Q1) Blade force has to balance the rate of
a)
change of momentum. For a given turning at a given mass flow rate higher pitch to chord gives a higher required blade force.

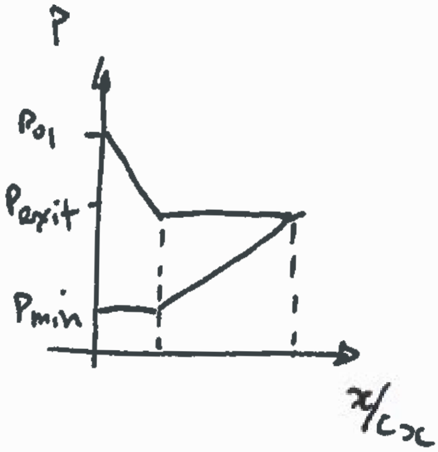
To match the higher blade force, the average velocity difference between the PS and SS must be higher, which leads to more diffusion as the peak velocity must diffuse back to match the exit conditions.

Blade shape not known at pre-lim design, so we use correlations based on angles only, such as Lieblein's.

+ Wetted area to BL loss trade.

Q1) b)

i)



$$\frac{F_\theta}{h C_x} = \frac{1}{3}\left\{(P_0 + P_{exit})\frac{1}{2} - p_{min}\right\} + \frac{2}{3}\left\{P_{exit} - (P_{exit} + p_{min})\frac{1}{2}\right\}$$

$$= \frac{1}{6}P_0 + \frac{1}{6}P_{exit} - \frac{2}{6}p_{min} + \frac{4}{6}P_{exit} - \frac{2}{6}P_{exit} - \frac{2}{6}p_{min}$$

$$= \frac{1}{6}P_0 + \frac{3}{6}p_{exit} - \frac{4}{6}p_{min}$$

$$= \frac{1}{6}P_0 - \frac{3}{6}(P_0 - P_{exit}) + \frac{3}{6}P_0 + \frac{4}{6}(P_0 - P_{min}) - \frac{4}{6}P_0$$

$$= \cancel{\frac{(1+3-4)}{6}}^{\,0} P_0 + \frac{2}{3}(P_0 - P_{min}) - \frac{1}{2}(P_0 - p_{exit})$$

$$= \frac{\cancel{2}}{3}\cdot\frac{1}{\cancel{2}}\rho V_{max}^2 - \frac{1}{2}\cdot\frac{1}{2}\rho V_{exit}^2$$

$$= \rho V_{exit}^2\left(\frac{1}{3}\frac{V_{max}^2}{V_{exit}^2} - \frac{1}{4}\right)$$

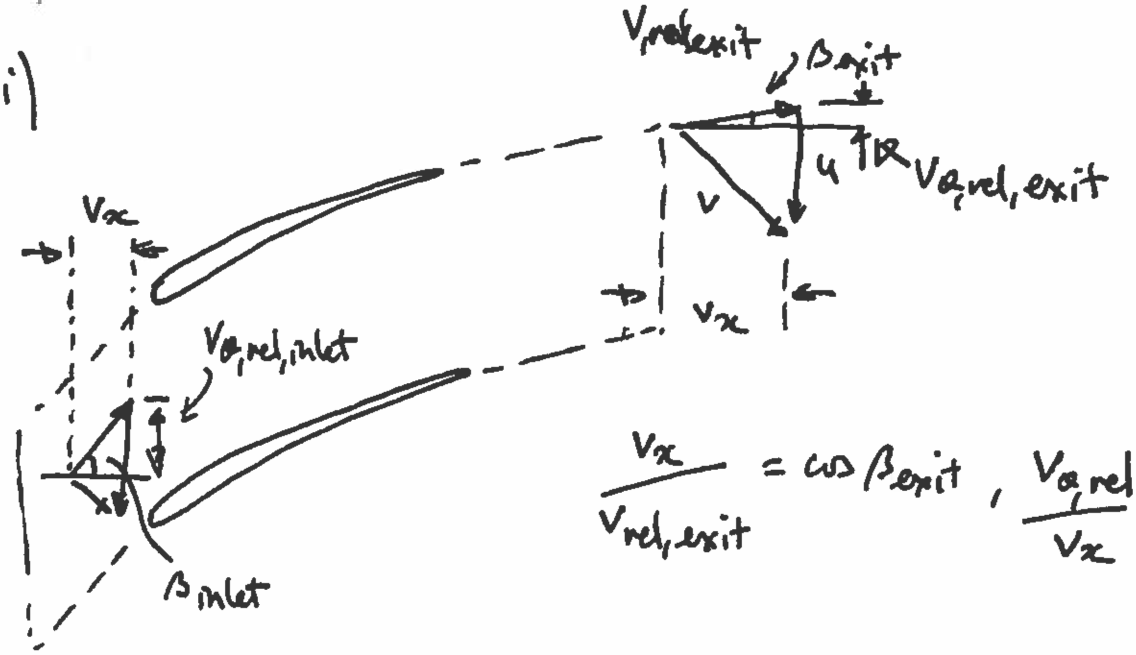
$$\frac{F_\theta}{h} = \rho V_{exit}^2\left(\frac{1}{3}\frac{1}{(1-DF)^2} - \frac{1}{4}\right)C_x$$

$$DF = \frac{V_{max} - V_{exit}}{V_{max}}$$

$$= 1 - \frac{V_{exit}}{V_{max}}$$

$$\Rightarrow \frac{V_{max}}{V_{exit}} = \frac{1}{1-DF}$$

ii)



$$\frac{V_x}{V_{rel,exit}} = \cos\beta_{exit}\ , \quad \frac{V_{\theta,rel}}{V_x} = \tan\beta$$

$$\frac{F_\theta}{h} = \frac{\dot{m}}{h}\left(V_{\theta,rel,exit} - V_{\theta,rel,inlet}\right)$$

$$= \rho \frac{V_x p \cancel{h}}{\cancel{h}}\left(V_x \tan\beta_{exit} - V_x \tan\beta_{inlet}\right)$$

$$= \rho V_x^2\, p\left(\tan\beta_{exit} - \tan\beta_{inlet}\right)$$

Equate!
(b)(i)

$$\cancel{\rho} V_{exit}^2\, \Pi\, c_x = \cancel{\rho} V_x^2\, p\left(\quad\right)$$

$$\frac{p}{c_x} = \frac{V_{exit}^2}{V_x^2}\cdot\frac{\Pi}{(\quad)}$$

$$\frac{p}{c_x} = \frac{1}{\cos^2\beta_{exit}}\cdot\frac{\Pi}{(\quad)} \quad // \quad \text{where } \frac{\Pi}{(\ )} = \frac{\frac{1}{3}\frac{1}{(1-DF)^2} - \frac{1}{4}}{\tan\beta_{exit} - \tan\beta_{inlet}}$$

c)

Treating blade passage a 1-D

$$\frac{dV}{V} = -\frac{1}{1-M^2}\frac{dA}{A}$$

for a given area change (i.e. passage shape)

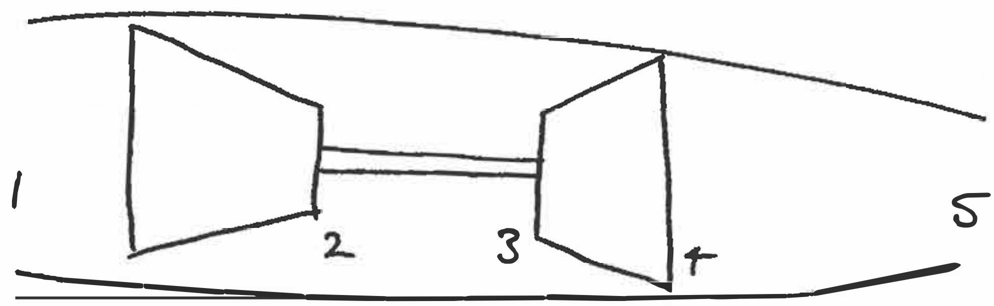
the velocity change increases with increasing

Mach No. so DF goes up as well.

Q1 Compressor blade diffusion factor - 10 Part IIB attempts

Not a popular question, however it was well answered by those who attempted it. For part (a) most students gave good answers, and most were able to evaluate the forces correctly in part (b). Most students were then able to apply the result of part (b) in part (c) spotting that they needed to use a control volume. Part (c) was less well answered with several straight guesses erroneously using the deviation versus exit Mach number graph from the notes.

Q2)



$P_{01} = 101\,kPa$

$T_{01} = 288K$

$\eta_{PC} = .85 \quad \eta_{PT} = .9 \quad \left.\frac{P_{02}}{P_{01}}\right|_{DES} = 6$

$T_{03}^{DES} = 1200K$

(a) Turbine choked and final nozzle choked

$$\Rightarrow \frac{\dot{m}\sqrt{c_{pe}T_{03}}}{P_{03}A_3} = \frac{\dot{m}\sqrt{c_{pe}T_{05}}}{P_{05}A_5} = \underset{\gamma=1.3}{f(1)}$$

$T_{05} = T_{04}$ (no work) & $P_{05} = P_{04}$ (losses in nozzle before throat negligible)

$$\therefore \frac{P_{03}}{P_{04}} = \left(\frac{A_5}{A_3}\right)\left(\frac{T_{03}}{T_{04}}\right)^{1/2} = \left(\frac{T_{03}}{T_{04}}\right)^{\frac{\gamma_e}{\eta_{PT}(\gamma_e-1)}}$$

$$\Rightarrow \frac{T_{04}}{T_{03}} = \left(\frac{A_3}{A_5}\right)^{\frac{1}{\frac{\gamma_e}{\eta_{PT}(\gamma_e-1)}-\frac{1}{2}}} = \left(\frac{A_3}{A_5}\right)^{.232} = \text{Const}$$

and $$\frac{P_{04}}{P_{03}} = \left(\frac{A_3}{A_5}\right)^{1.116}$$

Work balance for compressor & turbine (neglecting $\dot{m}_{fuel}$)

$$\Rightarrow \quad c_p(T_{02} - T_{01}) = c_{pe}(T_{03} - T_{04})$$

$$\Rightarrow \quad \frac{T_{02}}{T_{01}} = 1 + \frac{C_{pe}}{C_p}\frac{T_{03}}{T_{01}} \underbrace{\left(1 - \frac{T_{04}}{T_{03}}\right)}$$

$$k = \text{const} \quad \text{since} \quad \frac{T_{04}}{T_{03}} = \text{const}$$

$$\therefore \quad \frac{p_{02}}{p_{01}} = \left(\frac{T_{02}}{T_{01}}\right)^{\eta_{pc}\frac{\gamma}{\gamma-1}} = \left[1 + k\,\frac{C_{pe}}{C_p}\,\frac{T_{03}}{T_{01}}\right]^{\eta_{pc}\frac{\gamma}{\gamma-1}}$$

At design $\quad \frac{p_{02}}{p_{01}} = 6 \qquad \frac{T_{03}}{T_{01}} = \frac{1200}{288}$

$$\therefore \quad 6 = \left[1 + k\,\frac{1.244 \times 1200}{1005 \times 288}\right]^{\frac{1.4 \times .85}{.4}}$$

$$\Rightarrow \quad k = \frac{1005 \times 288}{1244 \times 1200}\left[6^{\frac{.4}{1.4 \times .85}} - 1\right] = .160$$

(b) $\quad k = .160 \quad \Rightarrow \quad \frac{T_{04}}{T_{03}} = .840 \quad \Rightarrow \quad \frac{A_3}{A_5} = (.840)^{\frac{1}{.232}}$

$$\text{i.e.} \quad \frac{A_3}{A_5} = .471$$

(c) If A_3 reduced by 5% $\quad \Rightarrow \quad \frac{A_3^{new}}{A_5} = .447$

$$\Rightarrow \quad \left.\frac{T_{04}}{T_{03}}\right)_{new} = .830 \quad \Rightarrow \quad k_{new} = .170$$

Hence $\frac{p_{02}}{p_{01}} = 6.507 \Rightarrow 8.5\%$ increase

$$\frac{\dot m^{new}\sqrt{c_{p_e}T_{03}}}{p_{03}^{new}A_3^{new}} = \frac{\dot m\sqrt{c_{p_e}T_{03}}}{p_{03}A_3} \Rightarrow \frac{\dot m^{new}}{\dot m} = \frac{A_3^{new}}{A_3}\frac{p_{03}^{new}}{p_{03}}$$

If combustion loss is negligible

$$\frac{\dot m_{new}}{\dot m} = .95\frac{p_{02}^{new}}{p_{02}} = .95\times\frac{6.507}{6} = 1.03$$

i.e. 3% increase

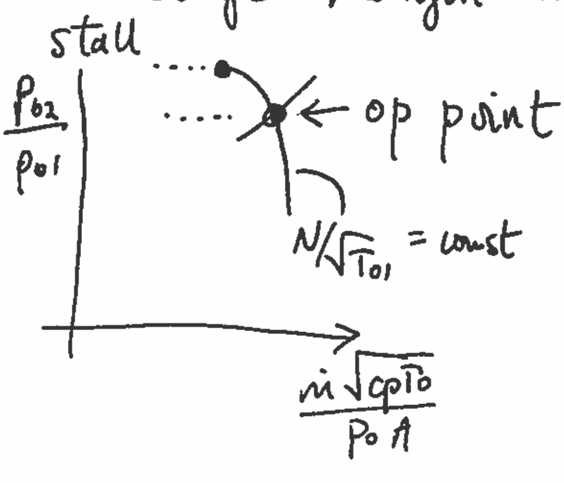
$$\frac{\dot m_f^{new}}{\dot m_f} = \frac{\dot m_{new}(T_{03}-T_{02}^{new})}{\dot m(T_{03}-T_{02})}$$

$$\frac{T_{02}}{T_{01}} = 6^{\frac{.4}{1.4\times.85}} = 1.826 \Rightarrow T_{02} = 526 \text{ K}$$

$$\frac{T_{02}^{new}}{T_{01}} = 6.507^{\frac{.4}{.85\times1.4}} = 1.877 \Rightarrow T_{02}^{new} = 540.5 \text{ K}$$

$$\therefore \frac{\dot m_f^{new}}{\dot m_f} = 1.03\frac{(1200-540.5)}{1200-526} = 1.008 \Rightarrow .8\% \text{ increase}$$

(d) Surge margin measures the "distance" that the compressor operating point is from stall at this speed.



It is necessary because of: engine-to-engine manufacturing tolerances; in-service deterioration which tends to raise the working line and reduce the stall pressure ratio; to allow for engine acceleration, when the compressor op point moves above the steady-state operating line. Some allowance is also made for the effect of ~~inlet~~ intake flow separation.

In practice, raising the pressure ratio by reducing the turbine nozzle area is likely to drive the compressor operating point to one with reduced efficiency and which is closer to the stall line.

Q2 Effect of volcanic ash on stall margin - 28 Part IIB attempts

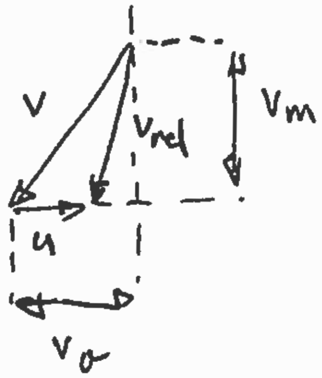
A very popular question attempted by all but 1 IIB candidate. Most could derive the *"show that"* but many simply forgot to evaluate the constant. This made it harder to do part (b), which was poorly answered. Most candidates made a good attempt at part (c) but algebra slips were common, either getting the exponent or the areas in the continuity match upside down. In part (d) a lot of students gave long winded answers describing the entire stall-surge cycle, rather than focussing on the basic and practical reasons why surge margin is required. When talking about the validity of the constant polytropic efficiency assumption, many candidates resorted to stock answers about part speed operation rather than the simple answer involving the change in pressure and mass flow, which was all that was required. During the exam, a candidate asked if the changes in mass flow of air and fuel in part of (c) should to be worked out together or separately. They were simply told to decide for themselves, to avoid giving a hint.

Q3)

a) Euler Equation: $h_0 - UV_\theta = \text{const.}$ (on a stream surface)

$$h + \frac{V^2}{2} - UV_\theta = \text{const.}$$

General velocity triangle.



$$V^2 = V_\theta^2 + V_m^2$$

$$V_{rel}^2 = (V_\theta - U)^2 + V_m^2$$

$$V_{rel}^2 = (V_\theta - U)^2 + V^2 - V_\theta^2$$

$$= \cancel{V_\theta^2} - 2UV_\theta + U^2 + V^2 - \cancel{V_\theta^2}$$

so $2UV_\theta = U^2 + V^2 - V_{rel}^2$

$$UV_\theta = \frac{1}{2}\left(U^2 - V_{rel}^2 + V^2\right)$$

so $h_0 - UV_\theta = h_0 - \frac{1}{2}\left(U^2 - V_{rel}^2 + V^2\right) = \text{const.}$

so $h_{02} - h_{01} = \frac{1}{2}\left(U_2^2 - U_1^2\right) - \frac{1}{2}\left(V_{2,rel}^2 - V_{1,rel}^2\right) + \frac{1}{2}\left(V_2^2 - V_1^2\right)$ //

Q3) a) cont.

$$\Delta h_0 = \tfrac{1}{2}(U_2^2 - U_1^2) - \tfrac{1}{2}(V_{2,rel}^2 - V_{1,rel}^2) + \tfrac{1}{2}(V_2^2 - V_1^2)$$

This can be written in terms of static enthalpy.

$$\Delta h_0 = h + \tfrac{1}{2}(V_2^2 - V_1^2)$$ ← This term is just the increase in abs. velocity. It needs to be recovered in a diffuser.

Static enthalpy rise comes from these two:

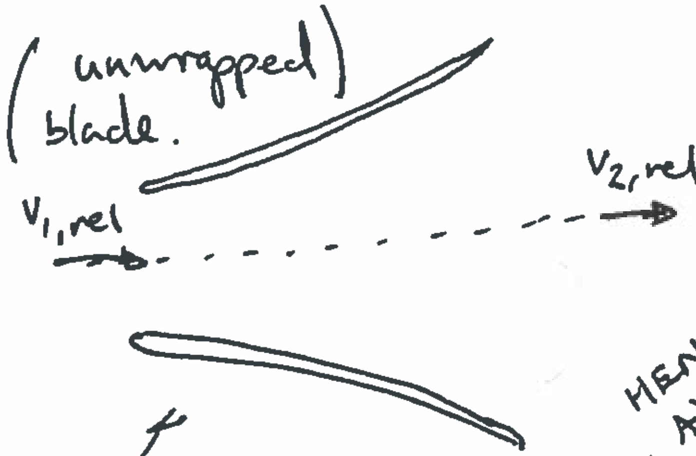
axials - not much of this

axials rely on this.

$$\tfrac{1}{2}(U_2^2 - U_1^2) \quad \text{and} \quad -\tfrac{1}{2}(V_{2,rel}^2 - V_{1,rel}^2)$$

This term just comes from the change in radius.

(no limit from BL.)



[30%]

This term is the diffusion in relative frame. Limited by BL separation.

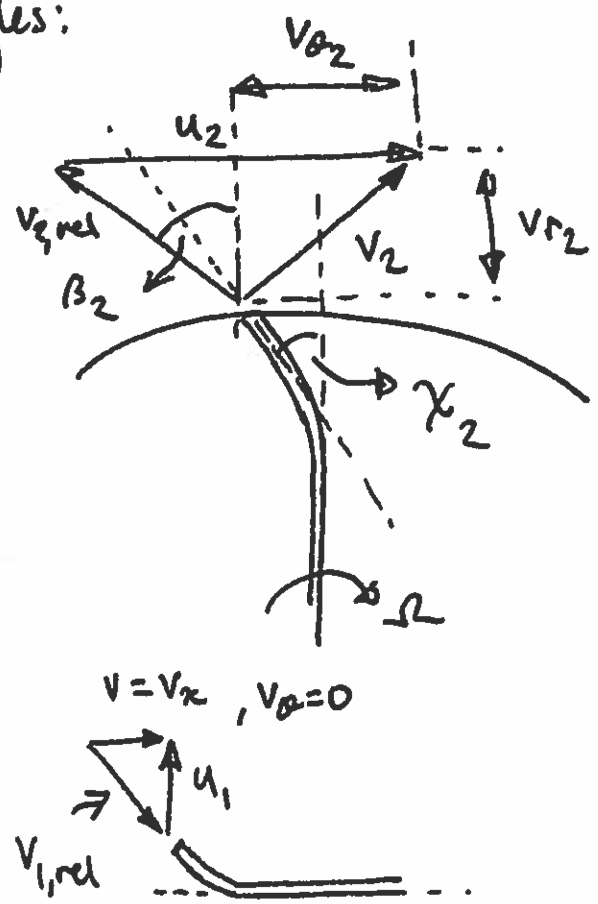
HENCE AXIALS CAN'T ACHIEVE HIGH PR PER STAGE

Q3) b)

Velocity triangles:

EXIT:

INLET:



$$\sigma = \frac{V_{\theta_2}}{u_2 + \tan\chi_2 V_{r_2}}$$

b) ii)

$$\dot{W}_x = \dot{m}\Delta h_0 = \dot{m}\frac{c_p T_{01}}{\eta}\left(\frac{T_{02s}}{T_{01}} - 1\right)$$

$$\frac{\dot{W}}{\dot{m}} = \Delta h_0 = \frac{\cancel{\dot{m}}}{\cancel{\dot{m}}}\frac{c_p T_{01}}{\eta}\left(PR^{\frac{\gamma-1}{\gamma}} - 1\right)$$

$$\therefore \Delta h_0 = \frac{c_p T_{01}}{\eta}\left(PR^{\frac{\gamma-1}{\gamma}} - 1\right) \quad //$$

$$\Delta h_0 = U\Delta V_\theta = U_2 V_{\theta_2} \qquad \text{NB. where } \chi = -ve$$

$$= r_2\Omega \cdot \sigma\left(r_2\Omega + V_{r_2}\tan\chi\right)$$

$$V_{r_2} = \frac{\dot{m}}{\rho_2 \cancel{\pi}\, 2\pi r_2 h}$$

$$\Delta h_0 = r_2\Omega \cdot \sigma\left(r_2\Omega + \frac{\dot{m}}{\rho_2 2\pi r_2 h}\cdot\tan\chi\right) \quad //$$

$$\frac{\Delta h_0}{\sigma} = r_2^2\Omega^2 + \frac{\dot{m}\,\Omega\tan\chi}{\rho_2 2\pi h}$$

$$r_2 = \frac{1}{\Omega}\sqrt{\frac{\Delta h_0}{\sigma} - c} \quad \text{where } c = \frac{\dot{m}\,\Omega\tan\chi}{\rho_2 2\pi h} \quad //$$

b ii) cont.

$\dot{m} = 7.836\ \mathrm{kg\,s^{-1}}$

$\mathrm{rpm} = 20{,}000\ \mathrm{rpm} \Rightarrow \omega = 2.0944\times10^{3}\ \mathrm{rad\,s^{-1}}$

$T_{01} = 300\mathrm{K}$

$\left.\begin{array}{l} P_{01} = 1\times10^{5}\ \mathrm{Pa} \\ P_{02} = 3\times10^{5}\ \mathrm{Pa} \end{array}\right\} \quad PR = 3$

$h = 0.01\ \mathrm{m}$

$\chi = -45^{\circ}$

$\eta_{t-t} = 0.9$

$\sigma = 0.9$

$R = 287.1\ \mathrm{J\,kg^{-1}\,K^{-1}}$

$c_p = 1005\ \mathrm{J\,kg^{-1}\,K^{-1}}$

given given..

$= c_p(1 - 1/\gamma)$

$$T_{02} = T_{01}\left(1 + \frac{\left(PR^{\gamma-1/\gamma} - 1\right)}{\eta}\right) = 422.91\ \mathrm{K}$$

$$\rho_{02} = \frac{P_{02}}{R T_{02}} = 2.471$$

$$P/P_0 = 0.6339 \ @ \ M=1$$

$$\rho_2 = 0.6339 \times 2.471 = 1.5663$$

$$C = \dot{m}\,\omega \tan(\chi) \Big/ \rho_2 2\pi h$$

$$r_2 = \frac{1}{\omega}\sqrt{\frac{c_p T_{01}}{\sigma\eta\left(PR^{\gamma-1/\gamma} - 1\right)} - C} = 0.263\ \mathrm{m}$$

Aliter.

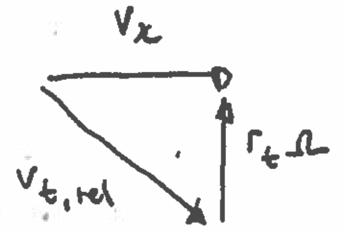
$$r_2 = \frac{\dot{m}\sqrt{c_p T_{01}}}{2\pi\, \rho_{02} \cos(\alpha_2)\, f_{\dot{m}}(M=1)}$$

$$r_2 = \frac{V_2}{\Omega}\left(\frac{\sin(\alpha_2)}{\sigma} - \cos(\alpha_2)\tan\ldots\right.$$

iterate $\alpha_2 = 36.5^{\circ}$

$r_2 = 0.263\ \mathrm{m}$

b) iii)



$$V_{t,rel}^2 = V_x^2 + r_t^2\Omega^2$$

$$V_x = \frac{\dot m}{\rho\pi(r_t^2 - r_h^2)} \Rightarrow V_{t,rel}^2 = \left(\frac{\dot m}{\rho\pi(r_t^2-r_h^2)}\right)^2 + r_t^2\Omega^2$$

As ρ & T vary slowly so minimise $V_{t,rel}$ wrt r_t

$$\frac{\partial V_{t,rel}^2}{\partial r_t} = \left(\frac{\dot m}{\rho\pi\Omega}\right)^2 \frac{-2\cdot\cancel{2}\cancel{r_t}}{(r_t^2-r_h^2)^3} + \cancel{2r_t}\cancel{\Omega^2} = 0$$

$$\Rightarrow \left(\frac{\dot m}{\rho\pi\Omega}\right)^2 2 = (r_t^2 - r_h^2)^3$$

$$\text{so } r_t^2 = \left[\left(\frac{\dot m}{\rho\pi\Omega}\right)^2 \cdot 2\right]^{\frac{1}{3}} + r_h^2$$

$$r_t = \sqrt{\left\{\left(\frac{\dot m}{\rho\pi\Omega}\right)^{\frac{2}{3}} 2^{\frac{1}{3}} + r_h^2\right\}}$$

This avoids choking, separations and loss due to shocks.

Q3 Radial compressor - 20 IIB attempts

Almost all could do the manipulation to derive the *"show that"* in part (a), but only a few candidates were able to spot that it was the difference in relative frame velocity change that distinguished the two types of machine (in addition to the more obvious change in radius). Part (b) had enough information to solve directly; a few candidates spotted this, whilst a few made impressive attempts and solved it iteratively by an alternate route. The last part was well answered except were the students had clearly run out of time.