given in question – to be changed)



Take moments about Q.

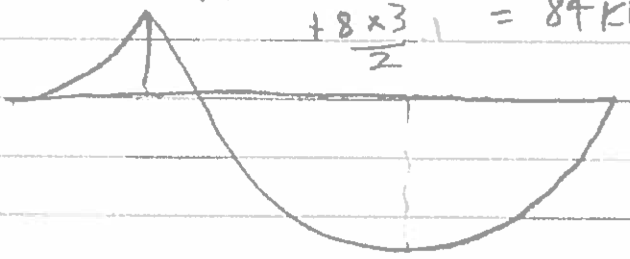
$$9R = 16 \times 12 \times 6 + 32 \times 12 \times \frac{1}{2} \times 4$$

$$\Rightarrow R = 213 \cdot 33 \text{ kN}$$

$$Q = \frac{(16 + 48)}{2} 12 - 213 \cdot 33 = 170 \cdot 67 \text{ kN}$$

B.M

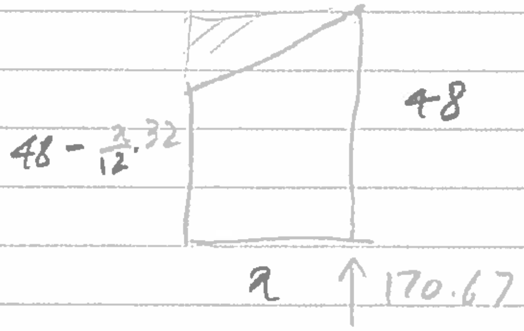
$$M = 12 \times 3 \times 1 \cdot 2 + \frac{8 \times 3}{2} = 84 \text{ kNm}$$



M max when S.F=0

3/2

Where is S.F zero?



$$S.F \text{ at } x = 170.67 - 48.x + \frac{x.\overset{4}{8}}{3}\frac{x}{2} = 0$$

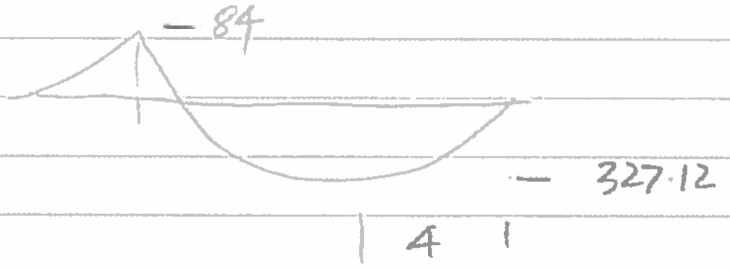
$$x = \frac{48 \pm \sqrt{48^2 - 170.67 . 16/3}}{8/3}$$

$$= 4.$$

$$B.M \text{ here} = 170.67 \times 4 - 48.4.2 + 10.67.\frac{4}{2}\frac{4}{3}$$

$$= 682.68 - 384 + 28.44 = \underline{\underline{327.12}}$$

BM



(b)(1) From data sheet

$$0.15 f_{cu} bd^2 = 327.12 . 10^6 \quad (N, mm)$$

$$0.15 . 30 . 500 . d^2 = 327.12.10^6 \Rightarrow d = 381 \text{ mm}^2$$

(b)(ii) At limit, neutral axis depth = $d/2 = 190$ mm

$$\therefore\ 327.10^6 = 0.87.460.\frac{3}{4}.380.A_s$$

$$\therefore\ A_s = 2866\ mm^2$$

Probably choose 4No 32 mm bars = 3216 mm^2
(but there are alternatives)

(c) Maximum moment is hogging = 84 KNm

If d fixed at 380 then x will be $< d/2$
Need to iterate

Guess $x = 0.2$ $\quad A_s = \dfrac{84.10^6}{360.380\left(1 - \frac{0.2}{2}\right)}$

$$= 682\ mm^2$$

Calculate $x = 2.175\left(\frac{360}{30}\right)\left(\frac{682}{380.500}\right) = 0.093$

$$\therefore A_s = \frac{84.10^6}{360.380\underbrace{\left(1 - \frac{0.093}{2}\right)}_{0.95}} = 646\ mm^2$$

(Close enough or could iterate further.

Probably choose 4 No 16 mm bars = 804 mm^2

(d) Shear capacity $v_c = 0.68.\underbrace{\left(\frac{100.804}{500.380}\right)^{0.33}}_{0.75}.\underbrace{\left(\frac{400}{380}\right)^{0.25}}_{1.01}$

$$= 0.51\ N/mm^2$$

Shear will be worst next to R.H. support

$Q = 170.67$ KN

$$\frac{Q}{bd} = \frac{170.67 . 10^3}{500 . 380} = 0.89 \text{ MPa}$$

v_s must resist $0.89 - 0.51 = 0.38$ MPa.

$$v_s = \frac{0.87 f_y A_{sq}}{b s}$$

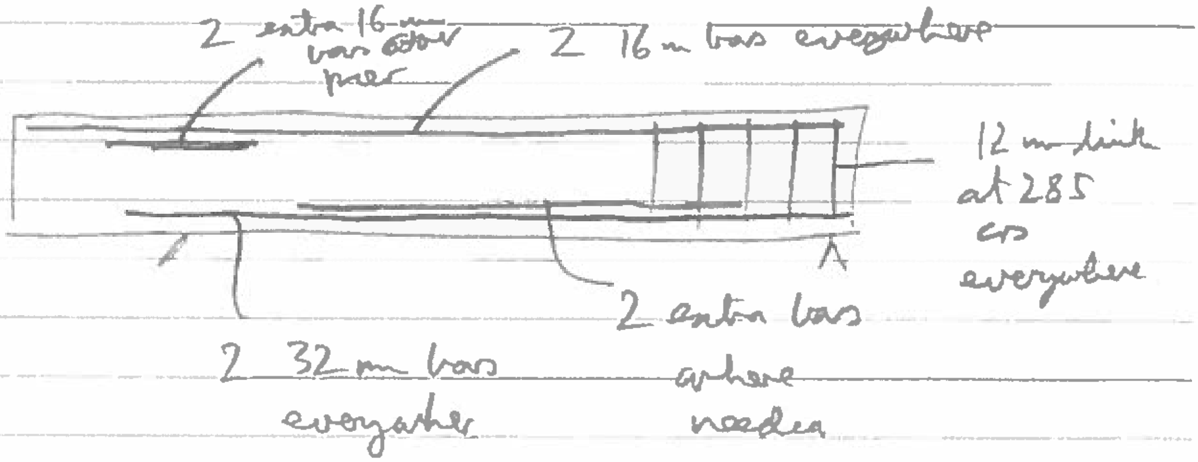
Choose $s = \frac{3d}{4}$ = (max allowed) = 285

$$\therefore 0.38 = \frac{0.87 . 360}{500 . 285} . A_{sq}$$

$\Rightarrow A_{sq} = 172 \text{ mm}^2$

$\therefore$ Two 12mm bars at 285 crs will suffice
(226 mm²

These can be used everywhere because same shear links needed.



Question solutions

4 (a) For loose sand, $K_a = \frac{1-\sin\phi'}{1+\sin\phi'}$ [active pressure coefficient]

$\sigma_h' = K_a \sigma_v'$ $\quad\quad \sigma_h = \sigma_h' + u$

where σ_h' = effective horizontal pressure

σ_h = total horizontal pressure

u = water pressure

$\phi' = 30°$ $\therefore$ $K_a = \frac{1-0.5}{1+0.5} = 0.33$

For stiff clay for temporary excavation assume undrained behaviour

Total active horizontal pressure $\sigma_h = \sigma_v - 2c_u$

where σ_v = total vertical pressure

c_u = undrained shear strength

- At the ground surface (level A):

  $\sigma_v = \sigma_v' = 20$ kN/m^2

  $\sigma_h = \sigma_h' = K_a\sigma_v' = 0.33 \times 20 = \underline{6.7 \text{ kN/m}^2}$

- At the water table (level B):

  $\sigma_v = \sigma_v' = 20 + 17 \times 1 = 37$ kN/m^2

  $\sigma_h = \sigma_h' = K_a\sigma_v' = 0.33 \times 37 = \underline{12.2 \text{ kN/m}^2}$

- At the bottom of sand (level C):

  $\sigma_v = 37 + 8 \times 19 = 189$ kN/m^2

  Pore pressure $u = 8 \times 10 = 80$ kN/m^2

  $\sigma_v' = \sigma_v - u = 189 - 80 = 109$ kN/m^2

  $\sigma_h' = K_a\sigma_v' = 0.33 \times 109 = 36.0$ kN/m^2

  $\sigma_h = \sigma_h' + u = 36 + 80 = \underline{116 \text{ kN/m}^2}$

- At the top of the clay (level C), behind wall

  $\sigma_v = 189$ kN/m^2 (as above)

  $\sigma_h = \sigma_v - 2c_u = 189 - 2 \times 75 = \underline{39 \text{ kN/m}^2}$

  (active pressure)

- At the toe of the wall (level D), behind wall:

  $\sigma_v = 189 + (20 \times 3) = 249$ kN/m^2

  $\sigma_h = \sigma_v - 2c_u = 249 - (2 \times 75) = \underline{99 \text{ kN/m}^2}$

  (active pressure)

- In front of wall (level C):

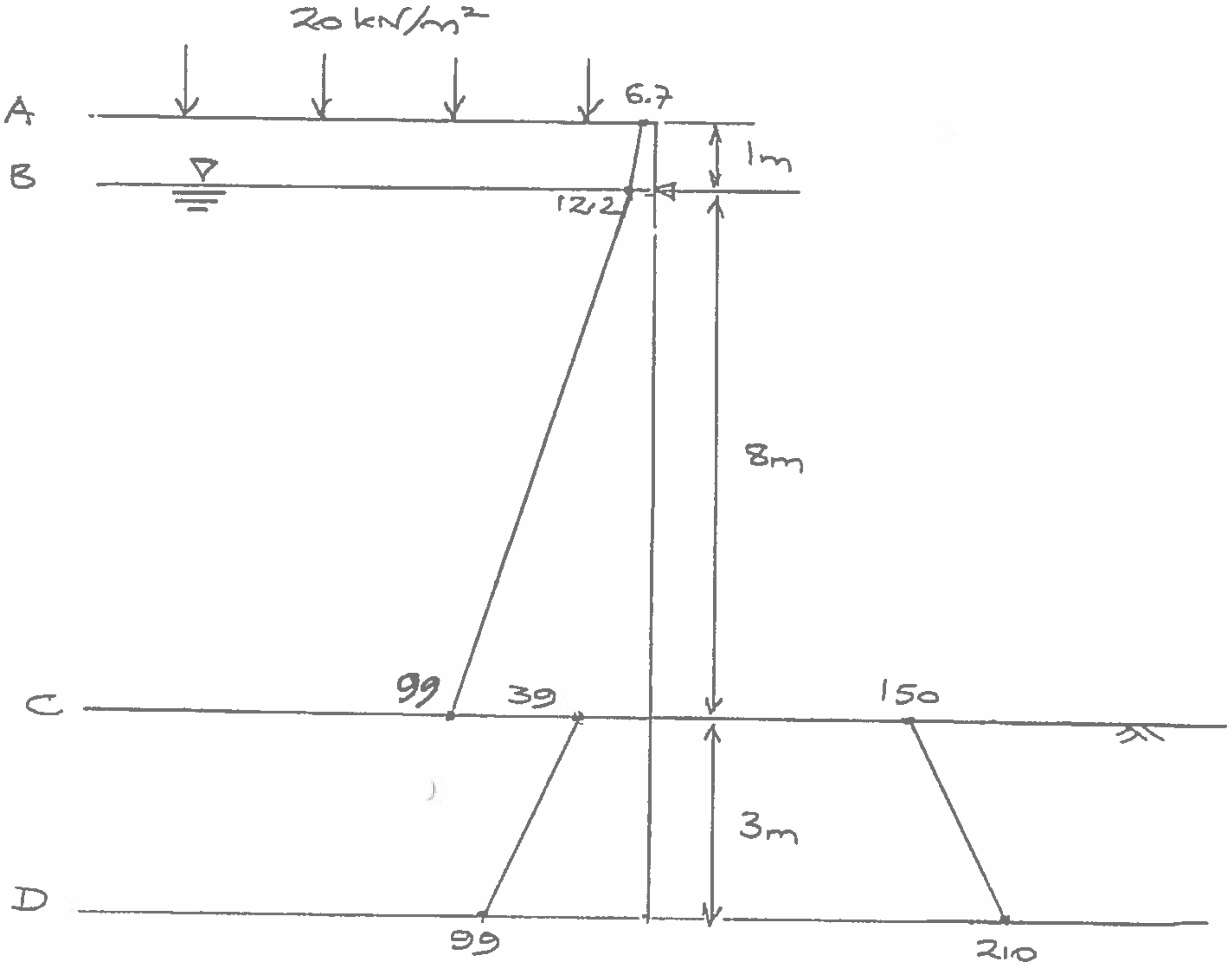
  at surface of excavation, $\sigma_v = 0$

  passive pressure $\sigma_h = \sigma_v + 2c_u = 2 \times 75 = \underline{150 \text{ kN/m}^2}$

- In front of wall at toe (level D):

  $\sigma_v = 3 \times 20 = 60$ kN/m^2

  passive pressure $\sigma_h = \sigma_v + 2c_u = 60 + 150 = \underline{210 \text{ kN/m}}$


20 kN/m²
A
B
C
D
6.7
12.2
99
39
150
99
210
1m
8m
3m

4(b) $B.M = 2000$ kNm/m

$$0.15\ f_{cu}\ \underset{\substack{\uparrow \\ 1000}}{b}\ d^2 = 2000.10^6$$

$\therefore d = 666$ mm

So overall thickness is 750 mm.

(c) As required with $d = 670$ mm

$$\therefore 2000.10^6 = 0.87 . 460 . \frac{3}{4} . 670 . A_s$$

$$\Rightarrow A_s = 9945 \text{ mm}^2/\text{m}$$

This requires 40mm bars at 125 crs
from data sheet.

Add comment that these might well be arranged in 2 layers to aid concrete flow

Solutions for Question 5

5. (a) Low permeability soils are *clays*, high permeability soils are *sands and gravels*, with silts in between. When tunnelling in clays, the permeability is low enough for there to be *no time for drainage* (assuming tunnelling is not stopped) and therefore the *undrained strength* is often high enough to ensure temporary stability of the tunnel face, even if completely unsupported. Whether or not the tunnel is below the water table in such cases is immaterial. But, if tunnelling in sands and gravels below the water table, the *water will flow into the face*, causing collapse and de-stabilising the tunnel.

Potential problems in tunnelling below the water table in sands and gravels can be overcome by (a) lowering the water table by *pumping from wells* installed for the purpose (b) *injecting grout* into the ground in advance of tunnelling – usually cement or chemical grouts – to reduce the permeability, or (c) *closed face* tunnelling machines, either *slurry* machines or *earth pressure balance* machines.

(b) Buildings made of masonry are susceptible to *differential* settlement and cracking is associated with *tensile strain*. Those subjected to *hogging* deformation are more susceptible than those subject to *sagging* - the tensile strains are induced at the top of the building whereas in the sagging zone they are in the foundations. The settlement trough induced at or near the ground surface by tunnelling is *Gaussian* in shape: a building directly above the tunnel may only be in a sagging mode, whereas to one side it is more likely to be in a hogging mode.

(c) *Compensation grouting* comprises injection of cement grout into the ground between the tunnel and the building foundation in a *highly controlled* manner. The grout is injected from *tube-à-manchettes* (TAM's) which are installed in the ground *prior to tunnelling*, usually from an adjacent shaft (or possibly from another tunnel). *Monitoring instrumentation* is installed on the building (electrolevels and/or levelling points) and also in the ground (extensometers) to measure ground movements. The grout is injected *in response to the measurements*. The primary aim is to reduce the potential *differential* settlement of the building, thereby limiting tensile strains and damage.

(d) (i) *Segmental linings*. These are often used for lining circular tunnels, constructed with tunnelling machines. The segments are usually made out of *pre-cast concrete*, but sometimes from *SGI (Spheroidal Graphite Iron)*. Advantages: constructed in factories under carefully controlled conditions, relatively easy to handle, erected within tunnelling machine, *robust*, very rare for collapse to occur. Disadvantages: usually only used for circular tunnels, therefore *lack of flexibility on shape*, difficult to vary thickness.

(ii) *Sprayed concrete linings (SCL)* - sometimes also known as *NATM (New Austrian Tunnelling Method)*. Concrete sprayed onto excavated soil surface, accelerators added, *hardens rapidly*, usually with *steel fibres added*. Advantages: very *versatile*, can easily change thickness, *excavated shape*. Disadvantages: needs careful quality control, *susceptible to poor workmanship*, collapse of tunnels has occurred where workmanship was poor, leading to errors (eg at Heathrow in 1994).

(e) A tunnel's stability in clay is determined by its *stability ratio*, N, defined as

$$N = (\sigma_v - \sigma_t)/c_u$$

where σ_v = total vertical pressure at tunnel axis level
σ_v = tunnel support pressure (if any, = 0 if open face)
c_u = undrained strength of the clay at tunnel axis level

If the value of N is *less than about 5* the tunnel face will be stable.

In London, c_u=*200 kPa* at 20m depth, so that *N=2* for an open face tunnel (assuming unit weight of 20 kN/m^3). In Bangkok or Singapore marine clay, c_u= *50kPa* at 20m depth so that *N=8*.

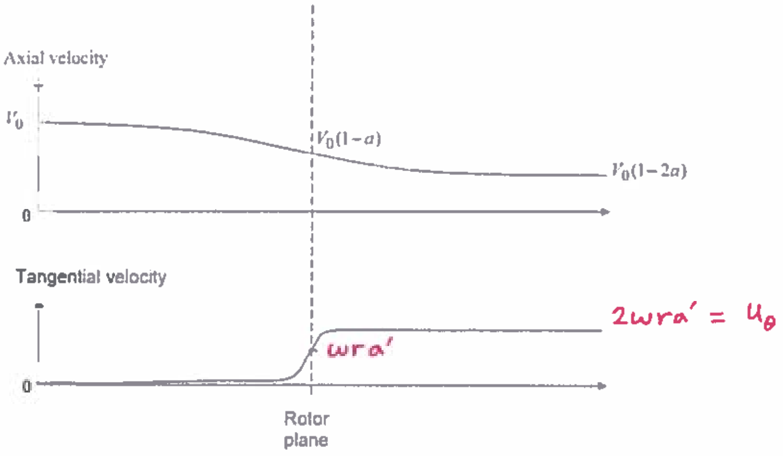
1B P8 Selected Topics, Section C

CRIB - 2017

Jonathan Cullen/Tim Flack/Michael Sutcliffe

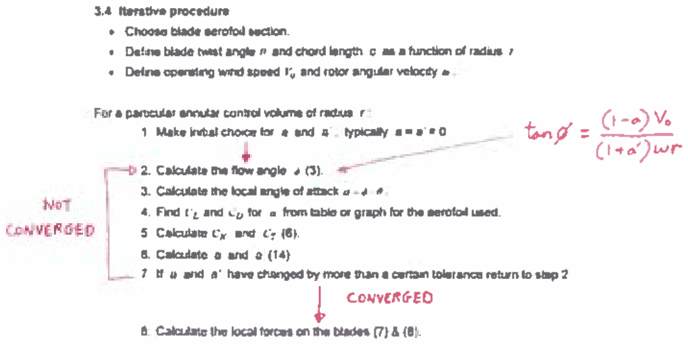
Question 6

a) From the notes:



b) BEM requires an iterative approach because the flow angle ϕ depends on the induction factors a and a' and vice versa.

From the notes



3.4 Iterative procedure

- Choose blade aerofoil section.
- Define blade twist angle θ and chord length c as a function of radius r
- Define operating wind speed V_0 and rotor angular velocity ω.

For a particular annular control volume of radius r:

1. Make initial choice for a and a', typically $a = a' = 0$
2. Calculate the flow angle ϕ (3).
3. Calculate the local angle of attack $\alpha = \phi - \theta$.
4. Find C_L and C_D for α from table or graph for the aerofoil used.
5. Calculate C_N and C_T (6).
6. Calculate a and a' (14)
7. If a and a' have changed by more than a certain tolerance return to step 2

8. Calculate the local forces on the blades (7) & (8).

NOT CONVERGED

CONVERGED

$\tan\phi = \frac{(1-a)V_0}{(1+a')\omega r}$

c),i) We are given

$B = 6$, $\theta = 18°$, r =2m to 8m, c =1.5m, $C_L \approx 2\pi\alpha$, $C_D \approx 0.01$,
V_0= 8 ms^{-1}, ω = 30 rpm = π rad s^{-1}, a = 0.114, and a' = 0.025.

Solidity $\sigma = \frac{Bc}{2\pi r} = \frac{3x1.5}{2\pi x5m} = 0.143$.

The BEM iterative procedure must be modified as we now know α so the flow angle ϕ can be calculated directly.

Blade stall typical occurs when the angle of attack α is greater than 15°, and $\alpha = \phi - \theta$, hence define a new flow angle ϕ =15 + 18 = 33°.

When $\alpha = 15°$ the new lift coefficient C_L = 1.64 (and C_D = 0.01.)

The normal force coefficient $C_N = C_L \cos\phi + C_D \sin\phi = 1.385$, while the tangential force coefficient $C_T = C_L \sin\phi - C_D \cos\phi = 0.888$.

New values of a = 0.143 and a'= 0.075 are found using:

$$a = \left[\frac{4\sin^2\phi}{\sigma C_N} + 1\right]^{-1}, \qquad a' = \left[\frac{4\sin\phi\cos\phi}{\sigma C_T} - 1\right]^{-1}$$

Repeating the procedure does not change these values, i.e. iteration is not required. Rearranging

$$\tan\phi = \frac{(1-a)V_0}{(1+a')\omega r}$$

we find the new incident velocity windspeed at stall is

$$V_0 = \frac{\tan\phi(1+a')\omega r}{(1-a)} = 12.8 \text{ ms}^{-1}$$

c),ii) To calculate the flap-wise and edge-wise bending moments we use the new incident wind speed V_0 under stall conditions, to calculate a new relative velocity seen by the blade V_{rel}.

$$V_{rel} = \frac{V_0(1-a)}{\sin\phi} = 20.1 \text{ ms}^{-1}$$

The normal and tangential forces are

$$F_N = \frac{1}{2}\rho V_{rel}^2 c C_N = 516 \text{ Nm}^{-1}, \; F_T = \frac{1}{2}\rho V_{rel}^2 c C_T = 330 \text{ Nm}^{-1}$$

Hence the flap-wise (M_N) and edge-wise (M_T) bending moments at the blade root are

$$M_N = \int_{4.5}^{5.5} F_N(r)dr = 516 \times 5 \times 1 = 2,580 \text{ Nm}$$

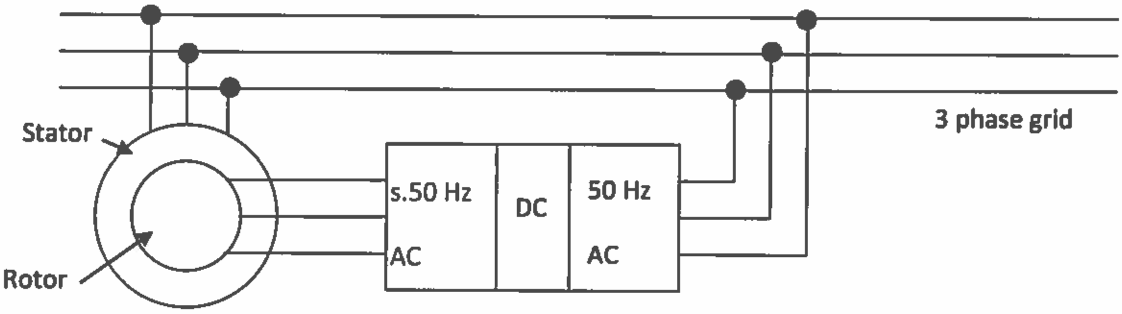
$$M_T = \int_{4.5}^{5.5} F_N(r)dr = 330 \times 5 \times 1 = 1,650 \text{ Nm}$$

This question on Wind Turbine Blade Design was a popular question. There was a wide range of marks for the question, with many students not completing all the sections probably due to time restrictions. For part (a), the answer required came straight from the notes or previous Tripos questions, and yet many students failed to replicate these simple graphs. In part (b) the students were required to describe the standard BEM iterative procedure, which was again straight from the notes. However, only a few students remembered to include the starting assumptions (choosing the blade aerofoil, twist angle, chord length, radius, wind speed, rotor velocity) and some forgot key equations. The key for part (c,i) was to realise that the BEM procedure needed to be modified as α *was given, and the flow angle* ϕ *could be calculated directly. This meant iteration of the BEM was not required. Students who worked this out scored highly in the question, as the remaining mathematics was simple. In part (c,ii) was answered well by students who had interpreted part (c,i), however only a few students remembered to convert the windspeed velocity* V_0 *into a new relative velocity* V_{rel} *before calculating* F_N *and* F_T*.*

7 (a) DFIGs allow variable speed operation of the wind turbine, which in turn optimizes power extraction by ensuring that the turbine always operates at its optimum tip-speed ratio.

They only require a fractionally-rated power electronic converter to achieve this, as opposed to other technologies which have to process the entire output electrical power.

They enable the induction generator to generate/absorb reactive power, as required by the power system, by varying the angle of the voltage supplied to the rotor via its slip-rings.



Schematic of the DFIG

The bi-directional converter supplies slip-frequency voltages to the rotor winding via the slip-rings. By varying the frequency, amplitude and phase of these voltages the no-load slip of the DFIG may be varied over a substantial range, enabling the generator to operate over a wide range of speeds. The principle of this is similar to speed control using rotor resistance control, but with the advantage that the power involved is returned to the grid rather than being dissipated.

(b) (i) With the slip-rings short-circuited the DFIG behaves as a standard induction machine i.e. it will operate with a slip close to zero and so at an angular speed close to synchronous speed ω_s.

The most probable wind speed is 6 ms^{-1}, the tip-speed ratio is 7 and the blade radius is 40 m so from:

$\lambda = \frac{\omega R}{v}$ the turbine angular speed will be 7×6/40 = 1.05 $rads^{-1}$

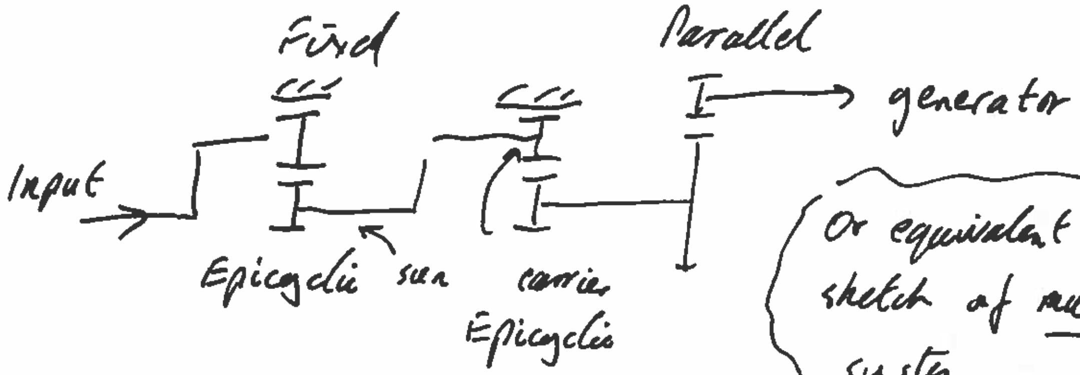
Thus the speed at the DFIG will be 30×1.05 = 31.5 $rads^{-1}$ and this will be very close to its synchronous speed $\omega_s = \omega/p = 2\pi \times 50/p$ giving p = 10 and so the DFIG will have 20 poles.

(ii) From $P = \frac{1}{2} C_p \rho A v^3$ and substituting in $C_p = 0.38$, $\rho = 1.23$ kgm^{-3}, $A = \pi R^2 = \pi \times 40^2 = 5030$ m^2 and $v = 10$ ms^{-1} gives P = 1.18 MW. Assuming no losses in the power train then the DFIG will need to have a power rating of 1.18 MW.

(iii) At rated wind speed the generator speed will be approximately 10/6 times its speed at the 6 ms^{-1} wind speed i.e. 52.5 $rads^{-1}$ giving a slip of (31.5 – 52.5)/31.5 = -2/3.

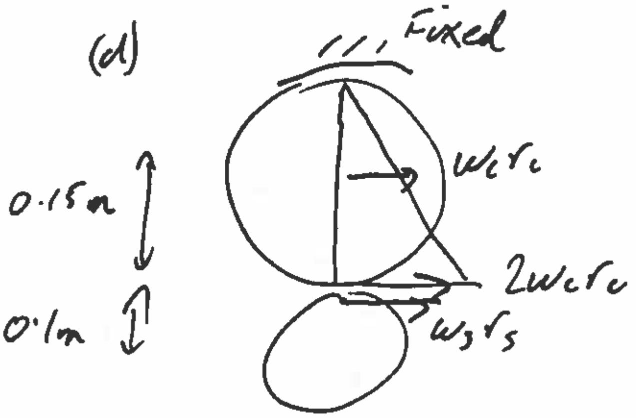
Many excellent attempts to part (a), most students understood why the DFIG is the generator of choice in large wind turbines and could explain its principles. For part (b), many students were able to find the pole number and power rating required for the generator, but the slip calculation, despite being very straightforward, eluded most candidates.

7(c) Epicyclics are used to give the required speed step-up as more compact (aerodynamics), lighter (vibrations) and cheaper.



Or equivalent sketch of multistage system

(d)



$2\omega_c r_c = \omega_s r_s$ ← critical equation often missed

$\Rightarrow \frac{\omega_c}{\omega_s} = \frac{r_s}{2r_c} = \frac{0.1}{2\times0.25} = \frac{1}{5}$

$\omega_s = 5\omega_c$

Input speed $\omega_c = 30 \text{ rpm} = \frac{30}{60}\times 2\pi = \pi$ rad/s

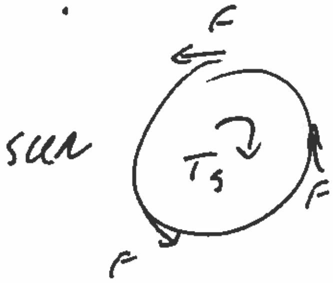
Output speed $\omega_s = 5\omega_c = 5\pi$ rad/s $= 15.71$ rad/s

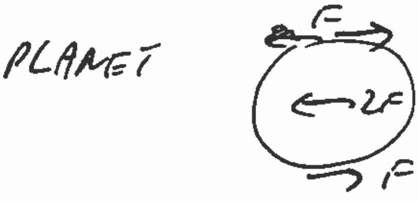
Torque out (neglect losses) $= \frac{\text{Power}}{\text{speed}} = \frac{200\times10^3}{5\pi} = \frac{40}{\pi} = 12.73$ kNm

At sun/planet $F = \frac{T_s}{3r_s} = \frac{40}{3\times0.1\pi} = \frac{400}{3\pi} = 42.4$ kN

3 contacts sometimes forgotten

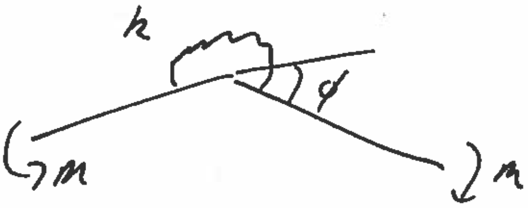
Forces on all tooth gear contacts are the same $= 42.4$ kN





[Mostly well executed, as long as the speed equation was correctly derived.]

8(a)

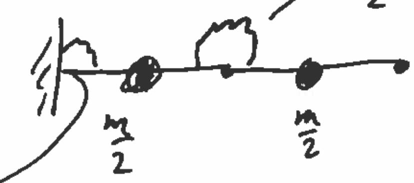


$$\frac{d^2y}{dx^2} = \frac{d\phi}{dx} = \frac{\phi}{L} = \frac{M}{EI} \qquad M = k\phi$$

Combining: $\frac{\phi}{L} = \frac{k\phi}{EI} \Rightarrow k = EI/L$

Care needed to define angles

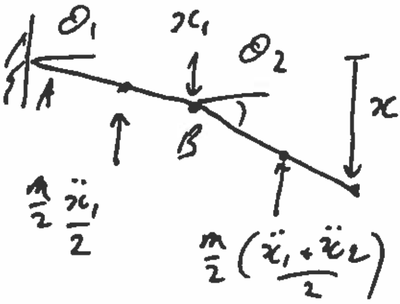
(b)



$k_2 = \frac{EI}{L/2} = \frac{2EI}{L}$

$k_1 = \frac{EI}{L/4} = \frac{4EI}{L}$

Other choices OK but need to specify masses and identify k of relevant springs



Again can set up equations in alternate ways but need to define symbols carefully

$$M_A \circlearrowright \quad k_1\theta_1 + \frac{m}{2}\frac{\ddot{x}_1}{2}\frac{L}{4} + \frac{m}{4}(\ddot{x}_1+\ddot{x}_2)\frac{3L}{4} = 0$$

$$M_B \circlearrowright \quad k_2(\theta_2-\theta_1) + \frac{m}{4}(\ddot{x}_1+\ddot{x}_2)\frac{L}{4} = 0$$

Also $\theta_1 = \frac{2x_1}{L}$, $\theta_2 = \frac{2}{\cancel{4}L}(x_2-x_1) \Rightarrow \theta_2-\theta_1 = \frac{2x_2}{L} - \frac{4x_1}{L}$

8(b) cont Assembling equations:

$$\cancel{\frac{16x}{L}}\ \frac{16}{L}\frac{2k_1x_1}{L}+\frac{m\cancel{L}}{\cancel{16}}\ddot{x}_1+\frac{3}{\cancel{16}}m\cancel{L}\ddot{x}_1+\frac{3}{\cancel{16}}m\cancel{L}\ddot{x}_2=0 \qquad \left(\times\frac{16}{\cancel{m}L}\right)$$

$$\frac{16}{L}\cdot 2k_2\frac{x_2}{L}-4k_2\frac{x_1}{L}\cdot\frac{16}{L}+\frac{m\cancel{L}}{\cancel{16}}\ddot{x}_1+\frac{m\cancel{L}}{\cancel{16}}\ddot{x}_2=0 \qquad \left(\times\frac{16}{L}\right)$$

$$k_1=2k_2$$

$$m\begin{pmatrix}4&3\\1&1\end{pmatrix}\begin{pmatrix}\ddot{x}_1\\\ddot{x}_2\end{pmatrix}+\frac{32k_2}{L^2}\begin{pmatrix}2&0\\-2&1\end{pmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}=0$$

$$[m^{-1}][k][x]=\omega^2[x] \text{ where } [m]^{-1}=\frac{1}{m}\begin{pmatrix}1&-3\\-1&4\end{pmatrix}$$

$$[m^{-1}][k]=\frac{32k_2}{mL^2}\begin{pmatrix}8&-3\\-10&4\end{pmatrix}$$

Eigenvalues: $|m^{-1}k-\lambda I|=0=\begin{vmatrix}8-\alpha&-3\\-10&4-\alpha\end{vmatrix}=0$ where $\alpha=\frac{mL^2\lambda}{32k_2}$

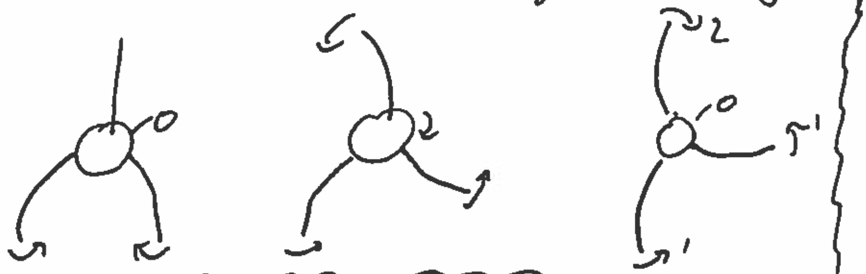
$$\alpha^2-12\alpha+2=0 \Rightarrow \alpha=11.83,\ 0.169$$

Lowest frequency $\omega=\sqrt{0.169\times\frac{32}{mL^2}\times\frac{2EI}{L}}=3.29\sqrt{\frac{EI}{mL^3}}$

(analytical value = 3.52)

(c) - Discretise blade with varying m and EI distribution
- include hub mass
- analyse each blade and hub
- solve for eigenvalues / eigenvectors

. OR could measure experimentally



Need to include hub in mode shape

A less popular question
Part (b) was all about method marks for all the critical steps

⑦

ROB MILLER

⑨ ⓐ $$P_{01} = \overset{0.26\text{ bar}}{P_{AMB}}\left(1+\frac{\gamma-1}{2}\overset{0.8}{M}^2\right)^{\frac{\gamma}{\gamma-1}} = \underline{\underline{0.396\text{ bar}}}$$

$$T_{01} = T_{AMB}\left(1+\frac{\gamma-1}{2}M^2\right) = \underline{\underline{250.4K}}$$

$$\boxed{3}$$

ⓑ $$\frac{T_{02}}{T_{01}} = \left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} = 20^{\frac{0.4}{1.4}} = 2.35$$

$$T_{02S} = 2.35 \times 250.4 = \underline{\underline{588.4K}} \quad \boxed{2}$$

$$R_C = \frac{W_{IDEAL}}{W_{REAL}} = \frac{T_{02S}-T_{01}}{T_{02}-T_{01}}$$

$$T_{02} = \frac{T_{02S}-T_{01}}{R_C} + T_{01}$$

$$= \frac{588.4-250.4}{0.9} + 250.4$$

$$= \underline{\underline{626.0K}} \quad \boxed{2}$$

ⓒ $$\frac{\Delta h_{0\,STAGE}}{U_m^2} = 0.4$$

$$\Delta h_0 = C_p(T_{02}-T_{01}) = 1.005(626-250.4$$

$$= \underline{\underline{377.5\text{kJ/kg}}}$$

$$\Delta h_{0\,STAGE} = \frac{377.5}{10} = 37.75\text{ kJ/kg} \quad \boxed{3}$$

$$U_m^2 = \frac{\Delta h_{0\,STAGE}}{0.4} = \frac{37748}{0.4}$$

$$U_m = 307.2$$

$$\boxed{3}$$

$U_m = r_m\Omega$ $\quad r_m =$ [illegible] $\dfrac{307.2}{6000\times 2\pi/60} = \underline{\underline{0.489m}}$

⑨ ⓓ

$$\frac{\dot{m}\sqrt{C_p T_{01}}}{A P_{01}} = \frac{\gamma}{\sqrt{\gamma-1}} M \left(1+\frac{\gamma-1}{2}M^2\right)^{-\left(\frac{\gamma+1}{2(\gamma-1)}\right)}$$

$$M = 0.8$$

$$\frac{\dot{m}\sqrt{C_p T_{01}}}{A P_{01}} = 1.2338$$

$$A = \frac{50\times\sqrt{1005\times 250.6}}{1.2338\times 0.396\times 10^5} = 0.513\,M^2 \quad \boxed{3}$$

$$A = \pi\left((r_m+\Delta R)^2 - (r_m+\Delta R)^2\right)$$

$$= \pi\left[\left(\cancel{r_m^2} + 2r_m\Delta R + \cancel{\Delta R^2}\right) - \left(\cancel{r_m^2} - 2r_m\Delta R + \cancel{\Delta R}\right.\right.$$

$$A = \pi\, 4 r_m \Delta R$$

$$\Delta R = \frac{0.513}{\pi\times 4\times r_m} = \frac{0.513}{\pi\times 4\times 0.489} = 0.0835$$

$$\text{BLADE HEIGHT} = 2\Delta R = \underline{\underline{0.167\,m}} \quad \boxed{3}$$

ⓔ

$$\frac{\Delta h_{0\,MID}}{U_m^2} = 0.4$$

$$U_H = r_H\,\Omega = \frac{r_H}{r_m}U_m = \frac{r_m-\Delta R}{r_m}U_M$$

$$\frac{\Delta h_{0\,HUB}}{U_H^2} = \frac{\Delta h_0}{U_m^2}\times\frac{U_m^2}{U_H^2} = 0.4\times\left(\frac{r_m}{r_m-\Delta R}\right)^2$$

$$= 0.582 \quad \boxed{3}$$

⑨ ⓔ MAX WORK COEFFICIENT SHOULD BE 0.5

THIS IS BECAUSE IN COMPRESSORS THE PRESSURE RISE PROVIDES AN ADVERSE PRESSURE GRADIENT FOR THE BLADE BOUNDARY LAYER. THUS BOUNDARY LAYERS BECOME THICKER AND SEPARATE $\boxed{3}$

10

ⓐ THE EXPRESSION $(F_G + P_a A_N)$ ONLY DEPENDS ON CONDITIONS INSIDE THE ENGINE

$$F_G = \dot{m}_A V_{19} + (P_{19} - P_a) A_N \qquad \boxed{2}$$

$$\text{So} \quad F_G + P_a A_N = \underbrace{\dot{m}_A V_{19} + P_{19} A_N}$$

FOR A CHOKED ENGINE NOZZLE THE RHS IS ONLY A FUNCTION OF ENGINE CONDITIONS

THE ENGINE CONDITION IS FIXED BY

$\dot{m}_F$, P_a, T_a, V_{19} FOR A CHOKED EXIT $\boxed{2}$

NOZZLE V_{19} ONLY DEPENDS ON THE ENGINES OPERATING POINT. FOR A FIXED P_a, T_a THE ENGINE OPERATION IS FIXED ONLY BY $\dot{m}_F$.

ONE NON-DIMENSIONAL PARAMETER THEREFORE FIXES THE ENGINE CONDITION. IN THIS CASE NON-DIMENSIONAL MASS FLOW HAS BEEN CHOSEN

$$\frac{\dot{m}\sqrt{C_p T_{02}}}{P_{02} A_N} \qquad \boxed{1}$$

ⓑ $$T_{02} = T_a\left(1 + \frac{\gamma - 1}{2} M^2\right) = 222\left(1 + 0.2 \times 0.8^2\right)$$

$$= 250.4\,K$$

$$P_{02} = P_a\left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} = 0.26\left(1 + 0.2 \times 0.8^2\right)^{\frac{1.4}{0.4}}$$

$$= [illegible] \; 0.396 \qquad \boxed{2}$$

$$\left(\frac{\dot{m}\sqrt{\cancel{C_p} T_{02}}}{P_{02} \cancel{A_N}}\right)_{TEST} = \left(\frac{\dot{m}\sqrt{\cancel{C_p} T_{02}}}{P_{02} \cancel{A_N}}\right)_{FLIGHT}$$

$$\frac{\dot{m}_{TEST}}{\dot{m}_{FLIGHT}} = \left(\frac{P_{02\,TEST}}{P_{02\,FLIGHT}}\right) \times \left(\sqrt{\frac{T_{02\,FLIGHT}}{T_{02\,TEST}}}\right)$$

$$= \frac{1.01}{0.396} \times \sqrt{\frac{250.4}{288}} = \underline{\underline{2.376}} \qquad \boxed{3}$$

10 (c)

THE GROSS THRUST IS THE THRUST ~~IS THE THRUST~~ WHICH WOULD BE PRODUCED UNDER THE SAME CONDITIONS WITH THE ENGINE STATIONARY. THE NET THRUST IS THE THRUST AVAILABLE TO THE AIRCRAFT INFLIGHT $\boxed{2}$

$$\left(\frac{F_G + p_a A_N}{P_{02} A_N}\right)_{TEST} = \left(\frac{F_G + P_a A_N}{P_{02} A_N}\right)_{FLIGHT}$$

$$F_{G\,FL} = \left[\frac{P_{02\,FL}}{P_{02\,TEST}} (F_G + P_a A_N)_{TEST}\right] - (P_a A_N)_{FC}$$

$$= \frac{0{\cdot}396 \times 10^5}{1{\cdot}01 \times 10^5} \times (30000 + 1{\cdot}01 \times 10^5 \times 0{\cdot}08)$$

$$- 0{\cdot}26 \times 10^5 \times 0{\cdot}08$$

$$F_{GFC} = \cancel{8448{\cdot}5N} \quad \underline{\underline{12{\cdot}85\,kN}} \qquad \boxed{3}$$

$$F_{N\,FC} = F_{G\,FC} - \dot{m}_{FC} V_{FC}$$

$$V_{FC} = M\sqrt{\gamma R T_A} = 0{\cdot}8\sqrt{1{\cdot}4 \times 287 \times 222}$$

$$= 238{\cdot}0\ ms^{-1} \qquad \boxed{2}$$

$$\dot{m}_{FC} = \left(\frac{\dot{m}_{FC}}{\dot{m}_{TEST}}\right) \times \dot{m}_{TEST}$$

$$\dot{m}_{TEST} = \frac{F_{G\,TEST}}{V_{J\,TEST}} = \frac{30000}{500}$$

$$= 60\ kg\,s^{-1}$$

$$m_{FC} = \frac{1}{238} \times 60 = 25{\cdot}2\ kg\,s^{-1} \qquad \boxed{2}$$

$$F_{N\,FC} = 12{\cdot}85kN - 25{\cdot}2kg\,s^{-1} \times 239 = \underline{\underline{6{\cdot}83\,kN}}$$

10d

$\dot{m}_f\,LCV$ IS A FLOW OF ENERGY
THIS IS NON-DIMENSIONALISED
BY A RATE OF DOING WORK
(FORCE × VELOCITY)

3

THE FORCE IS $P_{02}D^2$

THE VELOCITY IS THE SPEED OF SOUND BASED ON INLET CONDITIONS $\sqrt{\gamma R T_{02}}$ WHICH IS PROPORTIONAL TO $\sqrt{c_p T_{02}}$

1

$$\left(\frac{\dot{m}_f\,LCV}{\sqrt{c_p T_{02}}\,D^2 P_{02}}\right)_{TEST} = \left(\frac{\dot{m}_f\,LCV}{\sqrt{c_p T_{02}}\,D^2 P_{02}}\right)_{FLIGHT}$$

$$\frac{\dot{m}_{f\,TEST}}{\dot{m}_{f\,FLIGHT}} = \sqrt{\frac{T_{02\,TEST}}{T_{02\,FLIGHT}}} \times \frac{P_{02\,TEST}}{P_{02\,FLIGHT}}$$

$$= \frac{101}{34.63} \times \sqrt{\frac{288}{250.4}}$$

$$= \underline{\underline{2.73}}$$

2

(14) ⓐ THERE IS AN OPTIMAL C_L WHICH IS TYPICALLY 0.5

$$C_L = \frac{L}{0.5\times\rho\times A\times V_c^2} \qquad L = W_c$$

REARANGING

$$V_c = \sqrt{\frac{W_c}{0.5\times\rho\times C_L\times A}} \qquad \boxed{3}$$

THE AIRLINE WANTS TO OPERATE THE AIRCRAFT AT OPTIMAL M AND C_L ~0.85 AND ~0.5. ON A MEDIUM-HAUL ROUTE THE WEIGHT ↓W_c IS LOWER. THE AIRCRAFT THEREFORE NEEDS TO FLY AT LOWER ↓ρ. THE AIRCRAFT SHOULD THEREFORE FLY AT A HIGHER ALTITUDE↑

$\boxed{3}$

ⓑ $SFC = \frac{\dot{m}_f}{F_N}$

$$\frac{dw}{W} = -g\,\dot{m}_f = -g\,sfc\,F_N = -g\,sfc\,\overset{\text{DRAG}}{\overset{\downarrow}{D}}$$

$$= -g\,sfc\,\frac{L}{(L/D)} = -g\,\frac{sfc}{L/D}\times W \qquad \boxed{3}$$

ASSUMING CRUISE V CONSTANT AND LEVEL FLIGHT.

$$\frac{dw}{W} = -g\,\frac{sfc}{L/D}\times dt = -g\,\frac{sfc}{L/D}\times\frac{ds}{V}$$

ASSUMING L/D CONSTANT + OPTIMAL

$$\ln\left[\frac{W_{st}}{W_{end}}\right] \text{[illegible]} = +\,g\,\frac{sfc\,s}{V\,L/D} \text{[illegible]}$$

QED $\boxed{3}$

(11) ⓒ

$$SFC = \frac{\dot m_f}{F_N} = \frac{\dot m_f LCV}{V F_N} \times \frac{V}{LCV} = \frac{1}{R_{th} R_p} \times \frac{V}{LCV}$$

$\boxed{2}$

$$R_{th} = \frac{\Delta KE}{\dot m_f LCV}$$ ← FUEL HEAT RELEASE

$$R_p = \frac{V F_N}{\Delta KE}$$ ← POWER AIRCRAFT

← POWER TO JET

SUB INTO BREGUET'S RANGE EQUATION

$$s = \frac{\cancel{V}}{g} \frac{L/D}{\left(\frac{1}{R_{th} R_p} \times \frac{\cancel{V}}{LCV}\right)} \ln\left[\frac{W_{ST}}{W_{END}}\right]$$

$$s = \frac{R_{th} R_p}{g} \frac{L}{D} LCV \ln\left[\frac{W_{ST}}{W_{END}}\right]$$

$S_{OLD} = S_{NEW}$ AIRCRAFT FLIES SAME ROUTE $\boxed{2}$

$$\frac{\cancel{[R_{th} R_p]_{OLD}}}{\cancel{g}} \cancel{L/D} \, \cancel{LCV} \ln\left[\frac{W_{ST}}{W_{END}}\right]_{OLD} = \frac{\cancel{[R_{th} R_p]_{OLD}}}{\cancel{g}} \times 1{\cdot}1 \times 1{\cdot}05 \times \cancel{L/D} \times \cancel{LCV} \times \ln\left[\frac{W_{ST}}{W_{END}}\right]_{NEW}$$

$$\ln\left[\frac{W_{ST}}{W_{END}}\right]_{NEW} = \frac{1}{1{\cdot}1 \times 1{\cdot}05} \ln\left[\frac{W_{ST}}{W_{END}}\right]_{OLD}$$

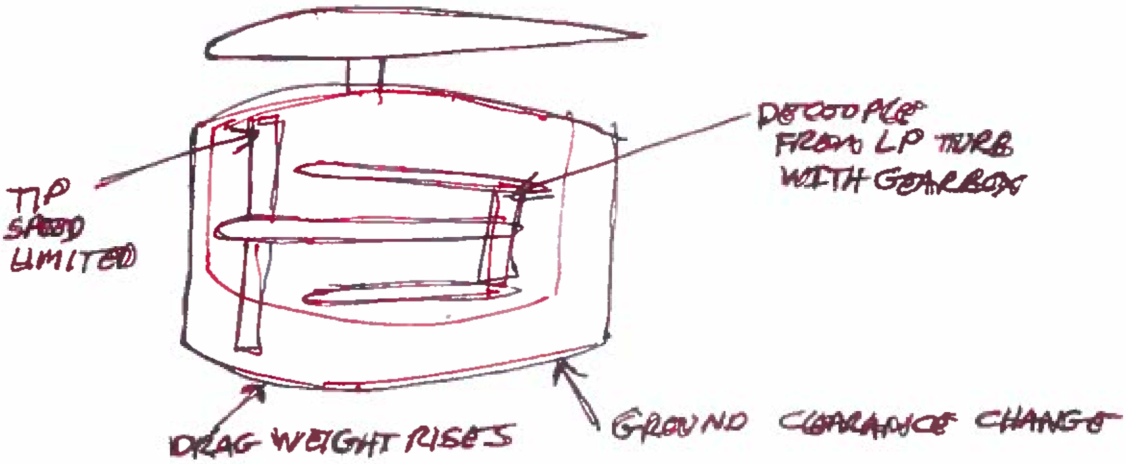
$$\left[\frac{W_{ST}}{W_{END}}\right]_{NEW} = e^{\frac{\ln\left[\frac{W_{ST}}{W_{END}}\right]_{OLD}}{1{\cdot}1 \times 1{\cdot}05}} = \underline{\underline{1{\cdot}42}}$$ $\boxed{3}$

(11) (a)

$\boxed{2}$

RAISE SIZE, THIS WILL RAISE PROPULSIVE EFFICIENCY $\boxed{2}$ THEORETICAL MAX 100% BUT REQUIRES INFINITE SIZE. NACELLE WEIGHT + DRAG RISES.

TIP SPEED OF FAN RISES AND LP TURBINE $\boxed{2}$ SPEED LIMITED SO NEED GEARBOX.



12 (a) The key element for a smartphone has always been the display. Indeed the display in the *iPhone 7* is one of the most superior in terms of contrast ratio and screen reflectance for liquid crystal based technology. If the display screen of the *iPhone 7* has a diagonal 4.7 inches with an aspect ratio 16:9, what is the display size in pixels if the resolution is 326 pixels per inch. [2 marks]

➔ **Answer:**

Since all pixels are the same and square-sized, we can approximate the screen size in terms of number of pixels as follows.

$y^2 + x^2 = 4.7^2$ where $y = (16/9)x$. This yields $x = 2.3$.

Hence display size = (16/9)*2.3 by 2.3 = (4.09 by 2.3) inches

➔ 326 (4.09 by 2.3) = 1334 x 750 pixels **(2 marks)**

(b) If the display screen yields a maximum contrast ratio of 1762 with a dark black luminance of 2.5 cd/m^2, what is the peak white luminance? Assume screen reflectance is negligible. [5 marks]

➔ **Answer:**

Maximum contrast ratio = (peak white luminance)/(darkest black luminance) = 1762

Thus peak white luminance = 1762x2.5 cd/m^2 = 4405 cd/m^2 **(5 marks)**

(c) Name three basic thin film transistor (TFT) technologies that are of current interest in display screens and tabulate their relative performance in terms of carrier mobility and current applications. [8 marks]

➔ **Answer:**

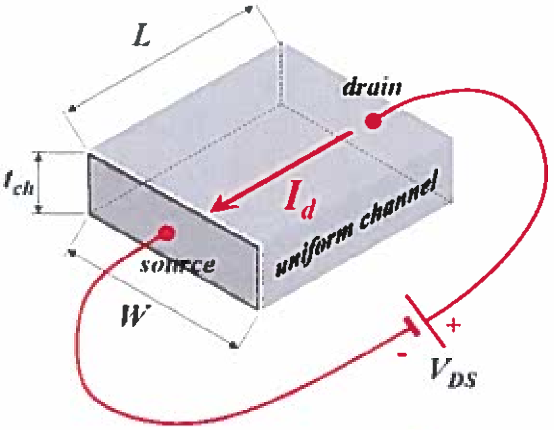
Attribute	**a-Si:H (hydrogenated amorphous silicon)**	**LTPS (low temperature poly-silicon)**	**AOS (amorphous oxide semiconductor)**
Mobility (cm^{2}V^{-1}s^{-1})	**< 1**	**30-100**	**<50**
Application	**Pads to TVs**	**Smart phones**	**OLED TVs**

***Top two-row answers are 1 mark each, and bottom row is 0.66 marks each (total: 8 marks).**

(d) A thin film transistor for eventual applications in OLED TVs is being fabricated using a new trial channel material, indium silicon oxide. Its electron mobility (μ) is 50 $cm^2V^{-1}s^{-1}$. The transistor will have channel width W = 10 μm, gate length L = 3.6 μm, and channel thickness (t_{ch}) = 20 nm. Assuming that the gate induces a uniform carrier density (N) of 5×10^{14} cm^{-3} in the channel, what is the drift current I_d for a source-drain voltage of 2 V. Note that elementary charge (e) is 1.6×10^{-19} couls.

[10 marks]

➔Answer:



Current equation based on drift process, i.e. using J = σE

$$I_d = W \cdot t_{ch} \cdot e \cdot N \cdot \mu \cdot \frac{V_{DS}}{L}$$

(5 marks)

Calculation of I_d

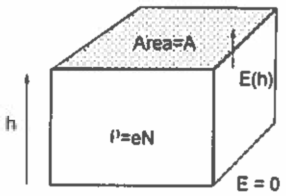
$$\therefore I_d = 10\mu m \cdot (20\times10^{-7}cm)\cdot(1.6\times10^{-19}coulomb)\cdot\left(5\times10^{14}cm^{-3}\right)\cdot 50cm^2V^{-1}s^{-1}\cdot\frac{2V}{3.6\mu m}$$

$$= 50\times10^{-9}\frac{coulomb}{s} = 50\times10^{-9}A = 44nA$$

(5 marks)

13 a) Explain how Gauss' law can be used to obtain the electric field created by a cube of uniform electronic charge. Be clear with your arguments about the direction of the field.

[5 marks]



➔ **Answer:**

By symmetry, for the gate stack geometry, E field cannot come out of sidewalls, can only come out of top or bottom. But E cannot come out of bottom either, as there are mobile charges there, so E(h=0) = 0.

Hence, Q = eN h.A, DA = $\varepsilon_0\ \varepsilon_r$ E(h) . A by Gauss, and $E = \dfrac{eN}{\varepsilon_0 \varepsilon_r} h$ **(5 marks)**

b) Hence derive the expression for the gate threshold voltage to turn off a depletion mode n-type FET with a channel doping density of N electrons /m^3, channel thickness h, and with a relative dielectric constant of ε_r.

[6 marks]

➔ **Answer:**

$$V = -\int E\,.dh = -\int \frac{eN}{\varepsilon_0 \varepsilon_r} h.dh = -0.5 \frac{eN}{\varepsilon_0 \varepsilon_r} h^2$$

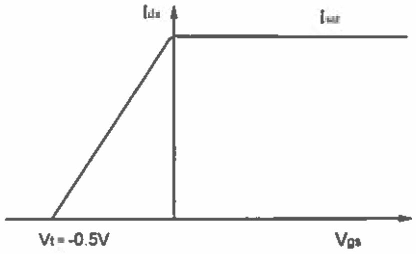
(6 marks)

c) From (b), calculate the gate threshold voltage V_t for this case where h = 50 nm, N= 2.5x10^{23} m^{-3} and $\varepsilon_0\varepsilon_r$ = 10^{-10} F/m. Sketch the I_{ds} vs. V_{gs} characteristic for this case.

[6 marks]

➔ **Answer:**

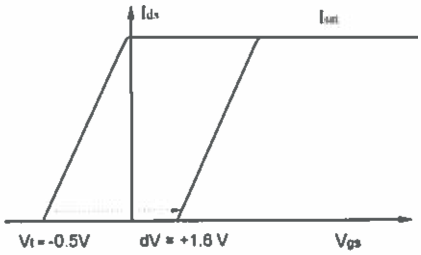
Leaving positive charge behind, V_{t1} = –½ x 1.6x10^{-19} x 2.5.10^{23} (/ 10^{-10}).(50x10^{-9})^2 = – 0.5 V



(6 marks)

d) A floating gate of Si ($\varepsilon_0\varepsilon_r = 10^{-10}$ F/m) in a Flash memory-like device has thickness t = 30 nm and a very thin, second gate oxide of SiO_2 is added on top of the floating gate as sketched in Fig. 5. The floating gate is charged up to charge density of $2.5\text{x}10^{24}$ m^{-3}. Calculate the new value of the gate threshold voltage in this case of a Flash memory-like device.

[8 marks]



➔ Answer:
new voltage = +½ x $1.6\text{x}10^{-19}$ x $2.5\text{x}10^{24}$ ($/10^{-10}$) x$(30\text{x}10^{-9})^2$ = 1.8 V
so nett V_{t2} = +1.8 V – 0.5 V = 1.3 V

(8 marks)

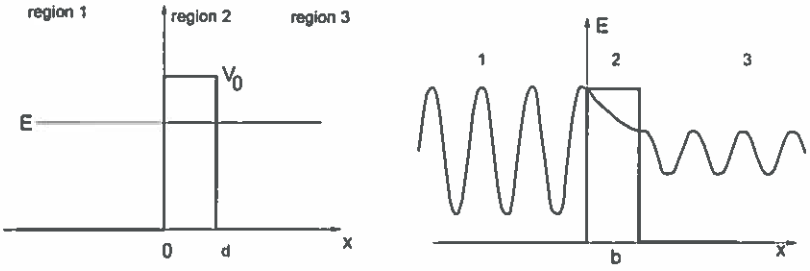
14 a) Derive an expression for the wavelength of an electron in terms of its kinetic energy T and effective mass m*. [6 marks]

$T = h^2k^2/(2m)$, so $k = (2mT)^{1/2}/h$ or wavelength $\lambda = 2\pi/k = 2\pi h/(2mT)^{1/2}$ (6 marks)

b) Explain what is meant by quantum mechanical tunnelling, for example by sketching the wavefunction of an electron travelling from left to right through a tunnel barrier of height V_0 and thickness d; the potential outside the barrier is V=0.

[6 marks]

Tunnelling is where kinetic energy is negative because total energy E is less than the barrier height V_0.



(6 marks)

c) Derive an approximate expression for the tunnelling probability P in terms of the parameters E, m*, V_0 and d.

[5 marks]

$k = [2m(E-V_0)]^{1/2}/ h = -[2m(V_0-E)]^{1/2}/ h$.

Decay of wavefunction after barrier length d is $P_0 = \exp(-kd)$,

So tunnelling probability is $P = |P_0|^2 = \exp(-2kd)$ (5 marks)

d) Calculate P for the case of a 1.5 nm thick SiO_2 layer assuming that the effective electron mass m* is half of the free mass, E = 0.5 eV, V = 3.5 eV, Planck's constant = 6.626×10^{-34} kg.m²/s, and the free electron mass is $m = 0.9 \times 10^{-30}$ kg.

[8 marks]

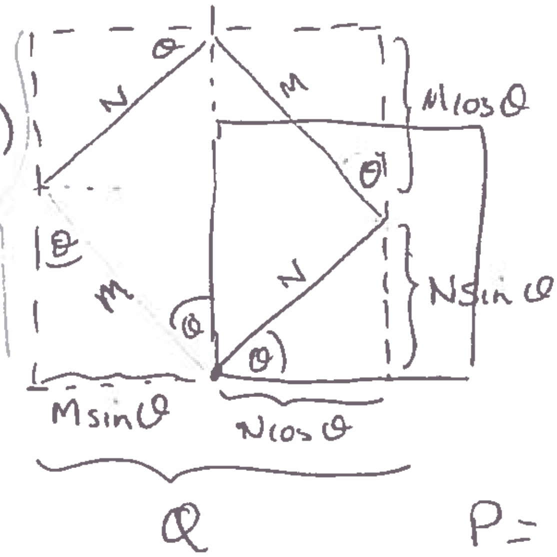
$2kd = 2x(2x\ 0.5\ x\ 0.9x10^{-30}\ x\ 3x1.6x10^{-19})^{1/2}\ (/6.63x10^{-34})\ x\ 1.5x10^{-9} = 3$

so $P = \exp(-3) = 0.05$ (8 marks)

Paper 8 – Section F (2017) – 10 pages Q15
Q16
Q17

⑮ a)

i)



$$P = M\cos\theta + N\sin\theta$$

$$Q = M\sin\theta + N\cos\theta$$

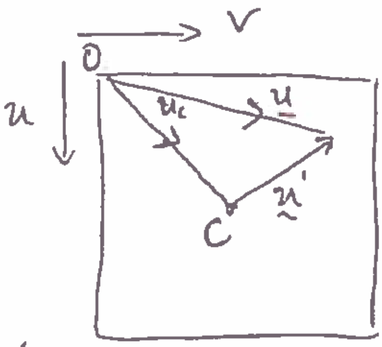
– strictly speaking, we need

$$P = M|\cos\theta| + N|\sin\theta|$$

$$Q = M|\sin\theta| + N|\cos\theta| \qquad [3]$$

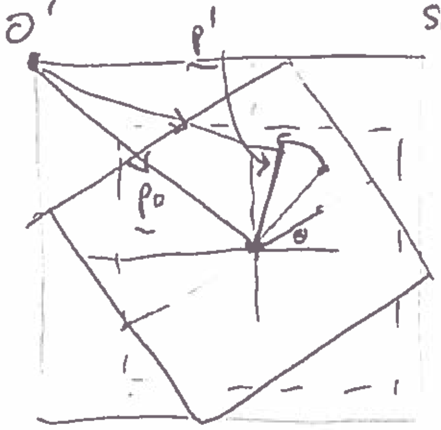
ii) $R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ [2]

iii) Suppose image origin is at top LH corner – let vector to centre be $\underline{u}_c = \begin{bmatrix} u_0 \\ v_0 \end{bmatrix}$



Any point in the image with vector $\underline{u}$ relative to O becomes $\underline{u}' = \underline{u} - \underline{u}_c$ relative to C.

We rotate this vector.



so: $\underline{p}' = R\underline{u}'$

The new coordinates in the rotated system will ∴ be

$$\underline{p} = \underline{p}_0 + \underline{p}' \qquad [3]$$

where $\underline{p}_0$ is the vector from top LHC to centre (see i) for coordinates)

5 cont.

iv) The vector p will not necessarily have integer coordinate values. The last step is therefore to interpolate onto the grid points — would generally use bi-linear interpolation, for speed.

[3]

(b) i) $$I_c(x,y) = \iint_{\substack{\text{all} \\ \text{image}}} I(x-u, y-v)\, g(u,v)\, du\, dv$$

– ie we convolve the image with the function g. We could put a guard-band of zeros around the image to deal with convolving at the edges.

[3]

ii) If $g(u,v) = g_r(x)\, g_c(y)$ we have

first: $I_1(x,y) = \int I(x, y-v)\, g_c(v)\, dv$ filter ~~with~~ on the columns

second $I_c(x,y) = \int I_1(x-u, v)\, g_r(u)\, du$ filter on the rows.

ie. 2 1D filters

[2]

iii) Full 2D ~ N×N

2×1D ~ 2N

} (more detail in proper crib)

[3]

iv) $H(\omega_1, \omega_2) = \alpha - \beta\, G(\omega_1, \omega_2)$

If $X(\underline{\omega})$ is the spectrum of our image, $G(\omega)X(\underline{\omega})$ is the lowpass filtering process. If we subtract this from X; is $X(\underline{\omega}) - G(\underline{\omega})\,X(\underline{\omega})$ we will obtain a high pass filter. The constants α and β allow us to tune the properties of the HP filter.

Gain of H → $(\alpha - \beta)$ at low freqs

H → α " high freqs

⇒ should choose α, β appropriately, eg $\alpha = 2$, $\beta = 1$

[3]

v). Illustrating the effects in 1D we have

If $\alpha - \beta = 1$ (unit gain at low freqs)

Step

(more detail in final ans)

If $(\alpha - \beta) = 0$ (zero gain at low freqs)

[3]

Q16

(a) Low pass filter — remove high frequency noise (amplified by d
— scale selection.

(i) large σ, low-pass filter w_c is reduced
small σ, low-pass filter w_c is increased

[3]

(ii) $S(x, y, \sigma_i) = G_{\sigma_i}(x, y,) * I(x, y)$.

Choise – $\sigma_i = \sigma_0 2^{\frac{i}{s}}$ logarithmically spaced $\sigma_{i+1} = \sigma_i \, 2^{\frac{1}{s}}$

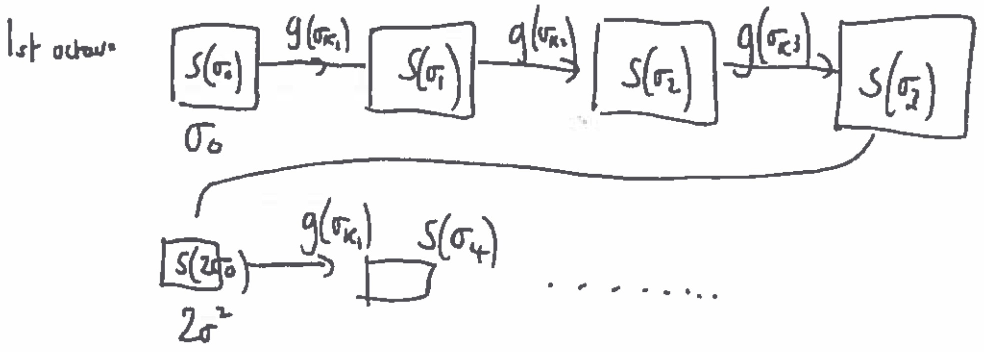
– s image per octave

sub-sample when σ is double ie $\sigma_i = 2^{\frac{1}{s}} \sigma_0$ etc
(ie after s images) to $\frac{1}{4}$ image size

– Incremental blur (reproductive property of gaussian convolution)
(same kernels re-used in each octave.)

$$\sigma_{i+i} = \sigma_i * \sigma_k$$

where $\sigma_k = \sigma_i \sqrt{2^{\frac{2}{s}} - 1}$.

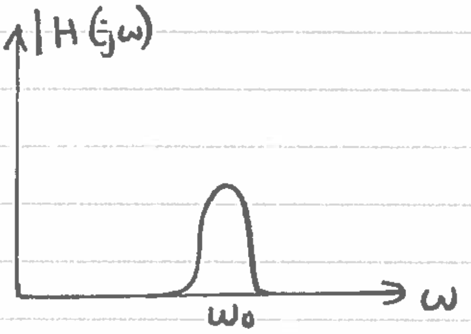


eg s = 3

[4]

Q 1ba (iii)

Band-pass filter — only small band of frequencies around ω_0 not attenuat
In 1D



$$\nabla^2 S(x,y,\sigma_i) = \nabla^2 G_{\sigma_i}(x,y) * I(x,y)$$

$$\simeq \left[G(x,y,k\sigma_i) - G(x,y,\sigma_i)\right] * I(x,y)$$

(ie. difference of gaussian blurred images where k= 1.2-1.6

$$\simeq \underline{S(\sigma_{i+1}) - S(\sigma_i)}$$

Note: Gaussians are low-pass filters. Difference is a band-pass filter since F.T. of gaussian is a gaussian in freq domain

Alternatively: In 1D

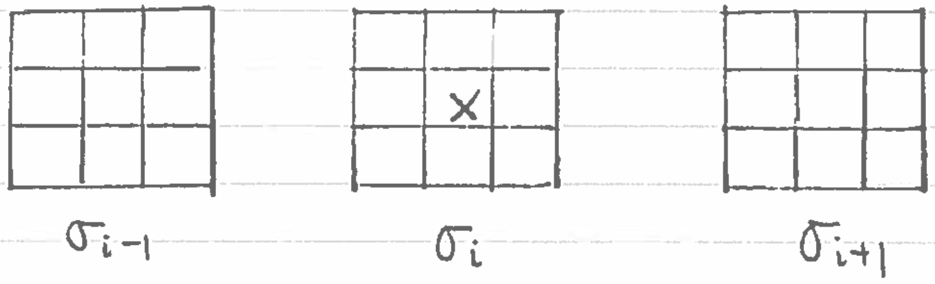
$$\underset{\text{gaussian}}{G(x)} \xrightarrow{FT} \underset{\text{gaussian}}{G'(\omega)} \quad \text{(this is low-pass)}$$

$$\frac{d^2G}{dx^2} \xrightarrow{FT} -\omega^2 G'(\omega) \quad \text{(this is band-pass)}$$

(4)

1b(a)(iv). $\nabla^2 S(x,y,\sigma_i) \simeq S(\sigma_{i+1}) - S(\sigma_i)$

Look for max\min in $\nabla^2 S$ responce for blob-centre
Scale is corresponding σ_i



Find local max/min by checking 26 neighbours (8 at σ_i, 9 at σ_{i-1} and 9 at σ_{i+1})

Position of blob is (x,y) and scale (σ_i) determined by the pixel with max\min of $\nabla^2 S$.

[3]

~~8~~
8

(b). Consider 2 patches of pixels 16x16

16 (i) $T_1(x,y)$ and $T_2(x,y)$

$$\therefore \text{cross-correlation} = \sum_{i=0}^{15}\sum_{j=0}^{15} T_1(i,j)\, T_2(i,j)$$

or over an Image $I(x,y)$

$$\text{Cross-correlation} = \sum_{i=0}^{15}\sum_{j=0}^{U} T(i,j)\ I(x+i,\ y+j)$$

[4]

(ii). Normalize by subtracting mean and dividing by $\sqrt{\text{variance}}$ for each pixel in templates.

$$N.CC = \frac{\sum_i\sum_j (T_1-\mu_1)(T_2-\mu_2)}{\sigma_1\ \sigma_2} = \frac{\text{covarc}(T_1,T_2)}{\sqrt{\text{Variance}(T_1)\ \text{var}(T_2)}}$$

$$= \frac{\sum\sum\left(T_1(i,j)-\mu_1\right)\left(T_2(i,j)-\mu_2\right)}{\sqrt{\sum\sum(T_1(i,j)-\mu_1)^2\sum\sum(T_2-\mu_2)^2}}$$

Unable to cope with misalignment and orientation changes and scale. [4]

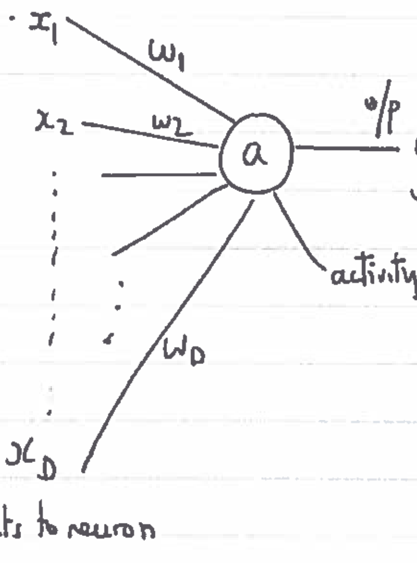
(iii) HOGs, SIFT use gradients of intensity. [3]

~~4~~

9

Q.17(a).

(i).



$y = f(a)$ where $f(\ )$ is typically $= \frac{1}{1+ \exp(-}$ logistic function

$\frac{1}{1+\exp-\{a\}}$

activity of neuron $a(t) = \sum_{D} w_i x_i$

[6]

(ii) Supervised learning: training data $\underline{x}$ and y pairs
data label

Look at error between neuron output and desired label and optimize (minimize) by estimating update to weights $\underline{w}$

[illegible]-Cost $(\underline{w})$ or loss function $= \sum_{n \in N} \underline{w}^T \underline{x}_n t_n$ where $t_n \in (-1, +1)$

or cross-entropy.

Use gradient of cost $\nabla C(\underline{w})$ to update $\underline{w}$

$$\underline{w}^{t+1} = \underline{w}^{t} + \eta \underline{x}_n \underline{t}_n$$

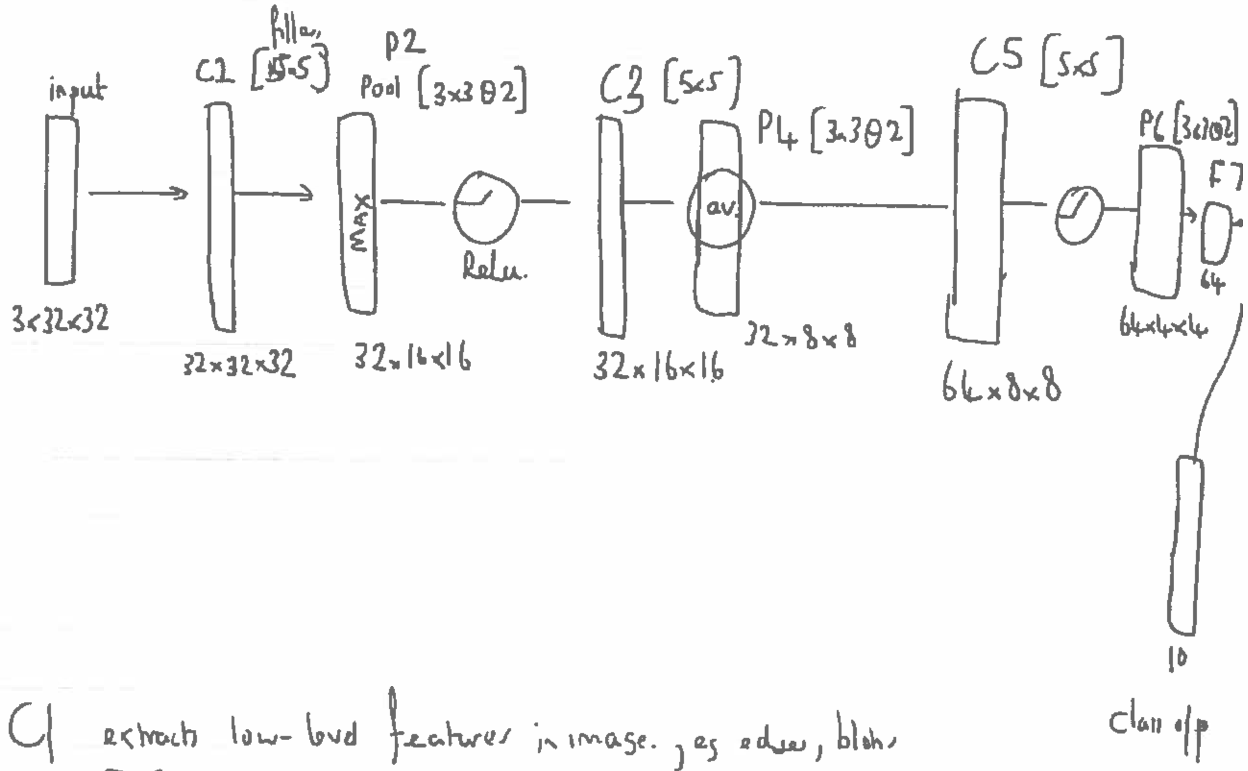
[6]

(iii) hyperplane in feature-space. Need data to be linearly separable. [3]

~~8~~

10

Q17 (b) Look at a typical CNN eg.



C1 extracts low-level features in image, eg edges, blobs
5×5 convolution

P2 Pooling - low-level flexibility to exact location

C3 Combination of low-level features

P4 Smooths response + subsamples feature map

C5 Parts + structure.

P6 Smooths + sub sample feature map (vector of length 1024)

F7 Sub-category classifier

F8 Final classifier (10 classes)

[10]

Version GMT/1

SECTION G

Bioengineering

19 (a) Describe the path of the illuminating and reflecting light from the fundus, when using both the Fundus Camera and the Scanning Laser Ophthalmoscope. Explain the reasons for each approach, and contrast the resulting optical properties. [6]

Answer:

Fig. 1

Illumination in the Fundus camera is shown top left: in this case the illumination is via an annulus and the reflections are observed via a small disc in the centre of this annulus. In the case of the SLO (top right), the illumination is via a small laser spot at the centre, and the reflections are observed everywhere except at the point of illumination by the laser.

There are two important issues: firstly, the illumination and reflection must not be coincident, since otherwise reflections will be dominated by those at the lens of the eye rather than at the fundus, which is actually what we want to visualise. Secondly, the optical efficiency is controlled by what percent of the illuminated light is seen in the reflected light. Both instruments solve the first of these issues, but the SLO is much more efficient since nearly all of the reflected light which passes back out of the lens is seen. This means the instrument can be used with a lower incident light power for the same image noise level.

(b) In Optical Coherence Tomography (OCT), the fundus is illuminated with a laser pulse E with n periods of a frequency ω_0:

$$E = \begin{cases} e^{j\omega_0 t} & -\frac{n\pi}{\omega_0} < t < \frac{n\pi}{\omega_0} \\ 0 & \text{otherwise} \end{cases}$$

(i) What is the bandwidth b of this pulse, defined as the width of the main lobe of the frequency response, and how does this relate to the spatial depth resolution of OCT? [7]

Answer: In order to answer this, the Fourier Transform of the pulse E must be calculated:

$$\mathscr{F}(E) = \int_{-\frac{n\pi}{\omega_0}}^{\frac{n\pi}{\omega_0}} e^{j\omega_0 t} e^{-j\omega t}\,\mathrm{d}t$$

Combining the exponentials, integrating and using Euler's formula gives:

$$\begin{aligned}
\mathscr{F}(E) &= \int_{-\frac{n\pi}{\omega_0}}^{\frac{n\pi}{\omega_0}} e^{jt(\omega_0-\omega)}\,\mathrm{d}t \\
&= \left[\frac{e^{jt(\omega_0-\omega)}}{j(\omega_0-\omega)}\right]_{-\frac{n\pi}{\omega_0}}^{\frac{n\pi}{\omega_0}} \\
&= \frac{2}{(\omega_0-\omega)}\sin\left(\frac{n\pi(\omega_0-\omega)}{\omega_0}\right) \\
&= \frac{2n\pi}{\omega_0}\mathrm{sinc}\left(\frac{n\pi(\omega_0-\omega)}{\omega_0}\right)
\end{aligned}$$

The main lobe extends from $-\pi$ to π, hence:

$$\begin{aligned}
\frac{n\pi(\omega_0-\omega)}{\omega_0} &= \pm\pi \\
\omega &= \omega_0 \pm \frac{\omega_0}{n}
\end{aligned}$$

So the bandwidth is:

$$b = \frac{2\omega_0}{n}$$

The spatial depth resolution of the pulse is just the length, which is $\frac{2n\pi}{\omega_0}c$, where c is the speed in the medium. Hence b is inversely proportional to the spatial resolution.

(ii) Suggest approximate values for n and ω_0 typically used in OCT. [2]

Answer: For OCT, the wavelength is typically 500 nm to 1300 nm, and the speed is roughly $2 \times 10^8 \mathrm{ms}^{-1}$, so ω_0 would typically be over 1×10^{16}. n would typically be of the order of 100.

(c) The images in Fig. 1 below show two types of visualisation from OCT data.

(i) For each image, explain what type of visualisation is being used, what it can display, and describe one feature of the eye which can be seen in this visualisation. [6]

Answer: The left hand image shows a *reslice*, which is a cross-sectional image

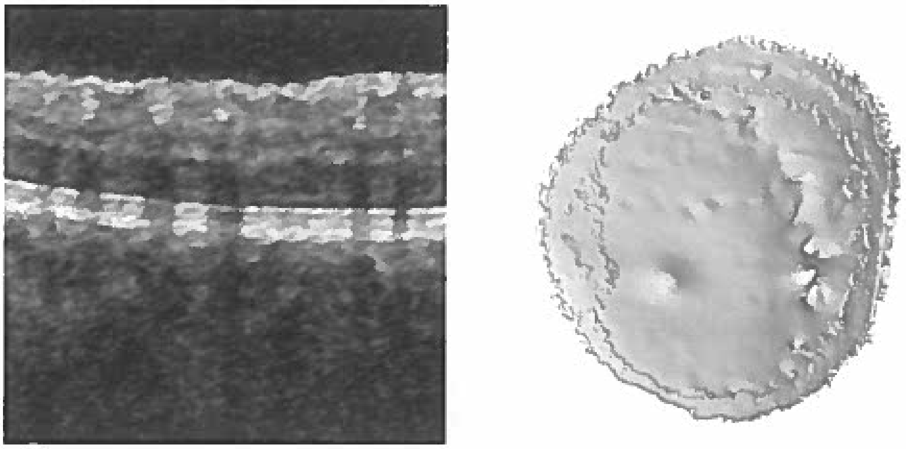


Fig. 2

through the 3D data, with the brightness representing the reflectivity of the tissue. This can be used to show the geometric distribution of the tissue within a particular slice. The right hand image shows a *surface rendering*. This is a 3D representation of the boundary between two different reflectivities in the 3D data, and shows the 3D geometry of that particular boundary.

The left hand image reveals various tissue layers in the fundus: the retinal layers, the retinal pigment epithelium (RPE) and at least the top of the choroidal layer can be seen, as well as possibly some blood vessels. The right hand image shows the surface of the retina: the macula surrounding the fovea can be seen on the left, and the top of the optic disc on the right.

(ii) For each image, describe one feature which is an artefact of the visualisation process, and briefly explain how this artefact arises. [4]

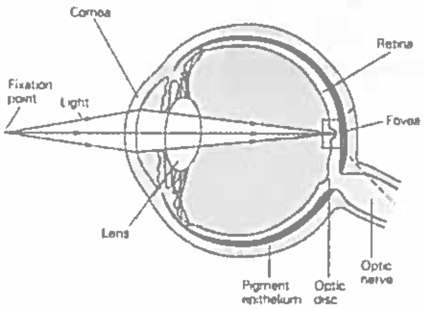
Answer: Some darker vertical stripes can be seen in the reslice image which are actually 'shadows' from areas of higher reflectivity above them. Equally the 'texture' of the reslice image is the result of coherent scattering rather than a real feature.

The edges of the surface in the surface rendering are not really surface edges: these are just the regions of data where the reflectivity has dropped below the threshold used to pick out the surface.

19 (a) Sketch a section of the human eye in the vertical plane that divides it into left and right halves, and annotate it by placing the following labels: cornea, lens, retina,

fovea, optic nerve. [4]

Answer: In more detail than expected:



(b) Explain the mechanism underlying the physiological blind spot in vertebrates, and give an example of an animal species that does not have such a blind spot. [4]

Answer: The physiological blind spot is a region of visual field that the eye cannot sense. It emerges from the relative spatial arrangement of retinal nerve fibers and photoreceptors in vertebrates. The axons of the retinal ganglion cells, which bundle up together to form the optic nerve, must leave the eye one way or another. In vertebrates, retinal ganglion cells are found closer to the light source (closer to the surface of the eye) and photoreceptors are at the back. Thus, there must be a hole in the sheet of photoreceptors to make space for the optic nerve to leave the eye through the back and reach the brain. This hole underlies the physiological blind spot. The octopus does not have a blind spot, because nerve fibers route *behind* the retina.

(c) Write short notes on

(i) simple cells in the primary visual cortex

(ii) cortical minicolumns

(iii) the transmission of visual information from photoreceptors to optic nerve fibers

[7]

Answer:

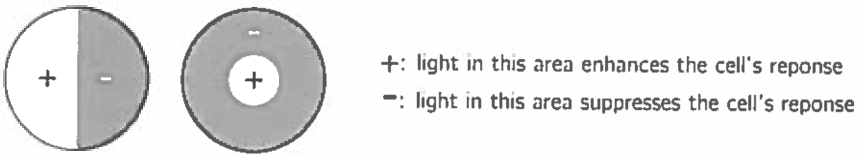
(i) Simple cells respond preferentially to small oriented stimuli (bars or gratings) in a particular location of the visual field, with a particular orientation and a particular spatial frequency. Their receptive fields have distinct excitatory and inhibitory regions that approximately balance out under uniform illumination. Simple cells differ from complex cells in that their responses to the superposition of multiple illumination patterns (e.g. bright spots in different locations within the receptive field) can be well predicted by the linear summation of responses obtained for each stimulus individually.

(ii) A cortical minicolumn refers to a block of neural tissue spanning all 6 cortical layers under a $\sim 40\mu\text{m} \times 40\mu\text{m}$ section of cortical surface. In primary visual cortex, such a block typically contains a few hundred neurons, each with similar response properties (e.g. similar orientation preference).

(iii) Photoreceptors transduce light (using special proteins): the more photons they absorb, the more hyperpolarised they become (unusual in sensory systems!). Hyperpolarisation leads to a drop in intracellular $[\mathrm{Ca}^{2+}]$ (closing of voltage-gated calcium channels), which in turn inhibits the release of glutamate released into ganglion cells (GCs) at synaptic terminals. This results in the depolarisation of some GCs, and hyperpolarisation of some others, which alters their rate of action potential firing. GCs also influence each other's activity through synaptic interconnections. It is the action potentials fired by GCs that are eventually sent down the optic nerve.

(d) Give two examples of convolution-based algorithms for the detection of luminance edges in an image. Sketch the associated spatial filters, and compare the advantages of the two algorithms. Which is closest to the processing of visual inputs performed by the retina? [5]

Answer: Luminance edges could be extracted by convolving the image by either of the two filters sketched below. The first one approximates the first derivative of the luminance along one direction (orthogonal to the orientation of the edge), and so will only detect edges that have the same orientation as the filter. Convolution with a whole bank of such filters with diverse orientations will need to be performed to extract all edges. Alternatively, edges can be detected as the zero crossings of the second derivative of the luminance. Curvature can be approximated by the radially symmetric filter shown below on the right. Edges of any orientation can now be detected simultaneously. This filter is similar to the receptive fields of retinal ganglion cells.



(e) What is an “orientation map” in the primary visual cortex? Describe an experiment that could be performed to reveal such maps. Include details of recording techniques and visual stimuli. [5]

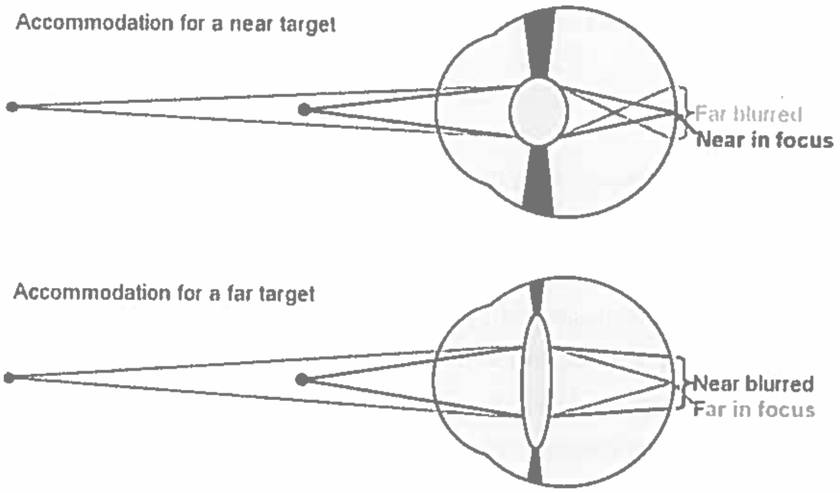
Answer: Most neurons of the visual cortex are tuned for specifically oriented stimuli, in specific positions in the visual field. Both position and orientation preference vary smoothly along the cortical surface, such that neighbouring neurons respond to similar orientations in nearby positions of the visual field, forming an “orientation map” with “pinwheels” at the junction of minicolumns with preferences for all orientations. To observe such maps, one can use optical imaging (e.g. using a voltage-sensitive dye) to monitor the activity (in superficial layers) of a large area of cortex. By measuring responses (averaging across multiple trials) to small oriented stimuli presented in various locations in the visual field (covering the retinotopic span of the whole imaged area), one can extract orientation preference across the surface and plot the resulting orientation map.

20 (a) Describe the tissue structure of the crystalline lens in the eye [5]

Answer: The crystalline lens sits behind the iris and contributes 1/3 of the total focussing power of the eye (about 20 Diopters out of 60 total for the eye, with the other 40 coming at the air-cornea interface). The lens is about a cm across and half a cm thick. The transparent, biconvex lens structure changes shape to change focus. There is an exterior capsule that contains the lens, which is in two parts, the nucleus and the cortex. The nucleus is older lens fibers and the cortex is the newer lens fibers; the capsule is the source of new lens fiber cells. The "lens fibers" are specialised elongated epithelial cells surrounded by unusual proteins called crystallines (30% by mass). The overall structure of the lens is complex and "onion-like" in terms of being in layers. There are no blood vessels or nerves in the lens.

(b) Describe lens accommodation in young, normal healthy eyes. Explain how the accommodation reflex differs from accommodation. [6]

Answer: Lens accommodation is the process of lens shape change that allows the eye to adjust for focus on objects nearer or further away. It is responsible for 1/3 of the total focussing power of the eye. The lens is more spherical to accomplish close up focus and becomes stretched/extended to accomplish far focus. Lens curvature is controlled by ciliary muscles, and by changing curvature, one can quickly re-focus the young and healthy eye on objects at different distances.



The accommodation reflex is the combination of accommodation (i.e. lens shape change) along with convergence and constriction of the pupils. Near point of convergence (NPC) is measured by bringing an object to the nose and observing when the patient sees double, or one eye deviates out. Normal NPC values are up to 10 cm. While it is well understood that proper convergence is necessary to prevent double vision, the functional role of the pupillary constriction remains less clear. Arguably, it may increase the depth of field by reducing the aperture of the eye, and thus reduce the amount of accommodation needed to bring the image in focus on the retina.

(c) Lenses are typically characterised by the quantity diopters (D) measuring lens power.

(i) How does diopter relate to focal length of a lens?

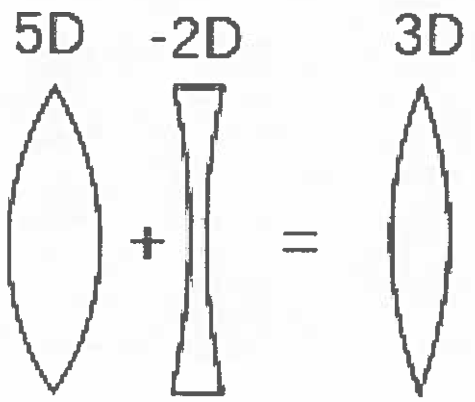
(ii) Draw the difference between +D and -D lenses.

(iii) A lens whose focal length is 50 cm is placed in contact with a second lens whose focal length is 10 cm. What is the power of the combination lens?

[6]

Answer: "Amplitude of accommodation" is the max amount that the lens can accommodate in diopters (D), equal to the reciprocal of the focal length measured in metres.

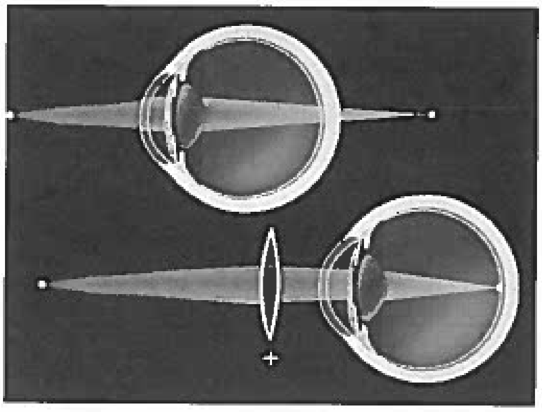
Examples of lens shapes from the lecture notes:



Diopters are extremely useful measures of lens power because they can be summed for compound lenses, so convert the two focal lengths to meters and then to diopters as 1/0.5 + 1/0.1 = 12 diopters

(d) Explain why reading glasses are increasingly common as the eyes age, and what type and strength of reading glasses are typically used to correct this. [8]

Answer: The lens continually grows throughout life, laying new cells over the old cells, which results in stiffening of the lens (by three orders of magnitude) as well as growth of the lens size (from about 90 mg to 250 mg in mass). It then becomes more difficult to nearly impossible for the lens to change shape under the action of the ciliary muscles, and thus the lens gradually loses accommodation ability with age, going from 10 diopters at age 20 to 0.25 diopters at age 70. This is called Presbyopia and it is part of the natural ageing process and happens to nearly everyone. The nucleus stiffens more than the cortex with ageing. The near point is the closest object that can be brought into focus naturally. This ranges from a few cm in children to several meters (longer than an arm's length!) in old age.



Convex lenses, typically in the range of powers of D = +1 to +3, are used to correct minor presbyopia and to allow for focus on things like reading materials held closer than the near point.

2016-2017 IB Paper 8, Section H – Post-exam Crib

21 (a) Explain, using examples to illustrate your answer, the advantages and disadvantages of each of the following types of business models:

(i) selling a product;

(ii) selling a product plus consumables; and

(iii) selling a product plus a service.

[6]

Sell a product: Pros = You can increase revenue with higher volumes; you control more of the value-adding activities yourself; your embed your knowledge within a product and thus have control over how your idea is developed in the future. Cons = Need to develop – or access - the capability to produce the product; potentially high levels of investment will be required; can be quite an inflexible business model; once you have sold the product you might have no further interaction with your customers (unless there is a problem); no opportunity for on-going revenue unless you sell customers another product.

Sell a product plus services: Pros = On-going revenues can be generated on the back of the sale of each product; you keep in contact with customers through the delivery of services – and this can help with the development of future products and services. Cons = You need to develop and manage the infrastructure to provide services – and this is a very different activity compared to developing, making and shipping a product.

Sell a product plus consumables: Pros = On-going revenues can be generated on the back of the sale of each product; you keep in contact with customers through the delivery of consumables; Con = You need to develop and manage the infrastructure to provide consumables.

Better answers *linked the points above to issues of:*

- ***Strategy*** *– What do the people setting up or running the business want to achieve? Do they want to make a quick return from an idea, do they want to run a small business, focused on local markets, or do they want to be the global leader in a particular market?*
- ***Technology*** *– What is the nature of the technology? Is it very novel and untested? Is it very complicated? Does it work on its own, or is it part of a larger system?*
- ***Market*** *– Who are the users of the product, and how buys the product (not always the same person)? How many potential buyers are there? Where are they?*
- ***Resources*** *– What do you need to make the business work? What do you have, and what do you need to get?*

(b) Compare, using examples to illustrate your answer, the characteristics of each of the following types of production system:

(i) project;

(ii) job shop;

(iii) batch production; and

(iv) mass production.

[9]

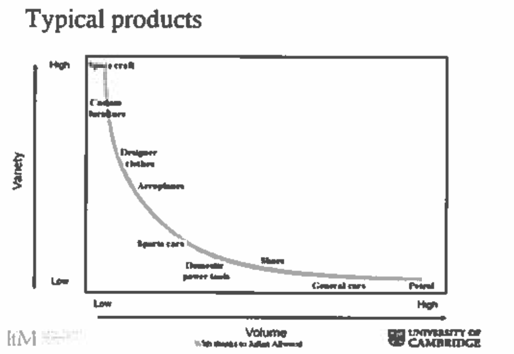
Project: Building or road construction; Shipbuilding; Open-heart surgery; High-class restaurant: Requires project management skills and coordination.

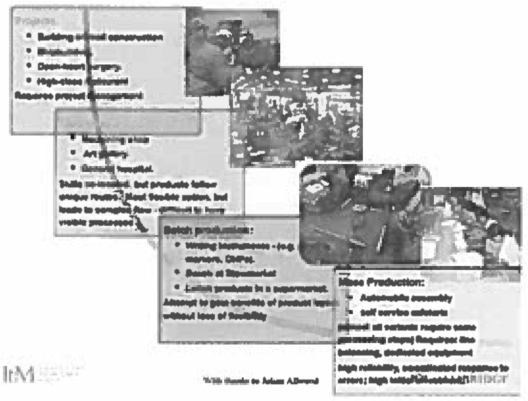
Machining shop: Art gallery; General hospital: Skills co-located, but products follow unique routes. Most flexible option, but leads to complex flow - difficult to have visible processes.

Batch production: Writing instruments - (e.g. pencils, pens); supercars; garden equipment; fresh prepared meals for supermarkets: Attempt to gain benefits of product layout without loss of flexibility.

Mass Production: car assembly; electronics, self service cafeteria; (almost all variants require same processing steps): Requires: line balancing, dedicated equipment high reliability, co-ordinated response to errors; high initial investment

Better answers *could describe these in terms of their relative position on the volume-variety curve:*


Typical products



(c) Discuss why there are conflicting objectives between supply and demand in a manufacturing firm, and describe strategies that can be used to manage these conflicting objectives.

[10]

First, you need to explain why the conflict exists: *The basic problem of supply and demand is: the process of producing the product takes time – both due to production and due to the need to acquire the materials and components required to make it. If the customer is not prepared to wait for this process to occur, a stock of finished goods must be held so that whenever a customer arrives, there is a complete product ready to be bought. However, this stock of finished goods is expensive – all the costs of production have been paid for, and extra costs are incurred due to the need to store the product. The ideal is clearly that a steady stream of customers should arrive at* *exactly the same rate as the factory produces the products – so all the factory's resources are continuously in use, but no stock is kept.*

The challenge is in designing the production system to reconcile two sets of conflicting objectives:

- *Customers would like to be able to buy any volume of a product without delay at any time*
- *Owners of the business would like to see all their resources fully and efficiently used all the time without having to pay any of the costs associated with holding stock*

Then, you can describe how these conflicts can be addressed: Clearly the resolution of this conflict is a complex. Firstly, demand (the time history of the rate at which customers buy products) is inherently variable and fluctuates, often in an unpredictable way. Therefore, in designing the production system, it is helpful to consider strategies to try to smooth the demand as much as possible.

Secondly, supply (the rate at which products can be made) is inherently smooth, and all production systems work best when tuned to work at a steady rate. The production system must therefore be designed to avoid wasted time, and to be as flexible as possible in changing the rate of output.

Possible strategies include:

Demand for air-cooling products is much greater in summer than winter, for instance. We know that ideally we would like to see a smooth pattern of demand at the average level, or at least we would like to be able to forecast demand. However, experience of forecasting is that it is extremely difficult, and it is difficult to be more accurate than saying that "tomorrow will be the same as today." Four general strategies have developed for trying to smooth the pattern of demand:

The simplest strategy is to design a system with capacity to supply the average demand, and use the quiet periods to build stock to meet the periods of peak demand.

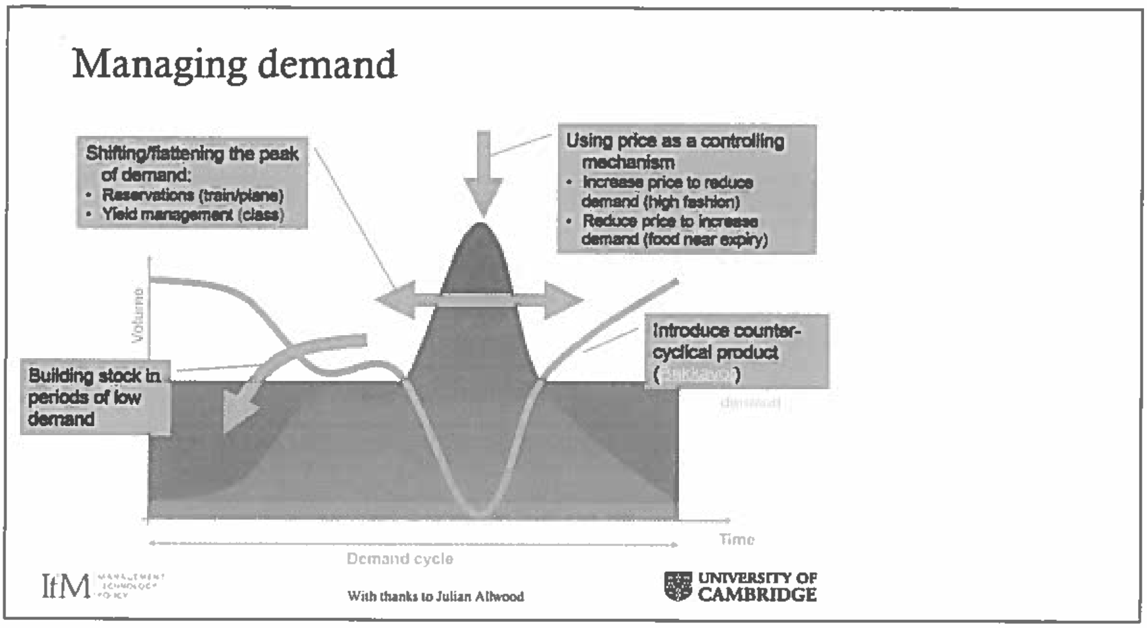
An attractive possibility is to find a second product with the inverse demand cycle so that the two combined have more level demand

Price can be used to reduce demand at the peak, and increase it at off-peak times - for instance by charging more for toys at christmas

Reservations can be used to allow advance booking of peak demand, so that later requests must be met at other times

A broader strategy made popular by the success of Dell Computers is to try to smooth demand by offering a range of products where the total demand is smoother than the demand for any individual item. If the difference between the products can be introduced late in the production system, the overall demand seen by most of the system will be smoother.

Better answers *put this in context with an illustration such as this:*



Managing demand

Shifting/flattening the peak of demand:
- Reservations (train/plane)
- Yield management (class)

Using price as a controlling mechanism
- Increase price to reduce demand (high fashion)
- Reduce price to increase demand (food near expiry)

Building stock in periods of low demand

Introduce counter-cyclical product ([illegible])

With thanks to Julian Allwood

22 A team of students has developed a robust, low-cost, digital medical heart monitor for use in disaster zones where normal medical facilities are not available. For this product, describe the processes by which the students could:

(a) assess the scale of the potential market;

*The basic answer needs to discuss: how to **define** a market, and how to **segment** it.*

*There are many ways in which you could divide, or segment the market place. This can be done via a **perceptual map**. How do customers, users and stakeholders perceive the product? There are four possible ways in which this can be considered.*

By the benefits that are delivered: what benefits do customers and users derive from the product? For a hedge trimmer, it might be a neat hedge, or perhaps versatility, or perhaps a great service deal.

By particular product attributes: perhaps the easiest way to segment a market is to compare product attributes. This however tends to say little about the customers and is often the weakest approach. For lawn mowers, this could include product performance, width of cut etc.

By characteristics of the consumer: this can be split to include demographics, and psychographics. Demographics relates to aspects such as social class, age, house size, sex etc. Psychographics relates to the user's attitudes and beliefs, what they feel, their lifestyle etc.

And by product use: describing ways in which a product is used. There might be strong customer loyalty, products might be used rarely or frequently, it could describe *purchase behaviour (e.g. Distress purchase, seasonal patterns or regular upgrades etc).*

Some perceptual maps use 'price' as a dimension. This is never preferable, an in many cases, price can be considered as an element of other dimensions.

Better answers *need to be able to add some context specific aspects: how would you find out about a market as challenging as medical devices for disaster zones? As it is unlikely that there are any market research reports, the data could be very patchy, so some assumptions would need to be made. This should be acknowledged in the answer*

(b) understand the requirements of stakeholders; and

Firstly, the stakeholders need to be identified. This could be done by viewing the external stakeholders in terms of who is involved in the purchasing decision.

- *Initiator(s) - Begins the buying process & gathers information*
- *Influencer(s) - Persuades or guides, has some role in influencing the decision, but is not the primary decision maker*
- *Decider(s) - Holds the power / purse strings. May not however be the ultimate user.*
- *Buyer - Conducts the transaction*
- *User - The actual end user, who interacts with the product*

The benefits to the users (such as usability, ergonomics, performance etc) may not be be visible to the buyer or decider. They may have a different set of goals.

It is important in beginning user research to identify the external stakeholders who influence the purchase decision and who are influenced by the end product.

In some cases, the same person may fulfil all roles. However, in most complex purchases, there are normally a number of stakeholders involved in the purchase decision. Each different stakeholder may have a different view on the elements of the design mix which are important to them.

We saw earlier how it is possible to identify unique purchase motivations for different market segments. It is also possible to do the same for the different stakeholders in the purchase / decision making process.

Then, you need to describe the possible different methods for capturing data from the stakeholders. This should include a description of the relative merits of the different qualitative and quantitative methods covered in the lectures.

Better answers *should be able to discuss these points in terms of the specifics of the situation, i.e. stakeholders might include multiple NGOs, contract manufacturing organisations, national/regional governments, volunteers, etc.*

(c) create a detailed product specification.

[25]

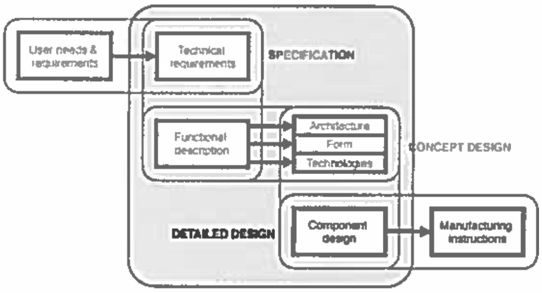
The basic answer needs to cover the key points of any product specification:

- *Define what needs to be designed*
- *Expresses customer needs in the '**language of the customer**'*
- *Does not limit ways in which the requirements are met*

- *Provides design targets*
- *Sets design constraints*
- *Provides precise, unambiguous, measurable detail about what the product must do*
  - *Quantifies and qualifies*
- *Enables the evaluation of solutions*
- ***Evolves as new information is learnt***

This should be placed in the broader context of going from user needs to final product manufacturing instructions:

Concept design



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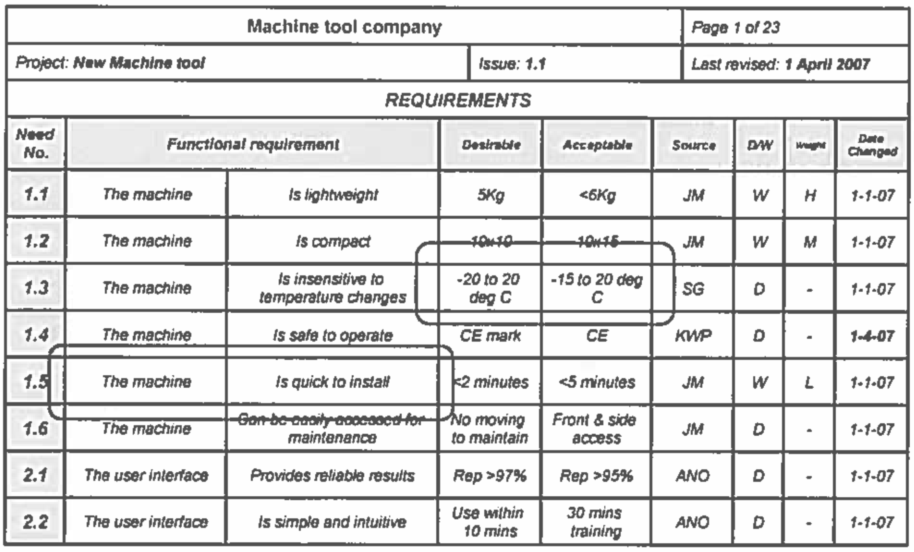
The detailed specification would look something like this:

The detailed specification …



Machine tool company								*Page 1 of 23*
*Project: **New Machine tool***			*Issue: 1.1*					*Last revised: **1 April 2007***
REQUIREMENTS								
Need No.	***Functional requirement***		***Desirable***	***Acceptable***	***Source***	***D/W***	***Weight***	***Date Changed***
1.1	*The machine*	*Is lightweight*	*5Kg*	*<6Kg*	*JM*	*W*	*H*	*1-1-07*
1.2	*The machine*	*Is compact*	~~*10x10*~~	~~*10x15*~~	*JM*	*W*	*M*	*1-1-07*
1.3	*The machine*	*Is insensitive to temperature changes*	*-20 to 20 deg C*	*-15 to 20 deg C*	*SG*	*D*	*-*	*1-1-07*
1.4	*The machine*	*Is safe to operate*	*CE mark*	*CE*	*KWP*	*D*	*-*	*1-4-07*
1.5	*The machine*	*Is quick to install*	*<2 minutes*	*<5 minutes*	*JM*	*W*	*L*	*1-1-07*
1.6	*The machine*	~~*Can be easily accessed for*~~ *maintenance*	*No moving to maintain*	*Front & side access*	*JM*	*D*	*-*	*1-1-07*
2.1	*The user interface*	*Provides reliable results*	*Rep >97%*	*Rep >95%*	*ANO*	*D*	*-*	*1-1-07*
2.2	*The user interface*	*Is simple and intuitive*	*Use within 10 mins*	*30 mins training*	*ANO*	*D*	*-*	*1-1-07*

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The process to getting to this could include decomposition of different functions required, and different possible configurations of elements.

***<u>Better answers</u>** would be able to provide a illustrative example of what the specification might look like for this heart-monitor, with rough requirements provided to show what issues would need to be considered for this particular application.*

23 (a) Describe the differences between these four types of Intellectual Property:

(i) copyright;

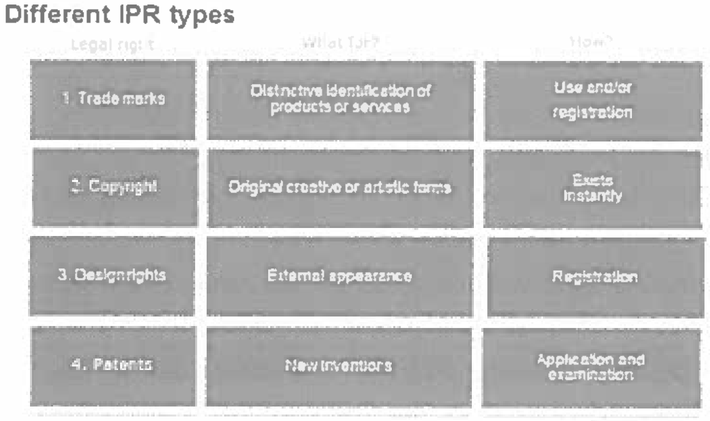
(ii) trademark;

(iii) design rights; and

(iv) patents.

[8]

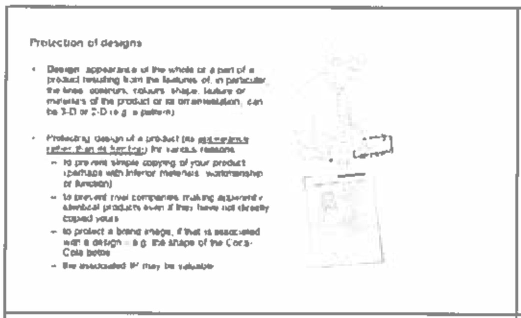
The basic answer should cover the information provided in this table:



Different IPR types

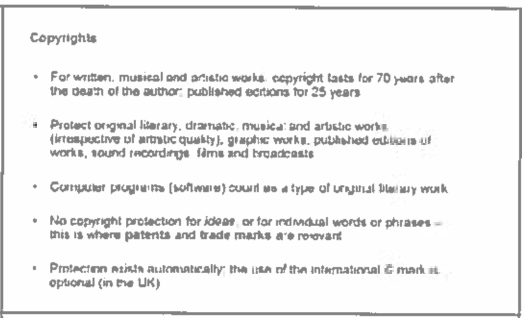
Legal right	What for?	How?
1. Trade marks	Distinctive identification of products or services	Use and/or registration
2. Copyright	Original creative or artistic forms	Exists instantly
3. Design rights	External appearance	Registration
4. Patents	New inventions	Application and examination

<u>Better answers</u> would provide more detail as shown below:



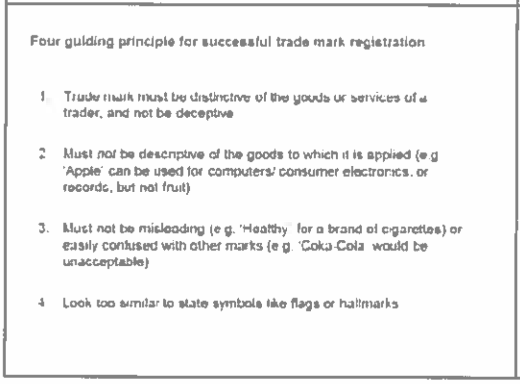
Protection of designs

- Design: appearance of the whole or a part of a product resulting from the features of, in particular, the lines, contours, colours, shape, texture or materials of the product or its ornamentation; can be 3-D or 2-D (e.g. a pattern)
- Protecting design of a product (its appearance rather than its function) for various reasons
  - to prevent simple copying of your product (perhaps with inferior materials, workmanship or function)
  - to prevent rival companies making apparently identical products even if they have not directly copied yours
  - to protect a brand image, if that is associated with a design – e.g. the shape of the Coca-Cola bottle
  - the associated IP may be valuable



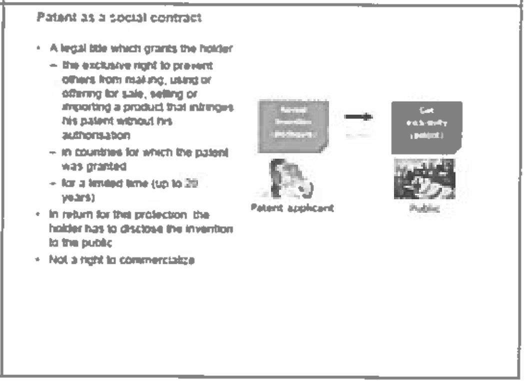
Copyrights

- For written, musical and artistic works, copyright lasts for 70 years after the death of the author; published editions for 25 years
- Protect original literary, dramatic, musical and artistic works (irrespective of artistic quality), graphic works, published editions of works, sound recordings, films and broadcasts
- Computer programs (software) count as a type of original literary work
- No copyright protection for *ideas*, or for individual words or phrases – this is where patents and trade marks are relevant
- Protection exists automatically; the use of the international © mark is optional (in the UK)



Four guiding principle for successful trade mark registration

1. Trade mark must be distinctive of the goods or services of a trader, and not be deceptive
2. Must *not* be descriptive of the goods to which it is applied (e.g. 'Apple' can be used for computers/ consumer electronics, or records, but not fruit)
3. Must not be misleading (e.g. 'Healthy' for a brand of cigarettes) or easily confused with other marks (e.g. 'Coka-Cola' would be unacceptable)
4. Look too similar to state symbols like flags or hallmarks



Patent as a social contract

- A legal title which grants the holder
  - the exclusive right to prevent others from making, using or offering for sale, selling or importing a product that infringes his patent without his authorisation
  - in countries for which the patent was granted
  - for a limited time (up to 20 years)
- In return for this protection, the holder has to disclose the invention to the public
- Not a right to commercialize

(b) For a patented technology, compare the advantages and disadvantages for a firm using a business model based upon:

(i) licensing the patented technology to another firm; and

(ii) manufacturing and selling a product that incorporates the patented technology.

[8]

Basic answer should cover:

Licence the idea to someone else

Pros = Get the cash quite fast, little responsibility for building the business/achieving success, comparatively little further funding /capital investment needed.

Cons = Get relatively small % of value, need to develop capability to manage licensing business model (i.e. how to find partners, how to negotiate contracts, and how to manage licensees), may have little control over how the licensees use your technology. Have to have the ability to get-on with other people / organisations.

Do it all yourself

Pros = You get 'all' the value generated – what you invest in the process can be directly recaptured by you (if all goes to plan), you are in 'full' control of what happens, you do not need to manage complex contracts.

Cons = You need to raise all the investment/capital, you take responsibility for all that goes wrong, you have to work within the limits of the resources you can access.

Better answers would compare these two, rather than just describing each one.

(c) (i) Describe the key elements of a business plan for commercialising an idea based around a patented technology using a partnership-based business model.

(ii) For the business plan described in part (i), discuss the specific issues a venture capitalist would focus upon when examining this business plan. Explain why you think these issues are particularly important to a venture capitalist.

[9]

(i) Basic elements of the business plan are:

The market
- Who has the problem that you attempting to fix?

Product or service
- What solutions are going to be used to address the problem?

Management team
- Who is going to do it? What is their track record?

Business operations
- How are you going to do it?

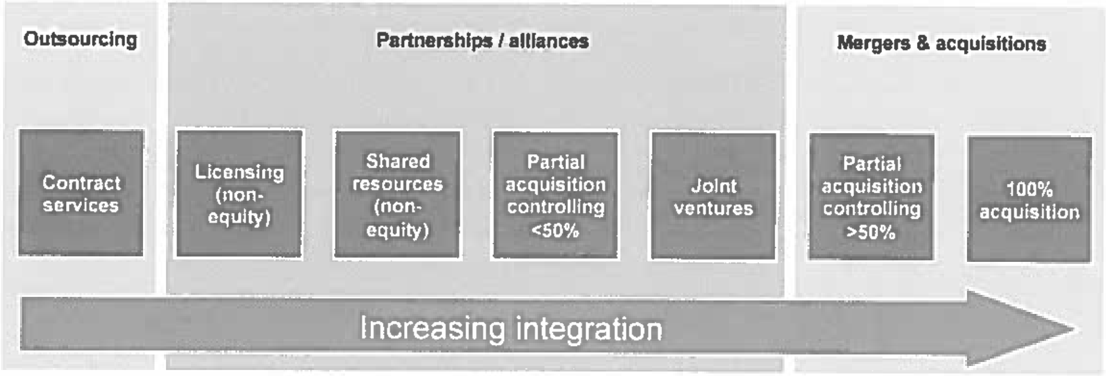
Financial projections
- How and when will money be made?

Marketing strategy
- How will get people to buy your product / service

Resources required
- What do you need to start your business?

Exit opportunities
- How will your investors get their returns?

For the specific case as described in question, there would need to be explicit reference to the ***types*** *of partnership that have been or need to be formed, and the role of the patented technology as part of the core value creation opportunity. Is all the required knowledge captured in the patent – or are there complementary assets i.e., the inventor – that are a key part of the plan?*



(ii) Any VC is particularly interested in:

Management *- especially where the team has not worked together before. They would look in detail at the CVs of the management team to make sure that there is both commercial and technical experience, appropriate for this industry.*

Access to markets *– they would look in detail not only at the projected market size but whether the team have a clear awareness of what needs to be done to access the specific market (e.g., if the plan is for a healthcare technology, do the team understand and have a plan for working with an organisation as complex as the National Health Service, with set procurement processes?).*

Exit *- what control, if any, will the VC have over a trade sale or a stock market listing? Do the team have a sense of who might buy the business in the future?*

For this particular situation, the VC would want to be convinced that the team have a clear plan for working with the partner. They would look in detail and who the potential partners are, what the current relationship between the start-up and the partner(s) is, and what the IP situation is. The VC would want to know whether the start-up team has any experience of managing partnerships.