4B11 2018 V3 cribs – answers are more verbose than expected

Q1 a) [30%] We want to calculate the field distribution at a arbitrary position away at the point *P*, which is a distance *R* from the aperture *A(x,y)*. If we consider an infinitely small differential of the aperture, *dS*, we can model this as a point source of light emitting spherical 'Huygens' wavelets with an amplitude of *A(x,y)dS*. The wavelet acts as a radiating point source, so we can calculate its field at the point *P*, a distance *r* from *dS*. The point source *dS* can be considered to radiate a spherical wave front of frequency ω. We consider only the part of the wavelets which are propagating in the forward (+z) direction and are contained in a cone of small angles away from the z axis, then we can evaluate the change in field *dE* at the point *P*, due to *dS*. As the wavelet *dS* acts as a point source, we can say that the power radiated is proportional to $1/r^2$ (spherical wavefront), hence the field *dE* will be proportional to *1/r*. We can see that for a real propagating wave of frequency ω and wave number *k*, ($k = 2\pi/\lambda$) we have the cosine component of a complex wave.

$$dE = \frac{A(x,y)dS}{r}\cos(\omega t - kr)$$

Now, we need to change coordinates

$$r = R\sqrt{1-\frac{2\alpha x + 2\beta y}{R^2}+\frac{x^2+y^2}{R^2}}$$

The final full expression in terms of *x* and *y* ($dS = dxdy$) for *dE* will now be.

$$dE = \frac{A(x,y)e^{j\omega t}e^{-jkR\sqrt{1-\frac{2\alpha x+2\beta y}{R^2}+\frac{x^2+y^2}{R^2}}}}{R\sqrt{1-\frac{2\alpha x+2\beta y}{R^2}+\frac{x^2+y^2}{R^2}}}dxdy$$

Such an expression can only be solved directly for a few specific aperture functions. To account for an arbitrary aperture, we must approximate, simplify and restrict the regions in which we evaluate the diffracted pattern. If the point *P* is reasonably coaxial (close to the *z* axis, relative to the distance *R*) and the aperture *A(x,y)* is small compared to the distance *R*, then the <u>lower section</u> of the equation for *dE* can be assumed to be almost constant and that for all intents and purposes, $r = R$. The similar expression in the exponential term in the top line of the original equation is not so simple. To simplify this section we must consider only the far field or Fraunhofer region where.

$$R^2 >> x^2 + y^2$$

In this case, the final term in the exponential ($(x^2 + y^2)/R^2$) can be considered negligible. To further simplify, we use the binomial expansion,

$$\sqrt{(1-d)} = 1 - \frac{d}{2} - \frac{d^2}{8}\ldots$$

Hence the simplified version of the field *dE*, can be expressed as:

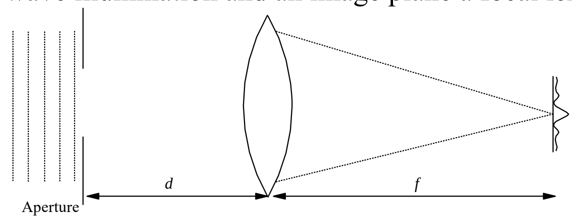
$$dE = \frac{A(x,y)}{R}e^{j(\omega t - kr)}e^{jk\left(\frac{\alpha x+\beta y}{R}\right)}dxdy$$

The total effect of the dS wavelets can be integrated across *dE* to get an expression for the far field or Fraunhofer diffraction pattern.

$$E(\alpha,\beta) = \frac{1}{R}e^{j(\omega t - kR)}\iint_{Aperture} A(x,y)e^{jk(\alpha x+\beta y)/R}dxdy$$

Fraunhofer region = Far field pattern = FT{ Aperture function}

b) [30%] Far field region = Focal plane of a positive lens = FT{ Aperture function }. This can be represented in the diagram below, assumning plane wave illumination and an image plane a focal length away from the lens.



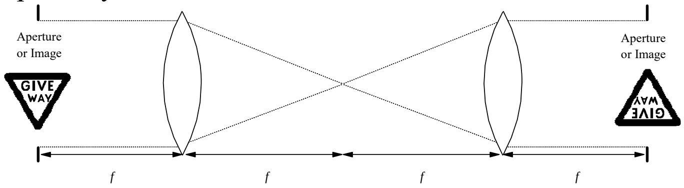
The mechanism that allow the control of diffraction is actually refraction at the air-glass interface. If the plane waves propagate through the aperture A(x,y) the result of the two glass interfaces is to add a quadratic phase term to the propagation.

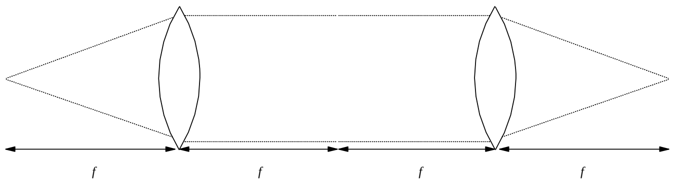
$$A(x,y)' = A(x,y)e^{-j\frac{k}{2f}\left(x^2+y^2\right)}$$

This then propagates the diffraction to form the far field in the focal plane of the lens. If the aperture is placed a distance *d* behind the lens, then there will be a corresponding change in the phase distortion term of the Fourier transform.

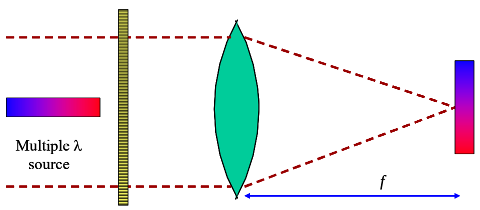
$$E(\alpha,\beta) = e^{\frac{jk}{2f}\left(1-\frac{d}{f}\right)\left(\alpha^2+\beta^2\right)} \iint_A A(x,y) e^{\frac{jk}{f}(\alpha x+\beta y)} dxdy$$

It looks like we are getting something for nothing, but this is not the case, as the lens introduces the quadratic phase distortion term in front of the transform. From the above equation there is a way of removing the phase distortion. If the distance is set so that $d = f$, then the phase distortion is unity and we have the full Fourier transform scaled by the factor of the focal length, *f*. This is a very important feature used in the design of optical systems and is the principle behind the *4f* system. In a *4f* system, there are two identical lenses separated by a distance *2f*. This forms the basis of a low distortion optical system.



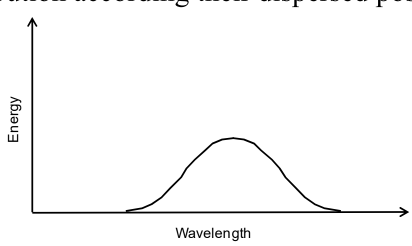


c) [20%] In the equation for the FT of the aperture there are three main control variable that scale the far field. The size of the aperture (x,y), the focal length of the lens f, and the wavelength as in the above equation $k = 2\pi/\lambda$. At one wavelength there is a single distribution of energy, whereas at multiple wavelengths there will be multiple far fields all overlaid on top of one another and each is scaled according to its particular wavelength. By changing the grating pitch *d*, we can vary the position of the diffracted light, for a wavelength λ. By placing a positive focal length lens after the grating, we can view the far field and see that the diffracted angle is converted into the position of the diffracted order (as we would expect from a grating). If we change the pitch of the grating, then the position of the spot generated by the grating will sweep across the far field. If we have a multiple wavelength input source illuminating the grating then each wavelength has a different angle of diffraction and will lead to a different position in the far field.

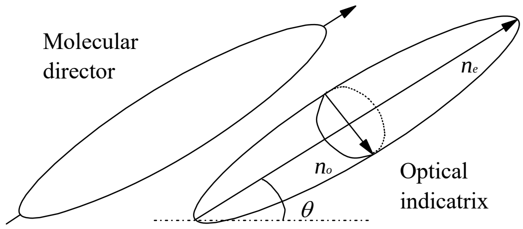


Hence, wavelength and position will vary with the grating pitch in the far field. If we monitor a fixed point in the far field, then the wavelength will scan across that point with the changing pitch, making a wavelength filter.

d) [20%] At any position in the far field there will be a dispersed distribution of wavelengths that depend on the grating pitch and focal length of the lens. At any fixed position there will be one particular wavelength that matches the diffraction properties of the system. A single mode fibre at telecoms wavelengths has a central core diameter of around 9um which means at a fixed position it will sample a range of wavelengths that match the launch conditions of the fibre (its NA). The launch of light down the fibre is roughly Gaussian in shape as the light is scanned across the central core/cladding area of the fibre. Hence the launch of the dispersed wavelengths will also have this same Gaussian distribution according their dispersed position.



Q2 a) [30%] The calimatic molecular shape also leads to an optical anisotropy in nematic LCs, with the two axes of the molecule appearing as the refractive index. The refractive index along the long axis of the molecules is often referred to as the extraordinary n_e and the short axis the ordinary n_o axis. The difference between the two is the birefringence. $\Delta n = n_e - n_o$



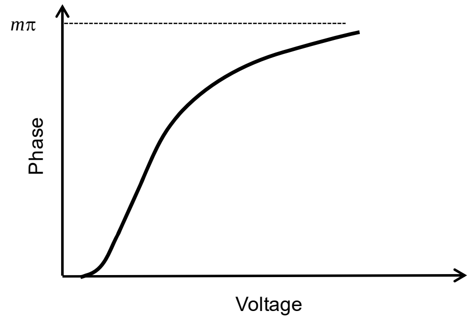
The combination of the flow allowing the molecules to move when an electric field is applied and the optical anisotropy means that we can effectively rotate the axes of the indicatrix as the molecules move, creating a moveable wave plate or optical retarder. This along with polarising optics makes the basis of most liquid crystal intensity and phase modulation characteristics. If we have an optical indicatrix oriented at an angle of θ to the plane of the cell (usually this corresponds to the plane with the glass walls and ITO electrodes), then we can calculate the refractive index seen by light passing perpendicular to the cell walls.

$$n(\theta) = \frac{n_o n_e}{\left(n_e^2 \sin^2\theta + n_o^2 \cos^2\theta\right)^{1/2}}$$

We can then calculate the retardance Γ of the liquid crystal layer for a given cell thickness d and wavelength λ. The retardance is the phase difference between a wave passing through the short axis and the wave passing through the material oriented and an angle θ.

$$\Gamma = \frac{2\pi d\left(n(\theta) - n_o\right)}{\lambda}$$

This is in effect a phase modulation as the retardance of the switched states allows for different pixels to propagate with different path lengths and phase difference.



The phase characteristic shows how a nematic LC would respond to an applied voltage (varying θ). There is an initial threshold (Freedrickz transition) then a roughly linear increase in phase to a level of $m\pi$ (where $m < 10$) before the LC molecules are all switched into the homeotropic state.

b) [30%] The phase modulation above is polarisation dependent. The orientation of the incident light (E-field) must be parallel with the long access of the molecule in the indicatrix above for it to see the full birefringence of the LC material (Δn).

Possible solutions:

1) Phase diversity. There are several techniques that allow both orthogonal polarisations to be aligned with the LC molecules. Using 2 orthogonal LC cells would allow both states to be modulated separately but there is some error due to the spacing of the two cells. A better technique is to pass the light through the cell twice (such as in reflection mode) but rotate the E-field by 90° between the two passes. This can be achieved using a half wave plane and a mirror.

2) Manually align the polarisation to the long axis of the molecules and then maintain this orientation. This limits the possible applications quite severely to only a few optical systems.

c) [25%] We can use this expression in a Jones matrix representation of the optical LC retarder to get the optical characteristics of the LC material. In the case of nematic LC materials, there is little restriction on the flow properties of the material, hence it is possible to continuously vary Γ through several rotations of π providing a thick enough cell. Based on the Jones matrix for a generalised retarder we get optimal phase modulation when the LC director is aligned parallel to the E-field of the incident light. This is effectively when $\psi = 0$ in the retardation matrix W

$$W = \begin{pmatrix} e^{-j\Gamma/2}\cos^2\psi + e^{j\Gamma/2}\sin^2\psi & -j\sin\frac{\Gamma}{2}\sin(2\psi) \\ -j\sin\frac{\Gamma}{2}\sin(2\psi) & e^{j\Gamma/2}\cos^2\psi + e^{-j\Gamma/2}\sin^2\psi \end{pmatrix}$$

Becomes

$$W = \begin{pmatrix} e^{-j\frac{\Gamma}{2}} & 0 \\ 0 & e^{j\frac{\Gamma}{2}} \end{pmatrix}$$

For optimal 2π phase modulation we set Γ = 2π.

$$W = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

Which will give zero transmission loss for a vertically polariser light sources passing through the LC cell. Hence for the LC switching between its two extreme states (neglecting the effect of the molecules attached to the surface of the cell due to anchoring forces):

$$\Gamma = \frac{2\pi d}{\lambda}\Delta n = 2\pi$$

For the LC and wavelength quoted: d = 1550nm/0.18 = 8.6um thick

d) If the device is limited to 2π total phase modulation, then there will be a limited performance as most devices will never fully achieve this modulation range due to surface effects within the LC device. At both low and high voltages, the modulation curve is limited and less linear, hence there should be margins allowed for these nonlinear areas of the curve. A better solution would be to design a device with 3-4π of phase range with allows room at either end of the voltage range to compensate for the non linear regions of the curve. It also allows for overshoot which helps with the drive electronics which run the device.

The penalty for this extended phase range is the fact that the devices must be thicker and therefore the LC will switch slower due to the reduced field across the device meaning a higher switching voltage is required. There are also bulk effects in the thicker LC which could lead to surface alignment problems and even possible synclinic domains.

Q3 a) [20%] The term *fan-loss* refers to the amount of optical power which is lost when light is launched down an optical fibre at the output end of the switch. Fan-in loss arises in holographic switch because the only fibre in the output which is on the main optical axis is the one in the centre of the array. When light is steered to the outermost fibres it is at an angle to the central axis which no longer satisfies the perfect launch condition of a single mode fibre. Hence there is a loss which depends on this angle and therefore the position of the output port. It is normally estimated to be proportional to the number of fibres *n*, at the output of the switch.

If the switch is configured to route light to the *k*th fibre in an output array of *n*, then the *crosstalk* is the ratio of light launched down the desired fibre to the light launched down one of the other fibres which are not being routed. Both are normally expressed as power ratios in decibels.

b) [30%] The total input power which appears in the output plane is P_{in}. The total power which is routed into a desired fibre by the CGH is P_{sp} and the remaining power is dissipated into the whole plane as the background or noise power P_{bk}.

$$P_{in} = 2P_{sp} + P_{bk}$$

The factor of 2 is due to the symmetry of the pattern due to binary phase. We can define the CGH efficiency η as the ratio between the power in the spot, P_{sp} and the input power P_{in}.

$$\eta = \frac{P_{sp}}{P_{in}}$$

For n fibres in the output array of a 1 to n switch, the power into a single fibre will be ηP_{in}. If the CGH has NxN pixels, then the replay field can also be assumed to contain NxN 'spatial frequency pixels'. If we assume that the background power is uniformly distributed over the N^2 spatial frequency pixels in the replay field then the background power at each pixel will be.

$$P_{bpix} = \frac{(1-2\eta)P_{in}}{N^2}$$

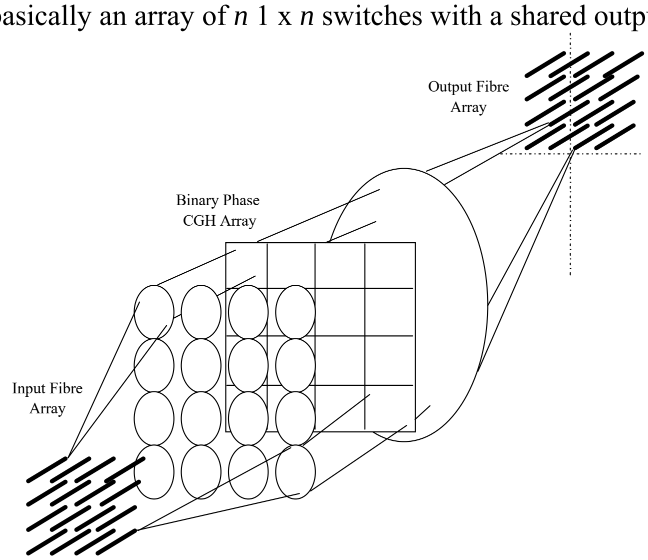
Hence the crosstalk is the ratio of the light routed to a fibre to P_{bpix}.

$$C = \frac{\eta}{1-2\eta}N^2$$

Assumptions:

- The distribution of the background power is uniform.
- The number of CGH pixels is infinite.
- The pixel pitch is effectively zero, hence no sinc envelope.
- The SLM used to display the CGH inevitably has no deadspace.
- The physical alignment of the fibres in the output array is perfect.
- Ferfect optics with no limitations or distortions.
- No fan in loss

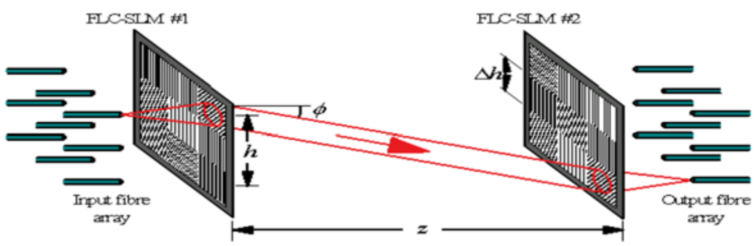
c) [25%] The n x n switch is basically an array of n 1 x n switches with a shared output lens



The n x n analysis for the crosstalk is the same except that we now have the background noise from each of the other n - 1 input fibres appearing at the each output fibre along with the ηP_{in} from the routed input. Hence the crosstalk will be.

$$C = \frac{\eta}{1-2\eta}\frac{N^2}{(n-1)}$$

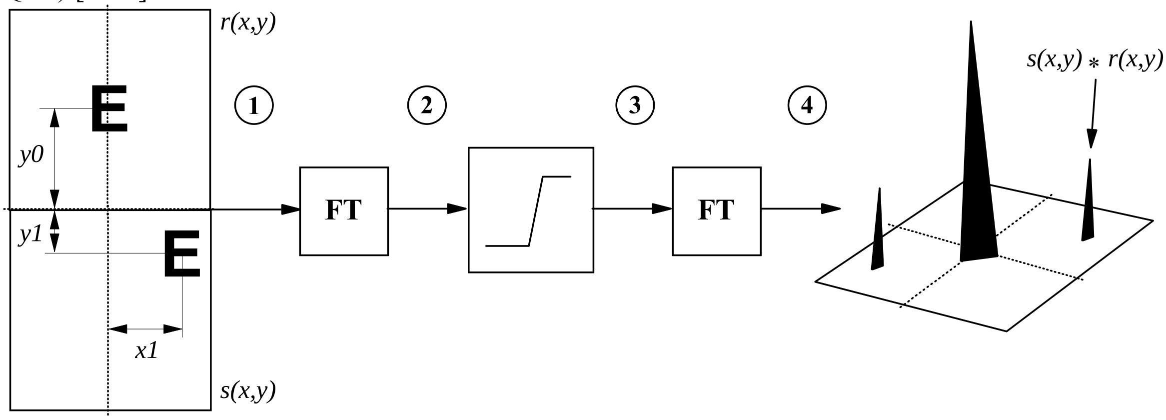
d) [25%] The single hologram n x n switch is limited in scalability as it can only diffract light over a limited angle. Also the crosstalk boundary becomes probative. We can rectify this by using two holograms to steer the light. The first steer light into the switch, whilst the second steers light out of the switch back onto the output fibre's axis. The most efficient combination for routing is if the second hologram is the complex conjugate of the first routing hologram.



The two hologram switch can be scaled to any size and the loss through the switch does not scale with the number of input and output ports, it does however increase the loss as there are now two holograms routing the same beam. The only parameters which scale with the number of ports are the crosstalk and the physical length *z*. The crosstalk of the two hologram switch is greatly improved as the crosstalk of the first hologram is multiplied by the cross talk of the second hologram.

$$C = \left(\frac{\eta}{1-2\eta} \frac{N^2}{(n-1)} \right)^2$$

Q4 a) [40%]



In plane 1, the input *s(x,y)* and reference *r(x,y)* are displayed side by side in an optical system and then transformed by a single lens into plane 2.

$$S(u,v)e^{-j2\pi(x_1u-y_1v)} + R(u,v)e^{-j2\pi y_0 v}$$

The nonlinearity between planes 2 and 3 creates the correlation and in its simplest form can be modelled by a square law detector such as photodiode or CCD camera which takes the magnitude squared of the light falling upon it.

$$S^2(u,v) + R^2(u,v) + S(u,v)R(u,v)e^{-j2\pi\left(x_1u-\left(y_0+y_1\right)v\right)} + S(u,v)R(u,v)e^{-j2\pi\left(-x_1u+\left(y_0+y_1\right)v\right)}$$

The final plane 4 is after the second FT, with the central DC terms proportional to FT $[R^2 + S^2]$ and the two symmetrical correlation peaks spaced by $(x_1\ y_1+y_0)$ and $(-x_1, -(y_1+y_0))$.

Components – 1) SLM 2) Lens 3) OASLM or CCD/SLM 4) Lens 5) CCD

b) [40%] Top E is reference r(x,y), Lower E is s1(x,y) lower F is s2(x,y)

Input plane (1) is r(0,y0) + s1(0,-y0) + s2(x1,y1)

JPS plane (2) is the FT of (1)

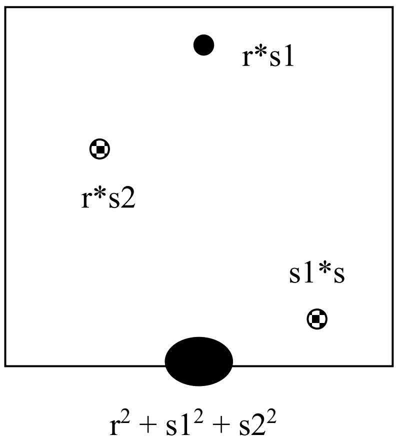
= R(u,v)exp(-j2py0v) + S1(u,v)exp(-j2p(-y0)v) + S2(u,v)exp(-j2p(x1u-y1v))

After square law non-linearity | a + b + c $|^2$ = a^2 + b^2 + c^2 + ab* + ac* + ba* + bc* + ca* + cb* which gives 3 cross correlation pairs: (ab* + ba*), (ac* + ca*) and (bc* + cb*)

(ab* + ba*) = RS1exp(-j2p(2vy0)) + RS1exp(-j2p(-2vy0))

(ac* + ca*) = RS2exp(-j2p(-x1u + (y0+y1)v)) + RS2exp(-j2p(x1u – (y0+y1)v))

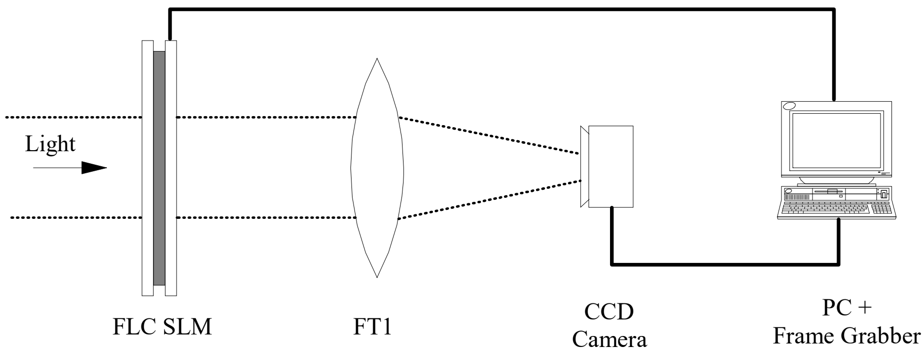
(bc* + cb*) = S1S2exp(-j2p(-x1u + (y1-y0)v)) + S1S2exp(-j2p(x1u + (y0-y1)v))



The main correlation peak will be r*s1 as is required by the correlation process. There will be a second peak r*s2 which will be a little smaller than the first as the E and F do not fully match. It will be around 80% of the full peak height. This could easily be mistaken for a letter E. There will also be a third erroneous peak s1*s2 which will also be 80% of full height which will lead to false object detections.

c) [20%] The JTC works on the basis of a non-linearity working on the spectrum of the input objects to create the product of the two Fourier transforms. This was modelled as a square law detector, but this gives undesirable broad peaks. A much better correlation peak is obtained when the degree of non-linearity is increased as high as possible. A simple square root function on the spectrum gives good narrow peaks, but the best performance is when the spectrum is thresholded. Hence the JTC was originally built using an optically addressed FLC SLM.

A better layout is to exploit the symmetry about the OALSM or non-linearity. If the optical system is split at this point, then the JTC just becomes two Fourier transforms and in fact can be done with a single laser, SLM and camera by doing two passes through the Fourier transform lens. This is know as the 1/f JTC.



The input and reference images are displayed side by side on a FLC SLM as in a full JTC. The SLM is illuminated by a collimated laser beam and the images are Fourier transformed by a single lens in its focal plane. This spectrum is then imaged onto a CCD camera. The spectrum is then non-linearly processed before being displayed onto the SLM again to form the correlation information. The 1/f JTC is a two-pass system, using the same lens to perform the second Fourier transform.

Q1 Diffraction and holograms
Mostly a straightforward question. First two parts were answered overall, quite a few forgot that the lens creates a curved wavefront due to Snell's Law and quite a few did not state assumptions. Part c) was quite badly answered with many not getting the wavelength ion the far field. Also part d) where few realised the fibre would have a Gaussian mode profiled.

Q2 Liquid crystal phase modulation
Pretty basic bookwork question that was not well answered. Most got the first part with the indicatrix and the polarisation dependence, some good solutions, especially the cholesteric. Most got the thickness wrong by a factor of 2 as they were calculating a retardance of π. Last part was weakly answered. But was a bit of an extension.

Q3 Optical correlators
Overall a popular question well answered. A few forgot the maths in a) and many did not point out the relationship between the NL process and the FLC chosen in b). c) was well answered in general.

Q4 Adaptive optics
A not so popular question, perhaps because not may recalled the board to board interconnect demonstrated but not detailed in the notes. Most assumed an optical fibre switch and managed to tie in the 180 degree symmetry of the FLC replay filed in some way.