1 (a) The amplitude on the surface is $\varepsilon e^{i\omega t}$

$\therefore$ the velocity ~~of~~ on the surface

$$u_a = \frac{d\varepsilon}{dt} = i\omega\varepsilon e^{i\omega t} = \hat{u}_a e^{i\omega t} \quad - (1)$$

Spherically symmetric field $\Rightarrow$

$$p'(r,t) = \frac{f(t - r/c_0)}{r} \quad - (2)$$

Momentum eq^n:

$$\rho_0 \frac{\partial u'}{\partial t} = -\frac{\partial p'}{\partial r} \quad - (3)$$

$$\text{let } p'(r,t) = \underbrace{\frac{A e^{-ikr}}{r}}_{\hat{P}} \cdot e^{i\omega t} \quad - (4)$$

$k = \omega/c_0$

Let $u' = \hat{u} e^{i\omega t}$

Substituting this and eq^n (4) in Eq (3), we get

$$i\omega\rho_0 \hat{u} = A e^{-ikr}\left(\frac{1}{r^2} + \frac{ik}{r}\right) \quad (5)$$

Substituting the BC, that $\hat{u} = \hat{u}_a$ on $r = a$ gives

$$A = \frac{i\omega\rho_0 \hat{u}_a a^2 e^{ika}}{1 + ika}$$

$$\therefore p'(r,t) = \frac{-\omega^2\rho_0\varepsilon a^2}{(1+ika)r} e^{-ik(r-a)+i\omega t} \quad - (6)$$

1 (b) We can find the power radiated in the far field, where $\frac{\hat{p}}{\hat{u}} \sim \rho_0 c_0$, i.e. the waves become plane

$\therefore$ the average intensity

$$\bar{I} = \frac{1}{2} Re\{\hat{p}\hat{u}^*\} \quad \text{becomes}$$

$$\bar{I} = \frac{1}{2}\frac{Re}{\rho_0 c_0}\left\{\frac{\omega^4 \rho_0^2 \varepsilon^2 a^4}{(1+ika)(1-ika)r^2}\right\}$$

$$= \frac{1}{2}\frac{1}{\rho_0 c_0}\frac{\omega^4 \rho_0^2 \varepsilon^2 a^4}{(1+k^2a^2)r^2}$$

$$\text{Power} = \bar{I} \times 4\pi r^2$$

$$= \frac{2\pi \rho_0 \varepsilon^2 (\omega a)^4}{c_0(1+k^2a^2)}$$

$\rho_0 = 1000\ kg/m^3$ $\quad \omega = 2\pi \times 5 \times 10^3\ rad/s$

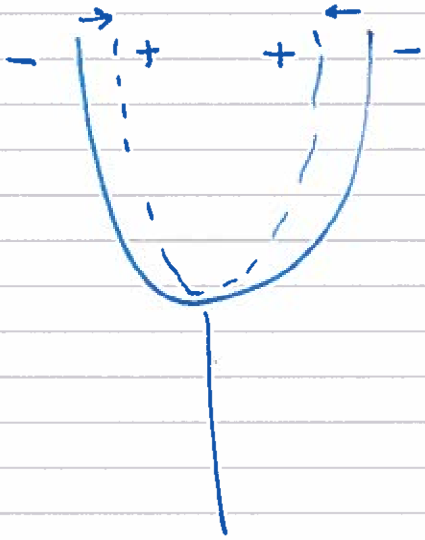
$c_0 = 1450\ m/s$ $\quad ka \ll 1$

$\therefore 1 + (ka)^2 \sim 1$.

$\varepsilon = 0.1 \times 10^{-3}\ m$ $\quad$, $a = 10^{-3}\ m$

$\therefore$ Power = 0.042 Watts

2(a)



The prongs move in and out in a symmetric motion. The motion of each prong creates a dipole. The two dipoles are out of phase, leading to a longitudinal quadrupole.

(b) Longitudinal quadrupole

2(c). $\Box^2 p' = \underbrace{A \frac{d^2 \delta(y_1)}{dy_1^2} \delta(y_2)\delta(y_3)}\, e^{i\omega t} \rightarrow S$

Using the Green's function,

$$p' = G * S$$

$$\therefore p' = \frac{A}{4\pi c_0} \int \frac{d^2 \delta(y_1)}{dy_1^2} \delta(y_2)\delta(y_3)\, e^{i\omega\tau} \cdot \frac{\delta\{|\underline{x}-\underline{y}| - c_0(t-\tau)\}}{|\underline{x}-\underline{y}|}\, d\underline{y}\, d\tau$$

Carrying out the integration w.r.t. τ :

$$p' = \frac{A}{4\pi c_0^2} \int \frac{d^2\delta}{dy_1^2} \frac{\delta(y_2)\delta(y_3)\, e^{i\omega\left(t - \frac{|\underline{x}-\underline{y}|}{c_0}\right)}}{|\underline{x}-\underline{y}|}\, dy_1 dy_2 dy_3$$

Carrying out the integrations wrt y_2 & y_3:

$$p' = \frac{A}{4\pi c_0^2} \int \frac{d^2\delta(y_1)}{dy_1^2} \frac{e^{i\omega\left(t - \frac{|\underline{x} - y_1 \underline{e}_1|}{c_0}\right)}}{|\underline{x} - y_1\underline{e}_1|}\, dy_1$$

where $\underline{e}_1$ is a unit vector in the x_1 direction.

By using a property of the convolution integrals:

$$p' = \frac{A}{4\pi c_0^2} \frac{d^2}{dx_1^2} \int \cancel{\frac{d^2\delta}{dy_1}}\, \frac{\delta(y_1)\, e^{i\omega\left(t - \frac{|\underline{x} - y_1\underline{e}_1|}{c_0}\right)}}{|\underline{x} - y_1 \underline{e}_1)}\, dy_1$$

$$\therefore \quad p' = \frac{A}{4\pi c_0^2} \frac{d^2}{dx_1^2} \left(\frac{e^{i\omega(t - \frac{|\underline{x}|}{c_0})}}{|\underline{x}|} \right)$$

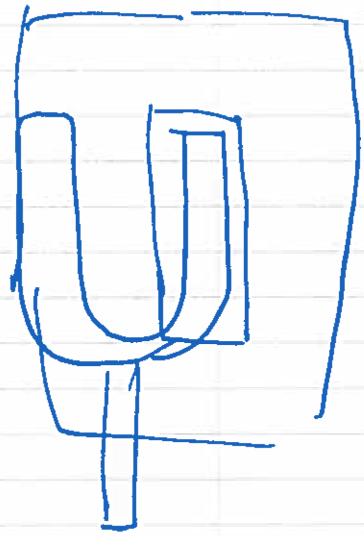
In the far. field (neglecting terms of the order of $\frac{1}{x^2}$), we get

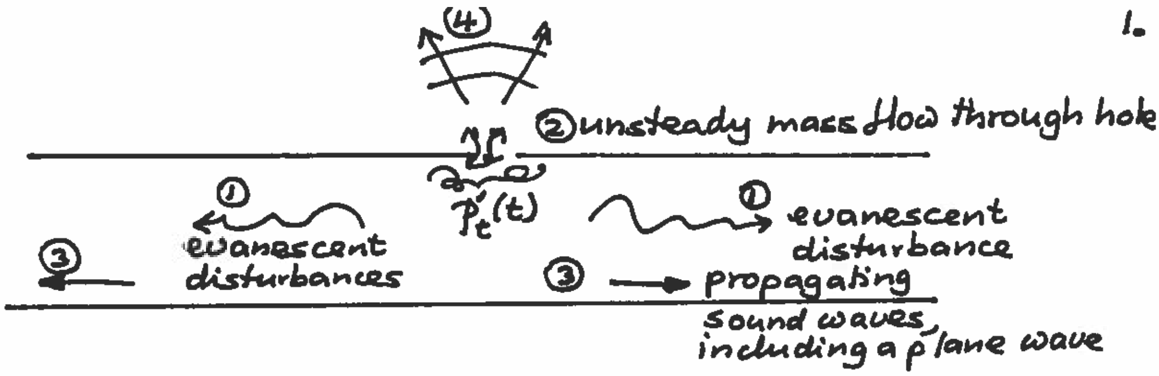
$$p' \sim \frac{-A\omega^2}{4\pi c_0^4} \frac{1}{x} \cdot \cos^2\theta \cdot e^{i\omega(t - \frac{x}{c_0})}$$

Where $\cos\theta = \frac{x_1}{x}$, $x = |\underline{x}|$

$$\therefore \quad |p'| = \frac{A}{4\pi} \frac{\omega^2}{c_0^4} \frac{\cos^2\theta}{x} .$$

2(d) When one of the prongs is held with the slot of a baffle, then we do not get a cancellation effect from the other dipole, so instead of a quadrupole, we get a dipole.





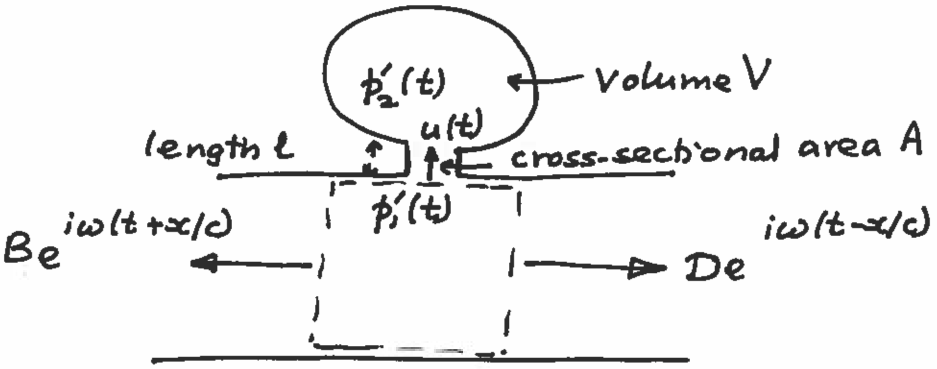
a) (1) $p_t'(t)$ produces evanescent waves which decay exponentially with distance from the patch of turbulent flow.

(2) $p_t'(t)$ causes unsteady mass flows in and out of the hole and this will generate additional disturbances

(3) In particular, since there is an unsteady mass flow there will be plane waves that propagate in the duct away from the hole, and possibly also higher order duct modes depending on the frequency.

(4) There will also be sound outside the duct in a way that depends on the external geometry.

b)



(i) Let $p_1'(t)$ denote the pressure at the opening of the hole.

$p_2'(t)$ is the pressure in the compact cavity

$u(t)$ is the velocity into the hole

$$p_1' - p_2' = \rho_0 l \frac{du'}{dt}, \qquad p_1' - p_2' = \rho_0 i\omega l u' \text{ (i) for frequency } \omega$$

Conservation of mass in the cavity

$$V\frac{d\rho_2}{dt} = \rho_0 A u, \qquad i\omega V \rho_2' = \rho_0 A u' \text{ for frequency } \omega$$

$$p_2' = c^2 \rho_2' = \frac{\rho_0 A u' c^2}{i\omega V}$$

Substituting in (i)

$$p_1' - \frac{\rho_0 A c^2}{i\omega V} u' = \rho_0 i\omega l u'$$

$$p_1' = \left(\rho_0 i\omega l + \frac{\rho_0 A c^2}{i\omega V}\right) u' = -\left(\omega^2 - \omega_0^2\right)\frac{\rho_0 l}{i\omega} u', \text{ where } \omega_0^2 = \frac{c^2 A}{V l} \quad \text{(ii)}$$

ii) It is given that $B = D = P$, where B and D denote the amplitudes of the waves upstream and downstream of the hole which is taken to be at $x = 0$.

Also given that $p_1'(t) = p_t'(t) + Pe^{i\omega t}$

Continuity of mass flow in the duct gives

$$u'A = (B + D)\frac{e^{i\omega t}}{\rho_0 c} S = \frac{2BS}{\rho_0 c} e^{i\omega t}$$

Hence substituting into (ii) we obtain

$$p_t'(t) + Be^{i\omega t} = -(\omega^2 - \omega_0^2)\frac{\rho_0 \ell}{i\omega} 2B \frac{S}{A\rho_0 c} e^{i\omega t}$$

$$Be^{i\omega t}\left[1 + \frac{(\omega^2 - \omega_0^2)\ell}{i\omega c}\frac{S}{A}\right] = -p_t'(t) = -\hat{p}_t e^{i\omega t}$$

Hence the waves are

$$\frac{-\hat{p}_t'(t)}{1 + (\omega^2 - \omega_0^2)\frac{\ell S}{i\omega c A}} e^{i\omega(t + x/c)} \quad \text{in } x < 0$$

$$\frac{-\hat{p}_t}{1 + (\omega^2 - \omega_0^2)\frac{\ell S}{i\omega c A}} e^{i\omega(t - x/c)} \quad \text{in } x > 0$$

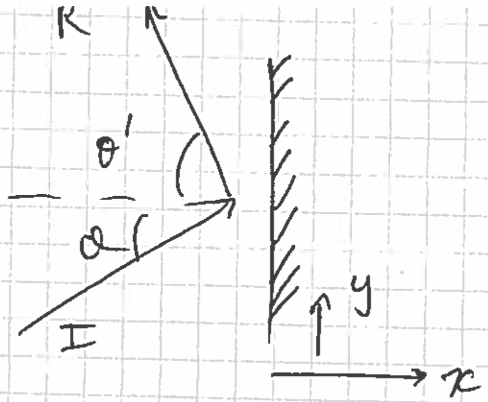
(iii) The damping mechanism in this model is the energy radiated to infinity in the sound waves in the duct.

(iii) In practice, there would be additional damping due to viscosity and nonlinear vortex shedding in the high speed flows through the hole on resonance. These lead to an additional term proportional to u in the pressure difference across the hole.

$$p_1' - p_2' = \rho_0 \ell \frac{du}{dt} + \alpha u.$$

4

(a)



$$p' = I e^{i\omega t - i\omega x\cos\theta/c_0 - i\omega y\sin\theta/c_0} + R e^{i\omega t + i\omega x\cos\theta'/c_0 - i\omega y\sin\theta'/c_0}$$

pressure

$$p' = -\rho_0 i\omega\phi \Rightarrow \phi' = \frac{i}{\rho_0\omega} p'$$

$$\underset{\sim}{u}' = \nabla\phi' = \frac{i}{\rho_0\omega}\left(\frac{i\omega}{c_0}\right)\left\{\left(-I\cos\theta\, e^{i\omega t - i\omega x\cos\theta/c_0 - i\omega y\sin\theta/c_0} + R\cos\theta'\, e^{i\omega t + i\omega x\cos\theta'/c_0 - i\omega y\sin\theta'/c_0}\right.\right.,$$

$$\left.\left. -I\sin\theta\, e^{i\omega t - i\omega x\cos\theta/c_0 - i\omega y\sin\theta/c_0} - R\sin\theta'\, e^{i\omega t - i\omega x\cos\theta/c_0 - i\omega y\sin\theta'/c_0}\right)\right\}$$

Boundary condition is $p' = Z(\nabla\phi')\cdot\hat{\underset{\sim}{x}}$ on $y=0$

Cancel $e^{i\omega t}$ throughout:

$$I e^{-i\omega y\sin\theta/c_0} + R e^{-i\omega y\sin\theta'/c_0} = Z\left(-\frac{1}{\rho_0 c_0}\right)\left(-I\cos\theta\, e^{-\frac{i\omega y}{c_0}\sin\theta} + R\cos\theta'\, e^{-\frac{i\omega y}{c_0}\sin\theta'}\right)$$

(2)

Must hold for all y, so exponentials cancel

$$\Rightarrow \quad \sin\theta = \sin\theta'$$

angle of incidence = angle of reflection

and
$$I + R = -\frac{Z}{\rho_0 c_0}\cos\theta\,(R - I)$$

$$R\left(1 + \frac{Z\cos\theta}{\rho_0 c_0}\right) = I\left(\frac{Z\cos\theta}{\rho_0 c_0} - 1\right)$$

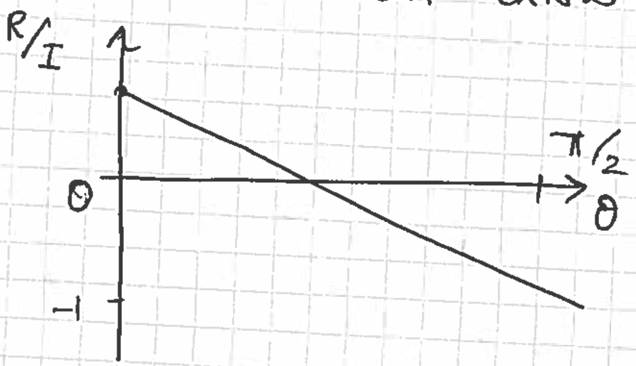
$$\frac{R}{I} = \frac{\dfrac{Z\cos\theta}{\rho_0 c_0} - 1}{\dfrac{Z\cos\theta}{\rho_0 c_0} + 1}$$

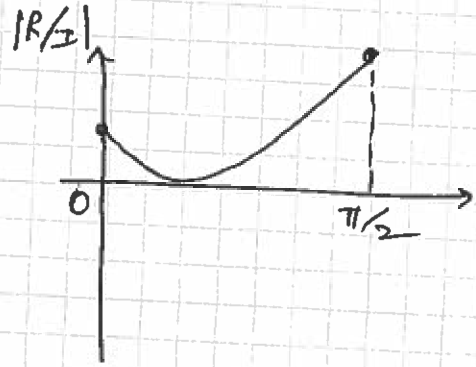
[5] (seen in lectures)

If Z pure real, call $Z = \bar{Z}\rho_0 c_0$

$$\frac{R}{I} = \frac{\bar{Z}\cos\theta - 1}{\bar{Z}\cos\theta + 1} = 1 - \frac{2}{\bar{Z}\cos\theta + 1}$$

If $\bar{Z} > 1$ then exists angle where $R = 0$





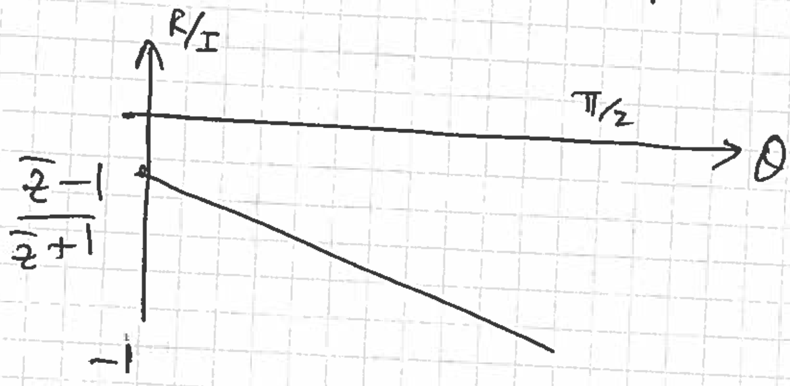
Hence
$$0 \le |R/I| \le 1$$

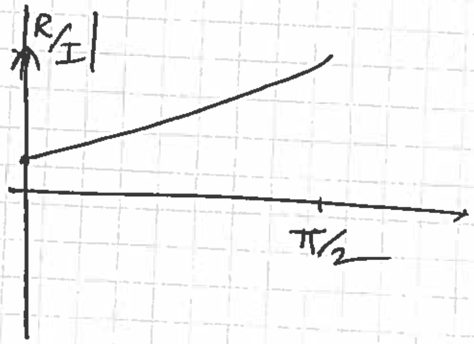
[2]

(unseen)

If $\bar{z} < 1$ then $R \neq 0$.

③

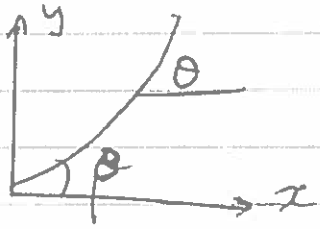




$$\frac{1-\bar{z}}{1+\bar{z}} \leq \left|\frac{R}{I}\right| \leq 1$$

[3]
unseen

$$c(x) = c_0 e^{\alpha x} \qquad x \geq 0$$



$$\frac{\sin\theta}{c_0(x)} = \text{constant} = \frac{\sin\beta}{c_0}$$

$$\frac{y'}{\sqrt{1+y'^2}} = \sin\beta \, e^{\alpha x}$$

$$y' = \frac{\sin\beta \, e^{\alpha x}}{\sqrt{1-\sin^2\beta \, e^{2\alpha x}}}$$

$$\int dy = \int \frac{\sin\beta \, e^{\alpha x} \, dx}{\sqrt{1-\sin^2\beta \, e^{2\alpha x}}}$$

$$\sin\beta \, e^{\alpha x} = u \qquad \sin\beta \, e^{\alpha x} \, dx = du/\alpha$$

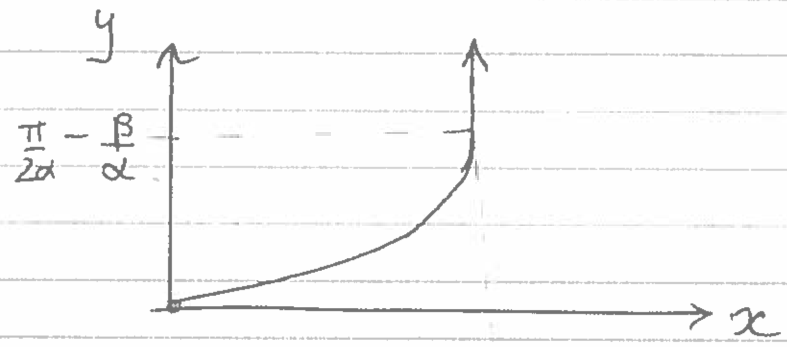
$$\rightarrow \int \frac{du}{\alpha\sqrt{1-u^2}} = \frac{1}{\alpha} \sin^{-1} u + k$$

$$y = \frac{1}{\alpha} \sin^{-1}\left[\sin\beta \, e^{\alpha x}\right] - \frac{\beta}{\alpha}$$ since $x=0, y=0$

[6]

seen similar.

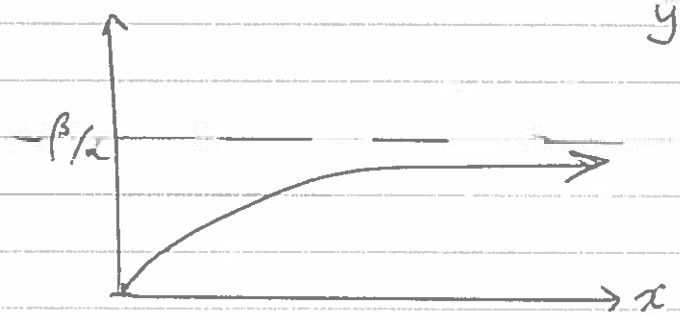
When $\alpha > 0$, $c_0(x)$ increases as x increases, and clearly have problem when $\sin\beta\, e^{\alpha x} = 1$, i.e. $x = -\frac{1}{\alpha}\log(\sin\beta)$, when $y' \to \infty$



[2]
unseen

When $\alpha < 0$ $c_0(x)$ decreases as x increases

$y \to -\beta/\alpha$ as $x \to \infty$



[2]
unseen