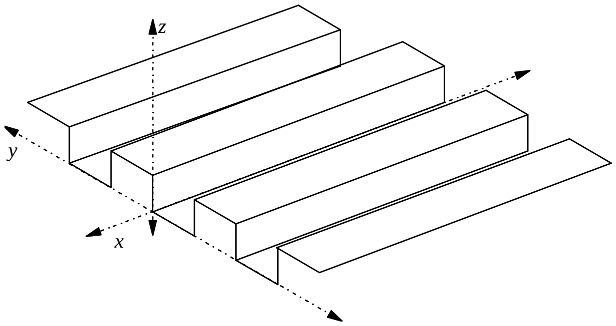
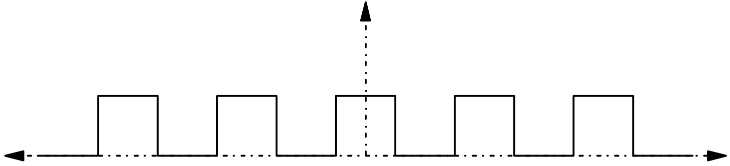
2017 4B11 Exam crib

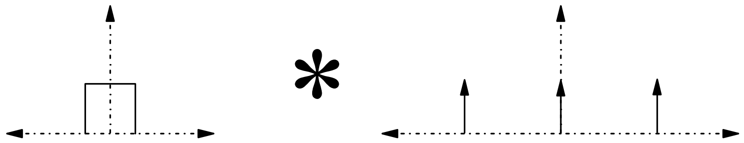
Q1 a) [35%] The exact structure of the far field depends on the shape of the ‘fundamental’ pixel and the number and distribution of these pixels in the grating. The pattern we generate with this distribution of pixels is repeated in each lobe of the sinc function from the fundamental pixel. The lobes can be considered as spatial harmonics of the central lobe, which contains the desired 2-D pattern. If the fundamental pixel goes to infinity then the replay field will be delta functions, if the grating has a periodic structure, then the replay filed will have Fourier harmonics. A good example is the 2-D grating or square wave.



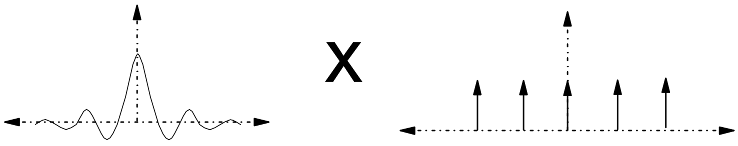
If viewed from the end (as a 1-D function).



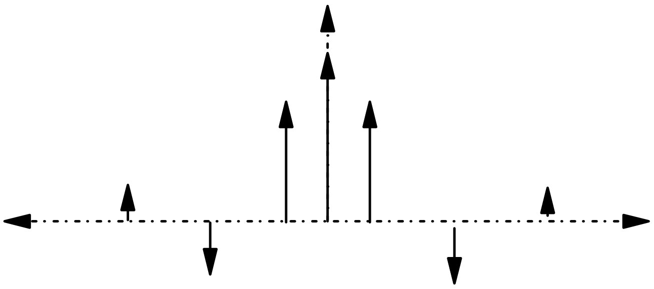
Which can be expressed as a convolution of two functions.



After the Fourier transform the convolution becomes a multiplication of the Fourier transforms of each function..

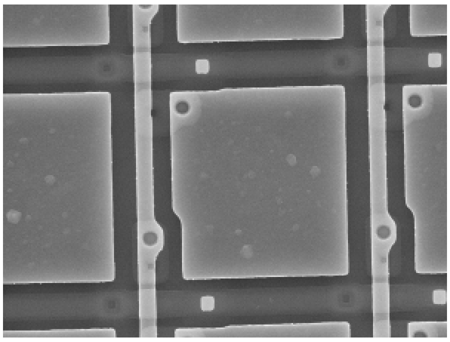


Gives the final result.

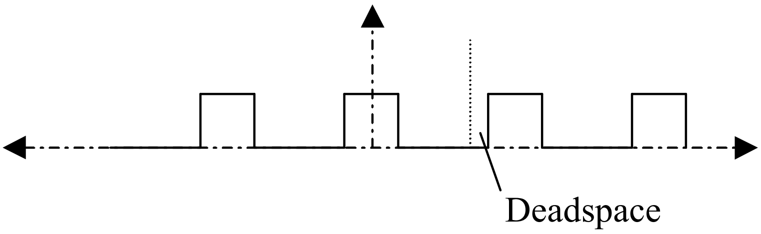


The train of delta functions has a sinc envelope and every second delta function is suppressed by the zeros of the sinc envelope. Assumptions: Infinite aperture, no aberrations.

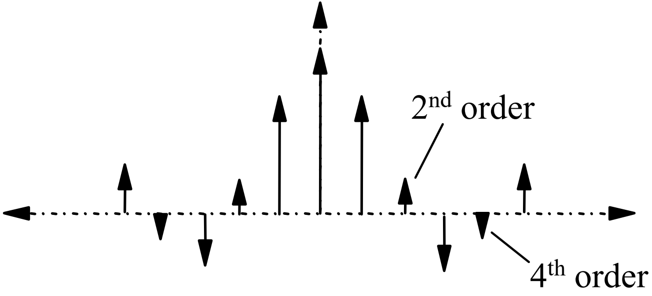
b) [25%] The key to LCoS is the silicon backplane which drives the LC pixels. As a result of this active backplane there will always be deadspace to isolate the pixels.



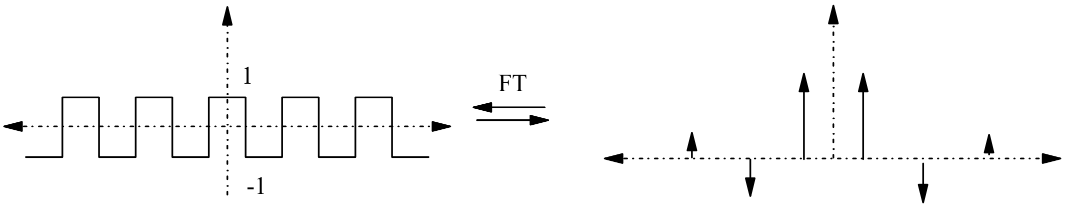
In part (a) we assumed that the pixel pitch matched the finite width of the pixels themselves. This is not the case in reality as each pixel must have a gap between itself and its neighbour to prevent short circuits and to allow for transistors etc to be used to drive the pixels. This region between the pixels is referred to as the deadspace and the ratio of pixel size to pixel pitch is called the fill factor.



When we calculate the replay field of the grating with deadspace, we can no longer assume that the zeros of the sinc envelope due to the fundamental pixel shape will be in the same spatial frequencies as the delta functions due to the repetition of the pixels, hence the second order is no longer suppressed by the sinc envelope. This can lead to unwanted noise and crosstalk in the replay field.



c) [20%] When a hologram is designed with binary phase modulation, it is possible to suppress the zero order by having the same number of +1 as -1 phase states.



When the modulator has deadspace this is not guaranteed so there may be some form of imbalance in the phase states creating an unwanted zero order. The root of this problem is that the phase state of the deadspace is unknown or may depend on the state of the pixels either side.

The problem could possibly be avoided if the state of the deadspace was known or in some way related to the pattern on the SLM. This could be done by either a measurement of the SLM deadspace states or through clever design of the pixels to make sure the deadspace phase state is fixed. If the modulation state of the deadspace is known then it can be put into the hologram design algorithm and compensated for by the other phase states of the hologram pixels.

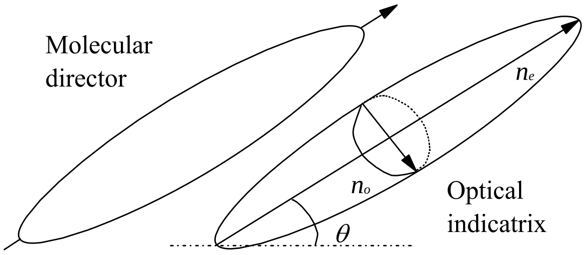
d) [20%] One of the most fundamental characteristics of an optical interconnect is its optical crosstalk. If the switch is configured to route light to the *k*th fibre in an array of *n*, then the crosstalk is the *ratio of light launched down the desired fibre to the light launched down one of the other fibres, which are not being routed.* If there is any significant unwanted orders in the replay field, then they will lead to crosstalk.

The effect of deadspace in a hologram replay field is the non suppression of the second order which also will contain other orders due to the multiple periodicities of the hologram. There could all end up launched down an unwanted fibre. This also applies to any extra zero order present which will also lead to significant increase in the crosstalk level. Both noise sources will effectively limit the number of low crosstalk positions in the replay field which limits the number of ports that can be switched.

The loss of light into unwanted orders due to deadspace also contributes the overall insertion loss of the optical switch.

[Well answered overall, a few forgot the delta train in a) and quite a few did not state assumptions. In b) most did the derivation ok, but quite a few said circuitry which would be under the pixel with LCoS. Not many got c) but most got the significance of the switch in d)]

Q2 a) [25%] The calimatic molecular shape leads to an optical anisotropy in nematic LCs, with the two axes of the molecule appearing as the refractive index. The refractive index along the long axis of the molecules is often referred to as the extraordinary n_e and the short axis the ordinary n_o axis. The difference between the two is the birefringence. $\Delta n = n_e - n_o$



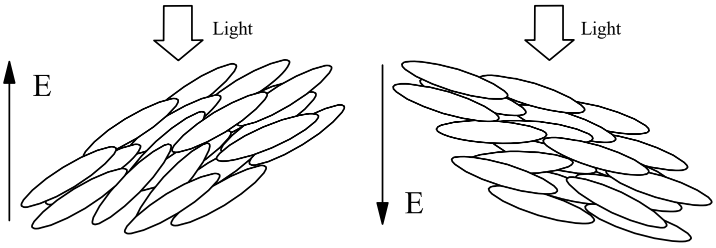
It is important to note that these parameters are bulk parameters which are based on a statistical average across billions of individual molecules. The bulk ordering of LCs such as nematics is not always inherent in the material beyond domains of molecules a few microns in size. The combination of the flow allowing the molecules to move when an electric field is applied and the optical anisotropy means that we can effectively rotate the axes of the indicatrix as the molecules move, creating a moveable wave plate or optical retarder. This along with polarising optics makes the basis of most liquid crystal intensity and phase modulation characteristics.

We can then calculate the retardance Γ of the liquid crystal layer for a given cell thickness *d* and wavelength λ. In the unswitched pixel, the molecules are aligned parallel to the cell electrodes and the electric field will see a refractive index n_e. When switched into the homeotropic state the LC molecules are perpendicular n_o the cell walls and the E field now sees no. If we compare the two pixels after passing through the layer of LC then one E field will have travelled faster than the other. As a result there will be a phase difference (or retardance) Γ between the pixels.

$$\Gamma = \frac{2\pi\ t}{\lambda}\left(n_e - n_o\right)$$

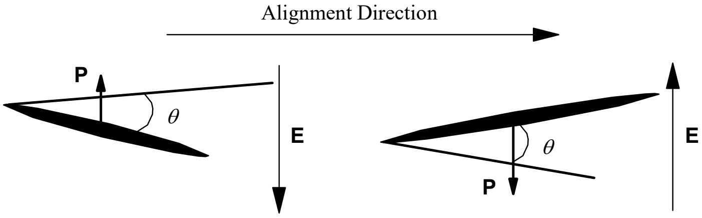
Assumptions: Monochromatic, coherent light, all the LC switches in the two states, The E field is aligned parallel to the long axis of the LC,

b) [35%] The application of optical switching in telecommunications has one major limitation when using liquid crystals as phase modulators in that we do not know the polarisation states of the incident light. The problem with nematic LCs is that the switching is out of the plane of the LC cell and requires the E field of the light to be parallel to the LC molecules which cannot be guaranteed.



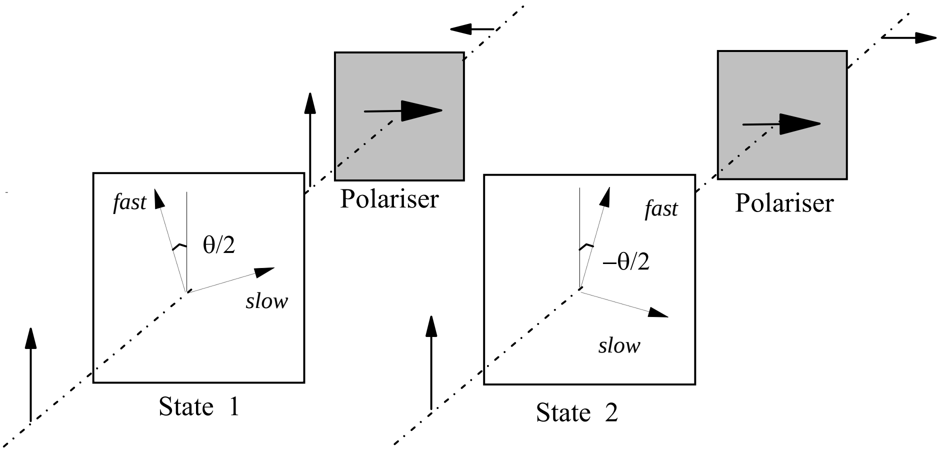
The out of plane switching is also limited by the viscosity of the LC which means that it is slow and will not be able to switch optical channels quickly (10-100msec).

FLCs are a more ordered LC phase and therefore fast to switch. If the FLC is restricted to a cell thickness of 2-5μm then the the molecules are bounded into two stable states either side of the director cone. This is referred to as a surface stabilised FLC geometry an creates a large birefringent electro-optical effect which is now switching in the lead of the plane of the cell. The penalty for doing this is that the molecules are only stable in the two states and therefore the modulation will only be binary. The up side to this binary modulation is that it can be very fast (~10μsec).



The main benefit of in plane LC switching is that it is inherently polarisation insensitive therefore will modulate any input E field in the same way which is ideally suited to optical switching applications there the polarisation of the incident light is unknown.

c) [35%] The input light should polarised so that its E field bisects the switching angle θ of the FLC and an analyser is placed after the pixel at 90º to the input light. If we start with vertically polarised light, then the FLC pixel fast axis positions must bisect the vertical axis and will be oriented at angles of $\theta/2$ and $-\theta/2$ respectively for each state.



State 1.

$$\begin{pmatrix}V_x'\\V_y'\end{pmatrix}=\begin{pmatrix}1&0\\0&0\end{pmatrix}\begin{pmatrix} e^{-j\Gamma/2}\cos^2\frac{\theta}{2}+e^{j\Gamma/2}\sin^2\frac{\theta}{2} & -j\sin\frac{\Gamma}{2}\sin(\theta)\\ -j\sin\frac{\Gamma}{2}\sin(\theta) & e^{j\Gamma/2}\cos^2\frac{\theta}{2}+e^{-j\Gamma/2}\sin^2\frac{\theta}{2}\end{pmatrix}\begin{pmatrix}0\\V_y\end{pmatrix}$$

$$=\begin{pmatrix}-V_y\,j\sin\frac{\Gamma}{2}\sin(\theta)\\0\end{pmatrix}$$

State 2

$$\begin{pmatrix} V_x^{'} \\ V_y^{'} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} e^{-j\Gamma/2}\cos^2\frac{\theta}{2} + e^{j\Gamma/2}\sin^2\frac{\theta}{2} & j\sin\frac{\Gamma}{2}\sin(\theta) \\ j\sin\frac{\Gamma}{2}\sin(\theta) & e^{j\Gamma/2}\cos^2\frac{\theta}{2} + e^{-j\Gamma/2}\sin^2\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} 0 \\ V_y \end{pmatrix}$$

$$= \begin{pmatrix} V_y\, j\sin\frac{\Gamma}{2}\sin(\theta) \\ 0 \end{pmatrix}$$

From these two expression we can see that the difference between the two states is just the minus sign, which means that the light has been modulated by 180º (π phase modulation). Moreover, the phase modulation is independent of the switching angle θ and the retardation Γ.

d) [15%] If the take the two states derived above we and calculate the transmission of the light through both states, The FLC parameters only effect the loss in transmission through the pixel which can be gained by squaring the above Jones matrix expressions.

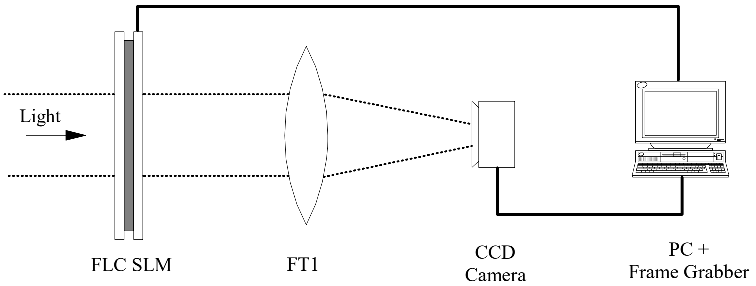
$$T = V_y^2 \sin^2(\theta)\sin^2\left(\frac{\Gamma}{2}\right)$$

Hence maximum transmission (and therefore minimum loss) occurs when $\Gamma = \pi$ and $\theta = \pi/2$. The retardance Γ can be controlled through cell gap (avoiding loss of SSFLC effects) and birefringence. The switching angle is more difficult to engineer as it arises directly from the materials properties of the LC and there are only a few materials that have such a high value and they are very difficult to keep in planar alignment.

[Popular question, but poorly answered. Part a) only 2 candidates actually derived the retardance rather than just quoting it. Part b) was ok, but a few omitted polarisation sensitivity. Part c) was mostly just quoted from the notes and it was surprising how many could not multiply matrices. In d) several did not square the expression but still got the right parameters]

Q3 a) [30%] In order to perform a correlation, we need to multiply the FT of *r(x,y)* and *s(x,y)* in the Fourier domain before the final FT creates the correlation of the original images.

$$s(x,y) * r(x,y) = F_T\left[F_T[s(x,y)]F_T[r(x,y)]\right]$$



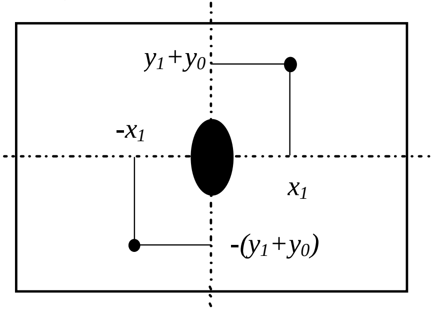
In the input pass, plane 1 is on the FLC SLM, the input *s(x,y)* and reference *r(x,y)* are displayed side by side in an optical system and then transformed by a single lens into the CCD camera.

$$S(u,v)e^{-j2\pi(x_1u - y_1v)} + R(u,v)e^{-j2\pi y_0 v}$$

The nonlinearity due to the intensity capture of the CCD creates the correlation and in its simplest approximate form can be modeled by a square law detector which takes the magnitude squared of the light falling upon it.

$$S^2(u,v) + R^2(u,v) + S(u,v)R(u,v)e^{-j2\pi\left(x_1u - \left(y_0 + y_1\right)v\right)} + S(u,v)R(u,v)e^{-j2\pi\left(-x_1u + \left(y_0 + y_1\right)v\right)}$$

This is the joint power spectrum which is then redisplayed on the input SLM again. This is the FT'ed again by the lens and then captured again by the CCD. The final plane is after the second FT, with the central DC terms proportional to FT $[R^2 + S^2]$ and the two symmetrical correlation peaks spaced by $(x_1\ y_1+y_0)$ and $(-x_1, -(y_1+y_0))$.



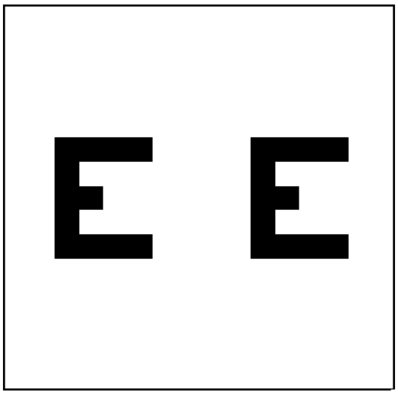
The central DC term is an unwanted source of noise, which degrades the optical system. There are always a symmetric pair of correlation peaks, so only half the output plane needs to be considered but the position of the correlation peak has to be decoded to gain the position of the reference object in *r(x,y)* if it correlates with *s(x,y)*.

Assumptions: Lenses do a simple FT, square law nonlinearity, modulation of SLM is for the full JPS.

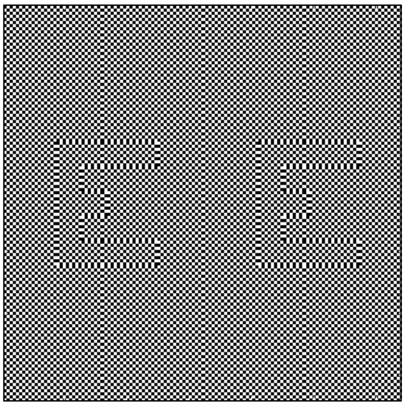
b) [30%] The type of applications where correlation are used nearly always require high speed operation, hence nematics are not ideal for the SLM and FLCs are preferred even though they are limited to binary modulation. The input and reference images are displayed side by side on a FLC SLM and is illuminated by a collimated laser beam and the images are Fourier transformed by a single lens in its focal plane. This spectrum is then imaged onto a CCD camera. The spectrum is then non-linearly processed before being displayed onto the SLM again to form the correlation information.

The quality of the correlation peaks and the zero order can be improved by non-linearly processing the spectrum which suits FLC SLMs as they are binary. A binarised spectrum produces good sharp correlation peaks and reduced zero order. If the binarised spectrum is converted to binary phase modulation [-1,+1], then the zero order can be reduced to around the height of the correlation peaks.

The problem is that the input or first pass displays the input images which are not suited to binary phase. The resulting spectrum has a huge dynamic range surpassing the available 8 bits of the CCD array and saturating the camera. To reduce the effects of the dynamic range, the input is multiplied pixel by pixel by aa alternate pixel binary phase chequerboard before displaying on the SLM.



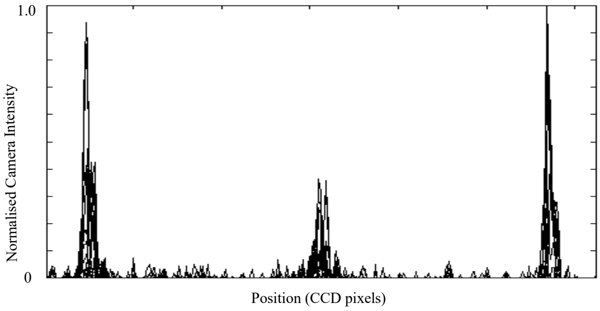
(a)



(b)

The multiplication of the input plane by the chequerboard ensures that the same number of -1 and +1 states (half of each) are always present in the input, independent of reference and input images. Hence, there will be less zero order present in the spectrum of the input plane. The result of the chequerboard modulation is to replicate the Fourier transform into the corners of the spectrum, which means that the dynamic range of the intensity sent into the corners will be reduced.

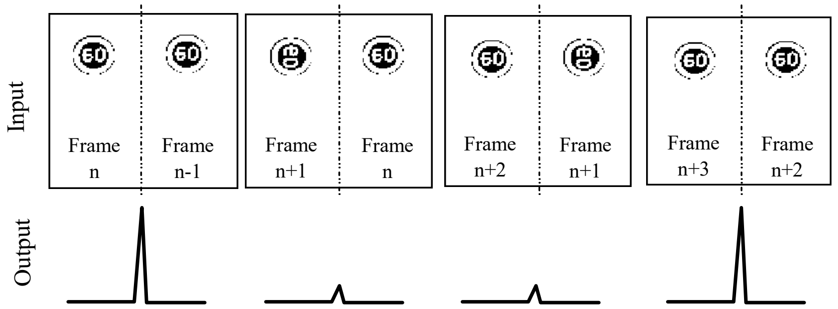
A variety of processing schemes can be used on the JPS as it is sampled from the CCD before being resent to the SLM (which is binary). A 3x3 convolution binarisation scheme works well, but the processing time can be slow due to the complexity of the filtering algorithm. The success of this algorithm is due to fact that the 3x3 convolution is a form of edge enhancement, which enhances the spectrum and the noise in the dark areas of the JPS and maintains the binary phase balance.



A major benefit of the 1/f JTC is that the non linearity can be programmed as part of the processing algorithm and hence it can be tuned to suit particular applications and input scenarios..

c) [30%] The traditional 1/f JTC can be used as a ‘recogniser’ or comparator, where the reference image is unknown or unspecified. This completely reverses the role of the correlator and opens up a whole plethora of applications that revolve around object tracking and motion analysis. Rather than having a pre-defined target or reference image, the input is made up from a sequence of frames from a video source.

One such scenario is when a correlator is used to compare sequential frames in a video stream in a production line. In such an application, the current frame is the unknown and the previous frame is the ‘reference’. Events that occur from frame to frame can now easily be tracked such as in an industrial inspection system as shown below.



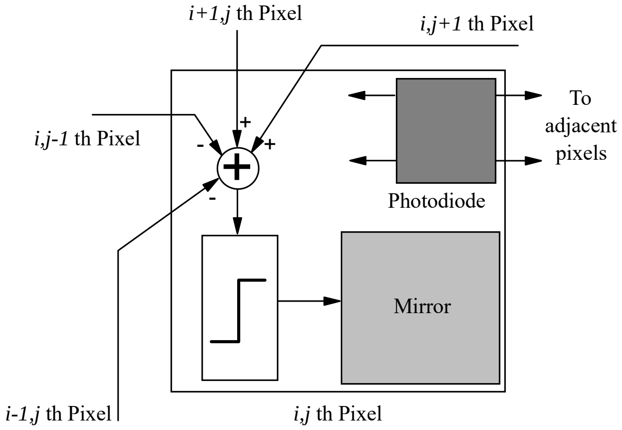
The current frame and previous frame are synchronised with the progress of objects through the system. If the sequence does not change, then the output correlations remain from frame to frame, however, when a change occurs (in the example a rotated roadsign), then the correlation between frames is interrupted. Moreover, the cycle of distortion can be detected by looking at the sequence of disturbances about the first detected defect.

This frame by frame comparison system can be further extended to allow the correlator’s inherent shift invariance to be exploited and a new dimension of object tracking added. If we use the comparator to compare two identical frames, then there will be a strong correlation, which will be centred in the middle of the output plane. If the object in frame *n*, shifts to the left, then the correlation between the two frames will also shift, and we can track the position of the object in frame *n* relative to its old position in frame *n-1*. Furthermore we can keep tracking the relative position of the object from one frame to the next, as long as it stays within the frame and does not rotate or change scale.

d) [10%] One of the best scheme used to binarise the spectrum was based on a nearest neighbour average comparison. The pixel to be binarised, P_{jk} is thresholded based on the average of its four nearest neighbours.

$$P_{j,k} = \begin{cases} +1 & \text{if} \quad P_{j,k} \geq \frac{1}{4}\left(P_{j-1,k} + P_{j+1,k} + P_{j,k-1} + P_{j,k+1}\right) \\ -1 & \text{Otherwise} \end{cases}$$

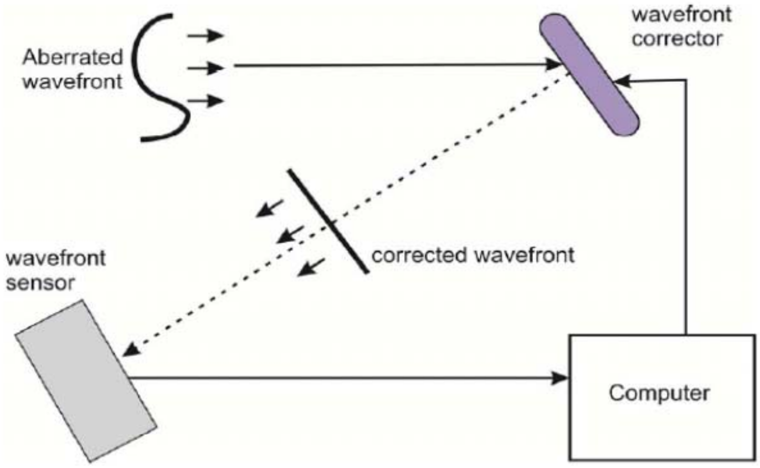
This type of algorithm could easily be programmed intot a smart camera or detector chip which would help speed up the detection and processing part of the JTC.



[Overall a popular question well answered. A few forgot the maths in a) and many did not point out the relationship between the NL process and the FLC chosen in b). c) was well answered in general]

Q4 a) [30%] Three examples of applications which would benefit from an adaptive optcal approach

1) Astronomy - images can be corrected for atmospheric aberration using an AO system.

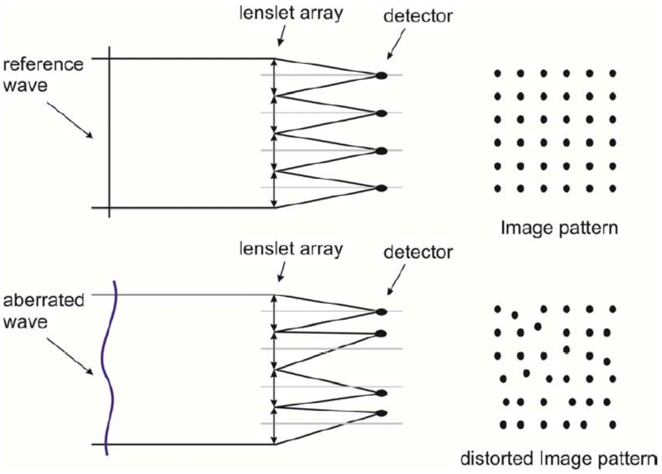


This would normally be a closed system where the aberration is fed back to the wavefront corrector to conjugate the incident light in the system.

2) Ophthalmic imaging or coherence tomography - similar to astronomy and using a closed loop system although it is more difficult to sample the aberrations of the eye die to the poor reflection of the retina and the saccadial motion of the eye (often drugged)

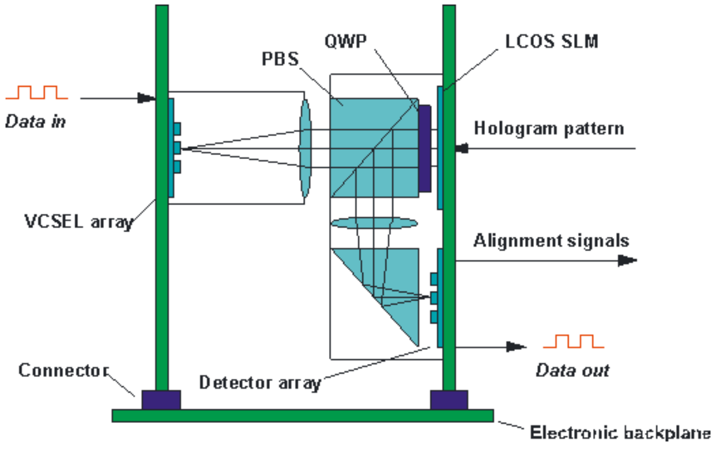
3) Optical tweezing - also similar to astronomy and using a closed loop system with a wavefront sensor to detect aberrations ij the optical path and allow accurate sampling of the optical field of the particles. It is also possible to use an open loop system as the aberrations of the tweezing system are unlikely to change significantly under operation.

b) [30%] The main problem with interferometric techniques is that they are based on a physical path length difference to form the optical fringes. This mean that the system is mechanically very unstable and any vibration between the components will lead to movement in the fringes. There are alternative methods which use a piecewise approach to the wavefront to measure the slope of deferment sections of the wavefront which can be combined to estimate the overall phase. One such method for measuring the slope of the wavefront, is the Shack-Hartmann wavefront sensor. For a plane wave, a spot will be focussed on the optical axis of each corresponding lenslet in the array, as shown in the diagram below as the reference wave. For a distorted wave each focussed spot is displaced and this displacement is proportional to the slope of the wavefront. The incoming beam, whether it is the reference or the measurement beam, passes through the Hartmann screen which divides the wavefront into many sub-apertures. These are then focused on the Hartmann grid by the lenslet array. By comparing the difference in the coordinates between the expected and measured beams, we can obtain the slope of the wavefront.



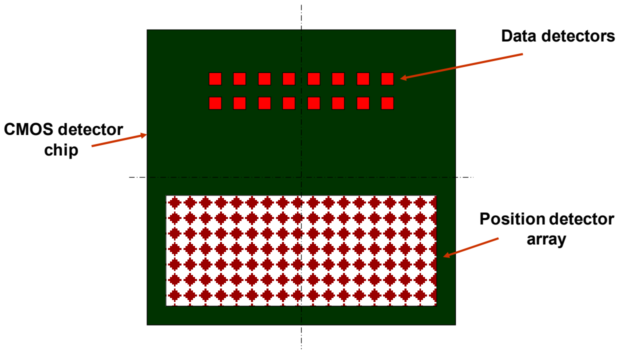
Three main limitation:
1) Stroke - if the wavefront varies too much over one lenslet, then the spot will move into a neighbouring quadrant and could be miss detected.
2) Number of lenslets - the limit on the spatial resolution of the detector is set by the number and aperture of the lenslets. The more lenslets, the more camera pixels require and also the higher degree of complexity of the decoding process of the sensor.
3) The sensor requires a rectangular grid of lenslets which are very difficult to fabricate without significant aberration of the lenslets themselves.

c) [25%] Board to board free space interconnect - In this case the aberration is mostly tip/tilt which leads to misalignment of the interconnect. In this case only open loop is possible due to the lack of a return path for feedback by the interconnect.



AO corrections would normally be encoded into the steering hologram for the interconnect. The wavefront detection could just be a simpler camera or a wavefront sensor or even the use of a multiplexed hologram to identify the aberrations within the optical interconnect itself.

d) [15%] The main purpose of this type of AO system is to correct for misalignment of the optical components which would prevent the beam from hitting the required photodetector. One of the features of using a FLC SLM as the steering hologram is that if would be used a binary phase modulator and would therefore have an inherent 180o symmetry in the replay field. This could be an advantage in this application as the symmetric order will move in conjunction with the misalignments and could be detected separately from the data channel. It would require some form of feedback for the position error.



[A not so popular question, perhaps because not may recalled the board to board interconnect demonstrated but not detailed in the notes. Most assumed an optical fibre switch and managed to tie in the 180 degree symmetry of the FLC replay filed in some way]