NRA and TPH

ENGINEERING TRIPOS PART IIB

2017

Module 4A3

TURBOMACHINERY I - CRIB

1 (a) A turbine stator with $p_{0,\mathrm{E}}$ and $T_{0,\mathrm{E}}$, is tested in a linear cascade, at scale, SF, at matched Reynolds and Mach number. The cascade flow is at $T_{0,\mathrm{C}}$. The cascade inlet, $p_{0,\mathrm{C}}$ can be found in the following way:

$$M_\mathrm{E} = M_\mathrm{C}$$

$$Re_\mathrm{C} = \frac{\rho_\mathrm{C} V_\mathrm{C} C_\mathrm{C}}{\mu_\mathrm{C}} \text{ and } Re_\mathrm{E} = \frac{\rho_\mathrm{E} V_\mathrm{E} C_\mathrm{E}}{\mu_\mathrm{E}}$$

Using the functions,

$$\frac{\rho}{\rho_0} = f_1\{M\} \text{ and } \frac{V}{\sqrt{c_p T_0}} = f_2\{M\}$$

the Reynolds numbers can be written as,

$$Re = \frac{\rho_{0,\mathrm{C}} f_1\{M\} \sqrt{c_p T_{0,\mathrm{C}}}\, f_2\{M\} C_\mathrm{C}}{\mu_\mathrm{C}} = \frac{\rho_{0,\mathrm{E}} f_1\{M\} \sqrt{c_p T_{0,\mathrm{E}}} f_2\{M\}\, C_\mathrm{E}}{\mu_\mathrm{E}}$$

given the Mach No. match,

$$\frac{\rho_{0,\mathrm{C}} \sqrt{c_p T_{0,\mathrm{C}}}\ C_\mathrm{C}}{\mu_\mathrm{C}} = \frac{\rho_{0,\mathrm{E}} \sqrt{c_p T_{0,\mathrm{E}}}\ C_\mathrm{E}}{\mu_\mathrm{E}}$$

$$\rho_{0,\mathrm{C}} \sqrt{c_p T_{0,\mathrm{C}}} = \rho_{0,\mathrm{E}} \cdot \frac{C_\mathrm{E}}{C_\mathrm{C}} \cdot \frac{\mu_\mathrm{C}}{\mu_\mathrm{E}} \sqrt{c_p T_{0,\mathrm{E}}}$$

$$\rho_{0,\mathrm{C}} = \rho_{0,\mathrm{E}} \cdot \frac{C_\mathrm{E}}{C_\mathrm{C}} \cdot \frac{\mu_\mathrm{C}}{\mu_\mathrm{E}} \cdot \sqrt{\frac{T_{0,\mathrm{E}}}{T_{0,\mathrm{C}}}}$$

as $\rho_0 = \dfrac{p_0}{RT_0}$,

$$p_{0,\mathrm{C}} = p_{0,\mathrm{E}} \cdot \frac{1}{SF} \cdot \frac{\mu_\mathrm{C}}{\mu_\mathrm{E}} \cdot \sqrt{\frac{T_{0,\mathrm{C}}}{T_{0,\mathrm{E}}}}$$

(b) A stator has axial inlet flow with $p_{01} = 35$ bar and $T_{01} = 1073$ K, $M_1 = 0.3$, $M_2 = 0.9$, $C_x = 5$ cm, $s/C_x = 0.8$. It's tested in a 5 passage linear cascade with $SF = 1.5$ and $h/C_x = 2$.

(i) A cascade with a greater number of passages is likely to be less affected by the non-cascade flow at the ends of the test section. As such, the central passages will be closer to the idealised 2-D cascade flow. A larger ratio of blade height to chord avoids the effects of secondary flows and boundary layer growth within the cascade row, as such the data is a good representation of the blade profile loss. The disadvantage is higher cost of manufacture as well as the higher cost of the greater mass flow required to reach the matched conditions.

(ii) For $T_{01} = 373$ K, the inlet stagnation pressure and total mass flow rate through the cascade for Mach and Reynolds number matched conditions can be calculated as follows.

The transport properties of air are in the CUED Databook, we note that at inlet M = 0.3, so $T_0 \approx T$

Engine, $T_{0,\mathrm{E}}$ = 1073 K - 273 = 800°, $\mu_{\mathrm{E}} = 44 \times 10^{-6}$
Cascade, $T_{0,\mathrm{C}}$ = 373 K - 273 = 100°, $\mu_{\mathrm{C}} = 22 \times 10^{-6}$

Using the expression derived in part (a),

$$p_{0,\mathrm{C}} = 35 \times 10^5 \frac{1}{1.5} \cdot \frac{22}{44} \cdot \sqrt{\frac{373}{1073}} = \mathbf{6.88\ bar}$$

The mass flow rate can be found from the flow function at inlet,

$$\begin{aligned}
\frac{\dot{m}\sqrt{c_p T_{01}}}{A p_{01}} &= f\{M_1 = 0.3\} = 0.6295 \\
\dot{m} &= \frac{A \cdot 0.6295 \cdot p_{01}}{\sqrt{c_p T_{01}}} \\
\dot{m} &= \frac{A \cdot 0.6295 \cdot 6.88 \times 10^5}{\sqrt{1005 \times 373}}
\end{aligned}$$

from the geometry given in the question,

$$\begin{aligned}
C_{x_C} &= SF \cdot C_{x_E} \\
s_C &= \left(\frac{s}{C_x}\right) C_{x_C} = \left(\frac{s}{C_x}\right) SF\ C_{x_E} \\
h_C &= \left(\frac{h}{C_x}\right) C_{x_C} = \left(\frac{h}{C_x}\right) SF\ C_{x_E}
\end{aligned}$$

The inlet area, A of the cascade is given by

$$\begin{aligned}
A &= 5 \times s_C \times h_C \\
&= 5 \times \left(\frac{s}{C_x}\right) SF\ C_{x_E} \times \left(\frac{h}{C_x}\right) SF\ C_{x_E} = 5 \times 0.8 \times 1.5 \times 0.05 \times 2 \times 1.5 \times 0.05 = 0.045\ \mathrm{m}^2
\end{aligned}$$

$$\dot{m} = \frac{0.045 \cdot 0.6295 \cdot 6.88 \times 10^5}{\sqrt{1005 \times 373}} = \mathbf{31.83\ kg\,s^{-1}}$$

(i) The compressor draws air at 1 bar, 300 K and has a total-to-total isentropic efficiency of 0.8. The flow is cooled and delivered to the cascade without loss. The shaft power required by the compressor is calculated from,

$$\begin{aligned}\dot{W}_x =& \dot{m}\,\Delta h_0 = \dot{m}\,\frac{c_p T_{\text{Comp,in}}}{\eta_C}\left(Pr^{(\gamma-1)/\gamma}-1\right)\\ =& 31.83\ \times\ \frac{1005\cdot 300}{0.8}\left(6.88^{(1.4-1)/1.4}-1\right)\\ =& 31.83\ \times\ 2.77\times 10^5 = \mathbf{8.82\ MW}\end{aligned}$$

(ii) For $D = 1.2$ m, the specific speed, N_s, for this duty is found by calculating the secific diameter, D_s, and reading the specific speed from the chart. First we calculate the compressor exit temperature and then density,

$$T_{0,\text{Comp,exit}} = T_{0,\text{Comp,inlet}} + \Delta h_0/c_p = 300 + 2.77\times 10^5/1005 = 575.6\ \text{K}$$

and

$$\rho_{\text{exit}} \approx \rho_{0,\text{exit}} = \frac{P_{0,\text{Comp,exit}}}{R\,T_{0,\text{Comp,exit}}} = \frac{6.88\times 10^5}{287.1\cdot 575.6} = 4.1624\,\text{kg m}^{-3}$$

So,

$$D_s = (\Delta h_0)^{1/4}\rho_{\text{exit}} D/\dot{m} = \left(2.77\times 10^5\right)^{1/4} 4.1624\cdot 1.2/31.83 = 3.6$$

$\implies N_s =$ **0.8 , from the graph, suggesting that at centrifugal machine would be best.**

$$\Omega = N_s\cdot(\dot{m}/\rho_{exit})^{-0.5}\cdot\Delta h_0^{3/4} = \mathbf{3.49\times 10^3 rad\ s}^{-1}$$

(iii) At lower exit pressure and higher flow rate conditions, the relative frame Mach numbers at the tip will increase. This will lead to poor performance at inlet due to shock losses, shock induced separations, and eventually choking, which prevents further increase in flow regardless of the outlet pressure. The higher flow rate increases the hub-to-tip pressure gradient, encouraging stronger secondary flows and hence loss within the passages.

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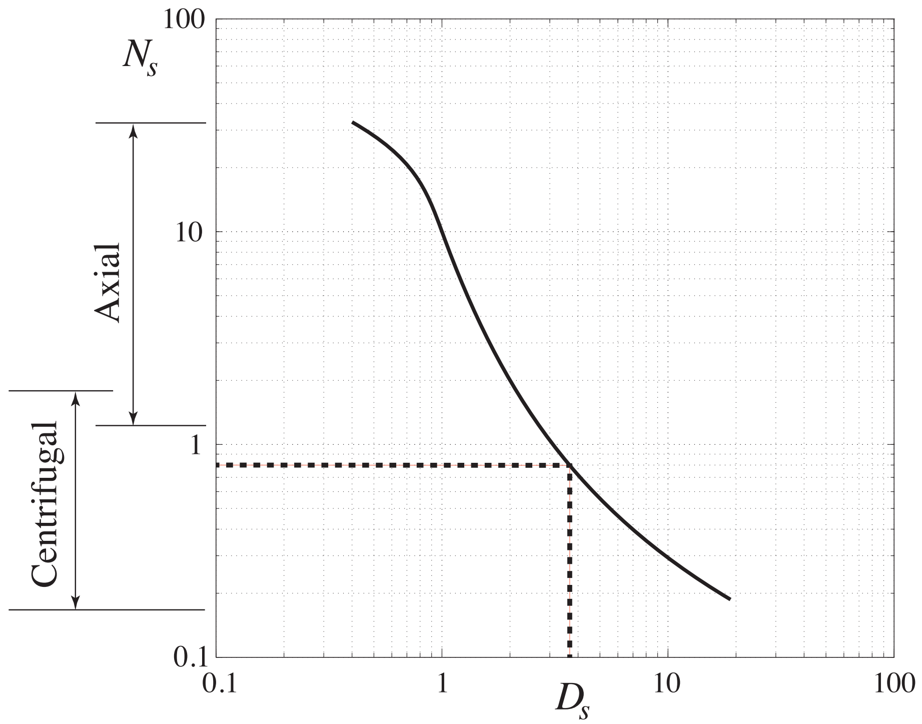


The Cordier line rises at low D_s reflecting higher mass per unit frontal area for axials, which produce less Δp_0 for a stage when compared to centrifugals.

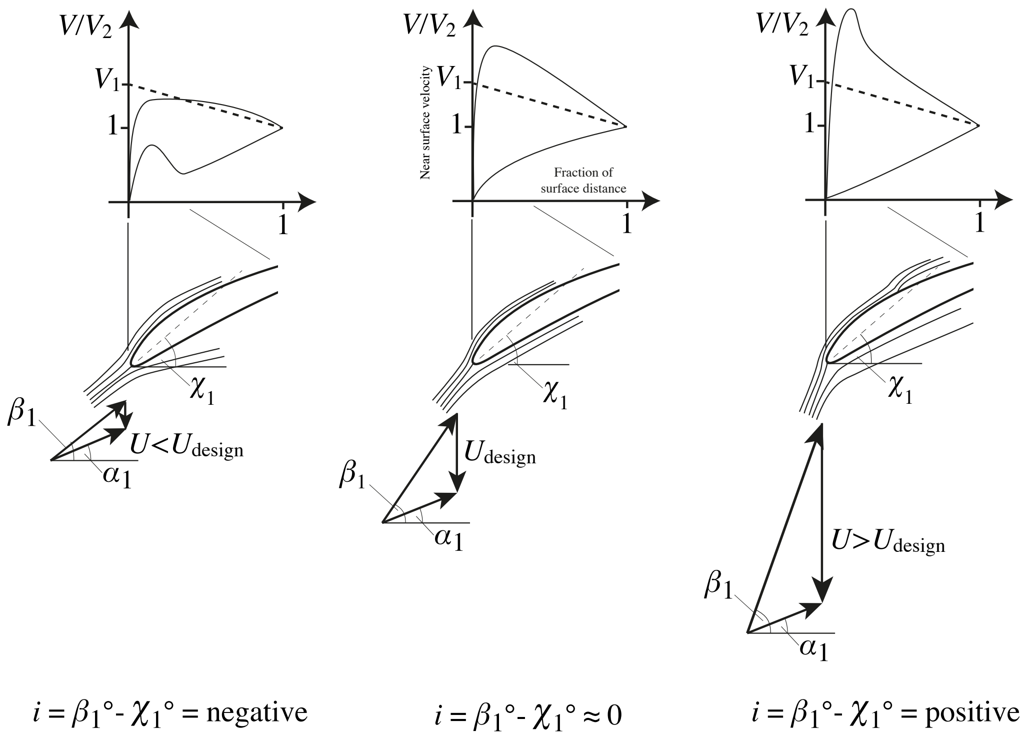
Assessor's Comments:

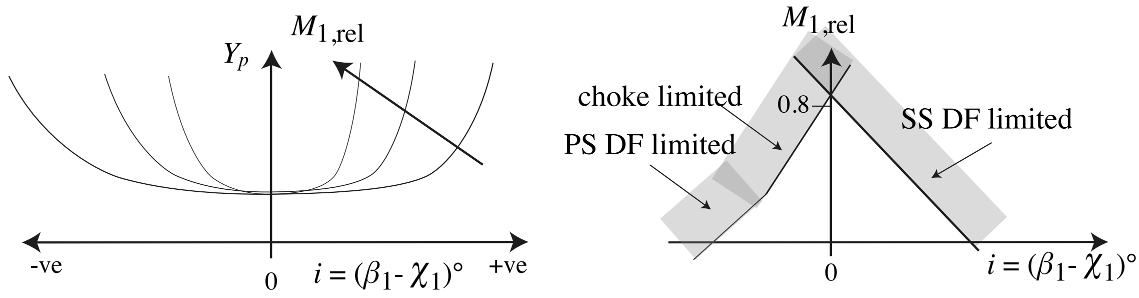
12 attempts, mean 67%, standard deviation 19%, max. 19/20, min. 8/20.
Deriving the relationship for Reynolds and Mach number scaling in part (a) was done well by all. In part (b), the question about the"design of the cascade" drew many answers about the "design of the profile" which had been regurgitated hopefully. The mass flow and inlet pressure calculations were done well by most. Part (c) was somewhat simpler, but many students made simple numerical slips.

2 (a) (i) As the blade speed increases, the incidence, $i = \beta_1 - \chi_1$, becomes negative. The Mach numbers on the early suction surface are higher than the design condition. This gives greater diffusion and hence loss generation on the later suction surface. There is also a risk of local supersonic regions, which are prone to separate the boundary layer.

As the blade speed decreases, the incidence, $i = \beta_1 - \chi_1$, becomes positive. The Mach numbers on the early pressure surface are higher than the design condition, the curvature of the pressure surface leads to significant local diffusion and boundary layer separation.

The deviation increases in both cases.

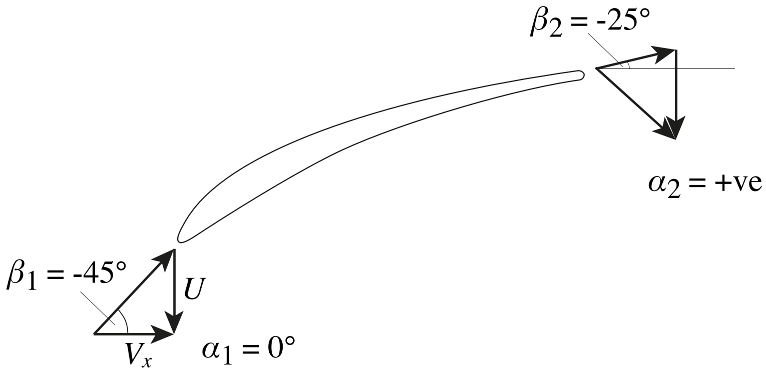




(ii) With increasing relative inlet Mach number: higher diffusion and shock induced separation on the suction surface occurs at lower values of positive incidence; at negative incidence, the whole passage starts to choke at the throat. The passage is not designed to cope with the supersonic flow downstream and boundary layers separate.

(iii) Sharp leading edges, thinner, flat blades, compression across a weak shock and turning through the change of reference frame give compressors with reasonable loss at moderate supersonic inlet Mach numbers. Such designs have limited incidence range compared to subsonic designs. However, they can provide useful pressure rise with fewer stages.

(b) Data from the question: $\Omega = 4180$ rpm; $p_{01} = 1.0$ bar; $T_{01} = 300$ K; no inlet swirl, so $\alpha_1 = 0°$; $\bar{r} = 0.35$ m; $\beta_1 = -45°$; $\frac{p_{0,\text{rel}}}{p_2} = 1.15$; $Y_p = 0.04$



(i) Start by calculating the blade speed,

$$U = \frac{4180}{60} \times 2\pi \times 0.35 = 153.21 \text{ m s}^{-1}$$

As $\beta_1 = -45°$, the axial velocity must be the same as the blade speed, and as there is no inlet swirl, the absolute inlet Mach number can be found from,

$$f_1\{M_1 = M_x\} = \frac{V_x}{\sqrt{c_p T_{01}}} = \frac{153.21}{\sqrt{1005 \times 300}} = 0.2790$$

$$\implies M_1 = M_x = 0.45 \text{ using the tables.}$$

Also, as $\beta_1 = -45°$, the blade Mach number must also be 0.45, so the inlet relative Mach number can also be found directly from the inlet velocity triangle,

$$M_{1,\text{rel}} = \sqrt{0.45^2 + 0.45^2} = \mathbf{0.636}$$

The relative exit Mach number can be found from the exit relative total to static pressure ratio,

$$f_2\{M_{2,\text{rel}}\} = \frac{p_2}{p_{02,\text{rel}}}$$

We can find an expression for this function in terms of the stagnation total-to-total pressure ratio as follows,

$$\frac{p_{02,\mathrm{rel}}}{p_{01,\mathrm{rel}}} = \frac{p_{02,\mathrm{rel}}}{p_2}.\frac{p_2}{p_{01,\mathrm{rel}}} = \frac{1}{f_2\{M_{2,\mathrm{rel}}\}}.\frac{1}{1.15}$$

giving

$$f_2\{M_{2,\mathrm{rel}}\} = \frac{1}{1.15}\ \frac{p_{01,\mathrm{rel}}}{p_{02,\mathrm{rel}}}$$

We can then rearrange the expression for Y_p,

$$Y_p = \frac{p_{01,\mathrm{rel}} - p_{02,\mathrm{rel}}}{p_{01,\mathrm{rel}} - p_1} = \frac{1 - p_{02,\mathrm{rel}}/p_{01,\mathrm{rel}}}{1 - p_1/p_{01,\mathrm{rel}}} = \frac{1 - p_{02,\mathrm{rel}}/p_{01,\mathrm{rel}}}{1 - f_1\{M_{1,\mathrm{rel}}\}}$$

$$p_{01,\mathrm{rel}}/p_{02,\mathrm{rel}} = \frac{1}{1 - Y_p\left(1 - f_1\{M_{1,\mathrm{rel}}\}\right)}$$

so

$$f_1\{M_{2,\mathrm{rel}}\} = \frac{1}{1.15}\left[1 - Y_p\left(1 - f_1\{M_{1,\mathrm{rel}}\}\right)\right]$$

to finish, linear interpolation from the tables gives

$$f_1\left\{M_{1,\mathrm{rel}} = 0.636\right\} = 0.7654 + (0.7591 - 0.7654) \times \frac{0.6364 - 0.630}{0.64 - 0.63} = 0.7614,$$

so

$$f_1\{M_{2,\mathrm{rel}}\} = \frac{1}{1.15}\left[1 - 0.04\left(1 - 0.7614\right)\right] = 0.8779$$

interpolating again we get,

$$\Longrightarrow\ M_{2,\mathrm{rel}} = 0.44 - (0.44 - 0.43) \times \frac{0.8779 - 0.8755}{0.8807 - 0.8755} = \mathbf{0.435}$$

(ii) To evaluate Lieblein's correlation for diffusion factor, we need to find the relative outlet flow angle β_2.
Having found the inlet and exit relative Mach numbers, we can use the flow function to find β_2,

$$\frac{\dot{m}\sqrt{c_p T_{01,\mathrm{rel}}}}{A_1 p_{01,\mathrm{rel}}} = f_2\left\{M_{1,\mathrm{rel}}\right\} \quad \text{and} \quad \frac{\dot{m}\sqrt{c_p T_{02,\mathrm{rel}}}}{A_2 p_{02,\mathrm{rel}}} = f_2\left\{M_{2,\mathrm{rel}}\right\}$$

 (cont.

so by continuity,

$$\dot{m} = f_2\left\{M_{1,\text{rel}}\right\} \frac{A_1 p_{01,\text{rel}}}{\sqrt{c_p T_{01,\text{rel}}}} = f_2\left\{M_{2,\text{rel}}\right\} \frac{A_2 p_{02,\text{rel}}}{\sqrt{c_p T_{02,\text{rel}}}}$$

$$f_2\left\{M_{1,\text{rel}}\right\} \frac{hs\cos\beta_1\ p_{01,\text{rel}}}{\sqrt{c_p T_{01,\text{rel}}}} = f_2\left\{M_{2,\text{rel}}\right\} \frac{hs\cos\beta_2\ p_{02,\text{rel}}}{\sqrt{c_p T_{02,\text{rel}}}}$$

$$\implies \cos\beta_2 = \cos\beta_1 \cdot \frac{f_2\left\{M_{1,\text{rel}}\right\}}{f_2\left\{M_{2,\text{rel}}\right\}} \cdot \frac{p_{01,\text{rel}}}{p_{02,\text{rel}}}$$

Putting the numerical values in, noting that we already calculated an expression for the ratio $p_{01,\text{rel}}/p_{02,\text{rel}}$ earlier

$$\cos\beta_2 = \cos\beta_1 \cdot \frac{f_2\left\{M_{1,\text{rel}}\right\}}{f_2\left\{M_{2,\text{rel}}\right\}} \cdot \left[1 - Y_p\left(1 - 0.7614\right)\right]$$
$$\cos\beta_2 = \cos -45^\circ \cdot \frac{f_2\left\{0.6364\right\}}{f_2\left\{0.435\right\}} \cdot \left[1 - 0.04\left(1 - 0.7614\right)\right]$$
$$\cos\beta_2 = \cos -45^\circ \cdot \frac{1.1152}{0.8618} \cdot \left[1 - 0.04\left(1 - 0.7614\right)\right]$$
$$\implies \beta_2 = -25.0^\circ$$

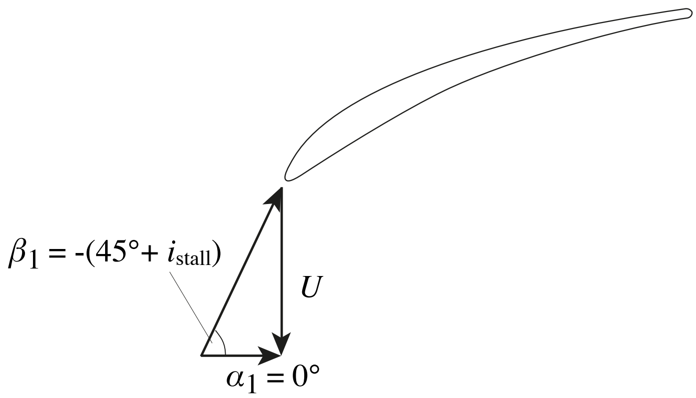
Lieblein's diffusion factor correlation from the question,

$$DF \approx 1 - \frac{\cos\beta_1}{\cos\beta_2} + \frac{1}{2}\left(\frac{s}{c}\right)\left|\tan\beta_2 - \tan\beta_1\right|\cos\beta_1$$

at design conditions, this gives an estimate of

$$DF \approx 1 - \frac{\cos(-45^\circ)}{\cos(-25.0^\circ)} + \frac{1}{2}(0.95)\left|\tan(-25.0)^\circ - \tan(-45^\circ)\right|\cos(-45^\circ) \approx \mathbf{0.45}$$

The incidence at stall can be found by iteratively increasing the inlet angle β_1 until the diffusion factor, $DF \approx 0.6$,



$\beta_1 = \beta_{1,\text{design}} + i$	DF
$i = 2°$	0.5083
$i = 4°$	0.5705
$i_{\text{stall}} = \mathbf{5}°$	0.6029

(c) The limiting value of negative incidence (relative to the design condition) with inlet relative Mach number held at the design condition can be found from continuity. In the following expressions, * denotes the throat value,

$$\frac{\dot{m}\sqrt{c_p T_{01,\text{rel}}}}{A_1 p_{01,\text{rel}}} = f_2\left\{M_{1,\text{rel}}\right\} \quad \text{and} \quad \frac{\dot{m}\sqrt{c_p T^*_{0,\text{rel}}}}{A^* p^*_{0,\text{rel}}} = f_2\left\{M_{*,\text{rel}} = 1\right\}$$

so

$$\dot{m} = f_2\left\{M_{1,\text{rel}}\right\}\frac{A_1 p_{01,\text{rel}}}{\sqrt{c_p T_{01,\text{rel}}}} = f_2\left\{M_{*,\text{rel}} = 1\right\}\frac{A^* p^*_{0,\text{rel}}}{\sqrt{c_p T^*_{0,\text{rel}}}}$$

$$f_2\left\{M_{1,\text{rel}}\right\}\frac{h s \cos\beta_{1,\text{choke}}\, p_{01,\text{rel}}}{\sqrt{c_p T_{01,\text{rel}}}} = f_2\left\{M_{*,\text{rel}} = 1\right\}\frac{h o\, p^*_{0,\text{rel}}}{\sqrt{c_p T^*_{0,\text{rel}}}}$$

$$\cos\beta_{1,\text{choke}} = \frac{f_2\left\{M_{*,\text{rel}} = 1\right\}}{f_2\left\{M_{1,\text{rel}}\right\}}\cdot\frac{o}{s}\cdot\frac{p^*_{0,\text{rel}}}{p_{01,\text{rel}}}$$

the flow upstream of the throat is isentropic and the throat opening to pitch is 0.68,

$$\cos\beta_{1,\text{choke}} = \frac{1.281}{1.1152}\cdot 0.68$$

$$\Longrightarrow \left|\beta_{1,\text{choke}}\right| = 38.6° \text{ so } i_{\text{choke}} = -(45° - 38.6°) = \mathbf{-6.36}°$$

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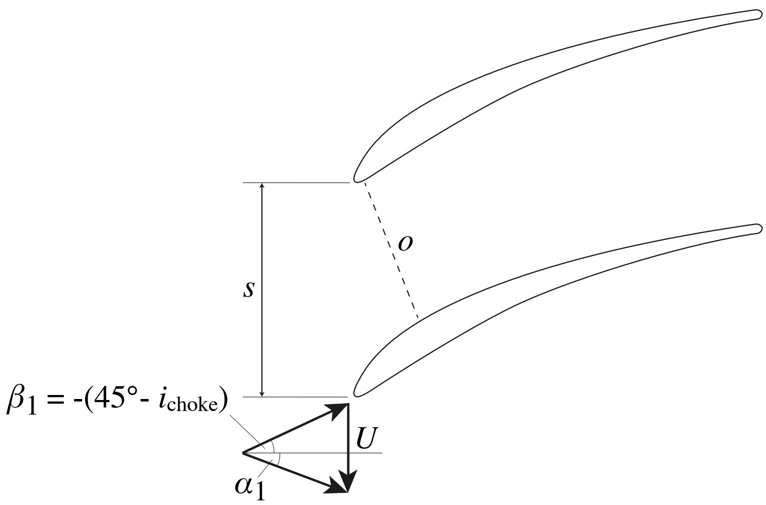


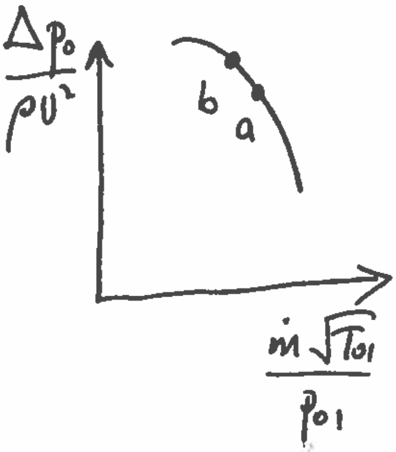
Assessor's Comments:

18 attempts, mean 59%, standard deviation 21%, max. 18/20, min. 6/20.
The descriptive part (a), on the behaviour of compressor rotors at off-design incidence, possibly made this the most popular question, but it polarised the candidates. Parts (b) and (c) were in effect hints for part (a), and it was either very well answered or rather poorly. There were numerous hopeful answers based on turbine behaviour. Perhaps some students had not expected to be tested on this part of the notes. Part (b) was well done by most. Part (c) was only managed by a few.

3. (a) For the stage

$$\frac{\dot{m}\sqrt{T_{02}}}{p_{02}} = \frac{\dot{m}\sqrt{T_{01}}}{p_{01}} \cdot \frac{p_{01}}{p_{02}}\sqrt{\frac{T_{02}}{T_{01}}}$$

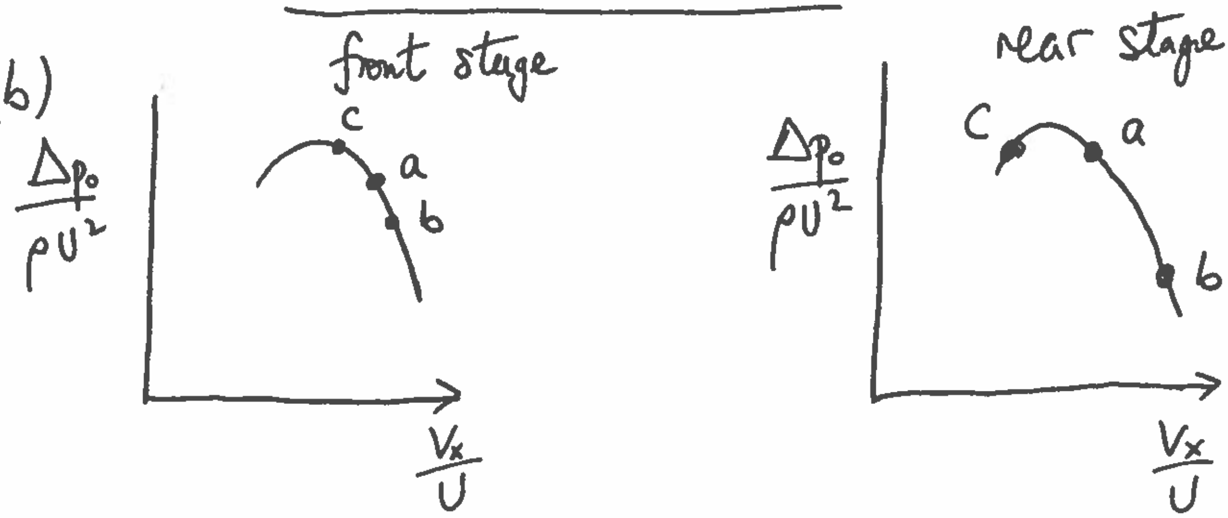
Since $\frac{p_{02}}{p_{01}} = \left(\frac{T_{02}}{T_{01}}\right)^{\frac{\eta_p \gamma}{\gamma-1}} \simeq \left(\frac{T_{02}}{T_{01}}\right)^{7}$ changes in $\frac{p_{02}}{p_{01}}$ are far more important than changes in temperature.



A <u>reduction</u> in $\dot{m}$ from a to b results in an <u>increase</u> in $\frac{p_{02}}{p_{01}}$

$\Rightarrow$ $\frac{\dot{m}\sqrt{T_{02}}}{p_{02}}$ reduces by more than $\frac{\dot{m}\sqrt{T_{01}}}{p_{01}}$

(b)

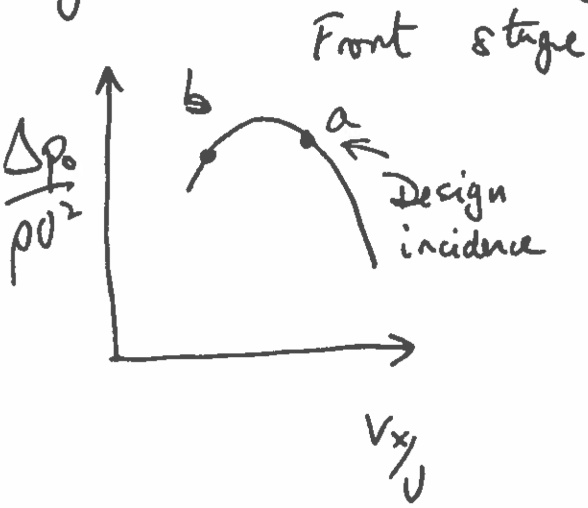


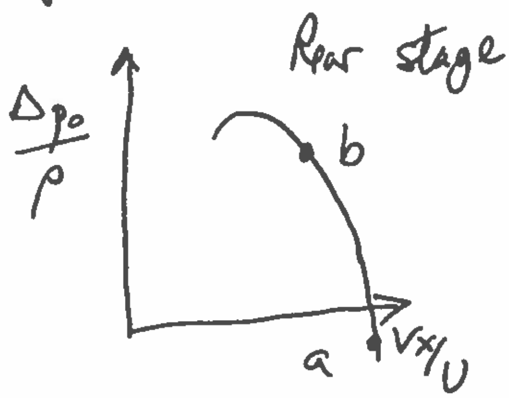
Because changes in flow fn (or $\frac{V_x}{U}$) are amplified through the compressor rear stages are driven towards choking by a small increase in compressor mass flow.

A small reduction in mass flow to the compressor pushes the rear stage towards stall.

(c) At design speed, as described in (b), it is the rear stages which are likely to stall first.

At part speed, $\frac{\Delta p_0}{\rho U^2}$ for each stage is reduced because of the fall in U. This means that increase in density across each stage is less than that for which the annulus contraction is designed and the flow corresponding to design incidence for the first stage is higher than the choking value for the rear stage (point a)





The rear stage sets the mass flow (point b) and the front stage is driven into stall

(d) • Front stages redesigned to operate at more negative incidence than optimal and rear stages at more positive.

Means reduction in peak η at design

• Vaiable inlet guide vanes & stators fitted to front stages.

Increases weight, cost & mechanical complexity. Impact possible on η because of difficulty sealing variable blades

• Split compressor into two.

Mechanical complexity of concentric shafts

But shorter, stiffer compression system maintains performance better over life.

Assessor's Comments:

14 attempts, mean 62%, standard deviation 21%, max. 18/20, min. 5/20.
A straight forward question, which required the students to work through the behaviour of multistage compressors. A fair amount of guessing was evident in the answers.