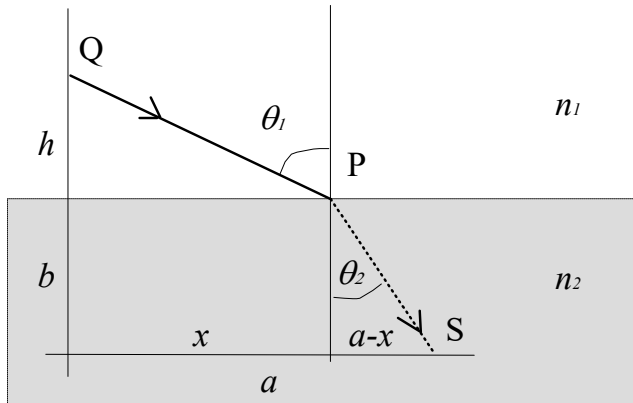


Q1 a) [30%] Pure ray theory only contains information about the direction the ray is travelling in, to solve even simple problems like reflection or refraction more information such as time and velocity are required. A ray travelling from one medium will be refracted as demonstrated in a way that is described by Snell's law. The proof comes from Fermat's principle which states the ray should transit through the system in the shortest possible time.



$$t = \frac{\overline{QP}}{v_1} + \frac{\overline{PS}}{v_2}$$

Which can be translated into physical co-ordinates.

$$t = \frac{\sqrt{h^2 + x^2}}{v_1} + \frac{\sqrt{b^2 + (a-x)^2}}{v_2}$$

This shortest time will be when t is minimised with respect to x , hence we set $dt/dx = 0$

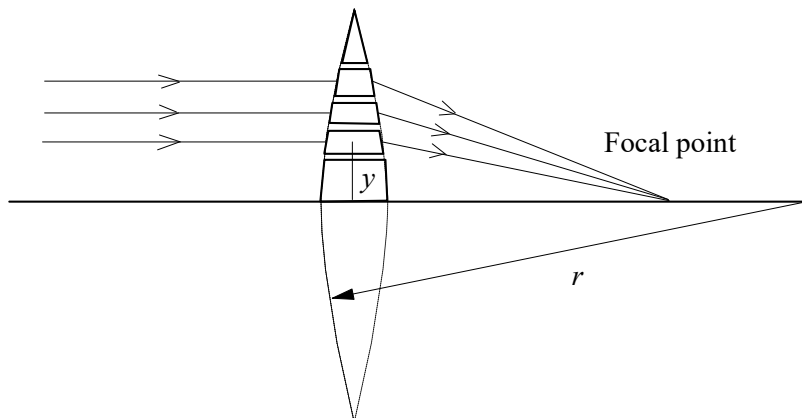
$$\frac{dt}{dx} = \frac{x}{v_1 \sqrt{h^2 + x^2}} + \frac{-(x-a)}{v_2 \sqrt{b^2 + (a-x)^2}} = 0$$

Which can be re-written to give.

$$\frac{\sin \theta_1}{v_1} = \frac{\sin \theta_2}{v_2} \quad \text{Which is in fact Snell's law} \quad n_1 \sin \theta_1 = n_2 \sin \theta_2$$

b) [30%] The principle of Snell's law can be used to solve the optical problem of light propagation through a thin wedge shaped prism. The refraction of the rays at each surface dictate the how light will pass through the thin prism. If we have a thin prism such that apex angle α is small and a small angle of incidence such that $\sin \beta \approx \beta$ then the total deflected angle can be approximated to: $\theta = (n-1)\alpha$

This is the basic principle used in all most geometric ray problems. Deviation from small values of α and β lead to aberration in the optical system, hence these values form a solid basis for good lens design and minimisation or potential aberrations. A good example of how this property can be used is shown below, where a thin lens is made from a series of thin prism sections.



The deviation of each ray means that if parallel rays are incident on the lens, then they will all converge to the same point called the focal point or focal length of the lens. This relationship forms the basis of most geometrical optical systems and is one of the fundamental relationships which are exploited on a regular basis in optical design procedures.

As has already been stated from the prism analysis, the rays must enter the prism at a shallow angle in order for the approximation of $\sin x \approx x$ to hold. This is often referred to as the *first order* analysis or the *paraxial ray* approximation as it only applies to rays which are close or parallel to the optical axis. The paraxial approximation is very useful for setting up basic systems. Hence any optical system must undergo a series of optimisations after the initial approximation to minimise the desired aberrations. No lens system will be perfect, so compromises must be made during the optimisation procedure, based on which aberrations will most effect the image quality of the lens system. This is usually done through a series of trial and error simulations using ray tracing software such as ZEMAX and CODE V.

c) [20%] There are many techniques that can be used to make the optical design as paraxial as possible. This is often done in order to decrease the overall optical length of the system as the shallow angles of the paraxial requirements lead to long optical paths.

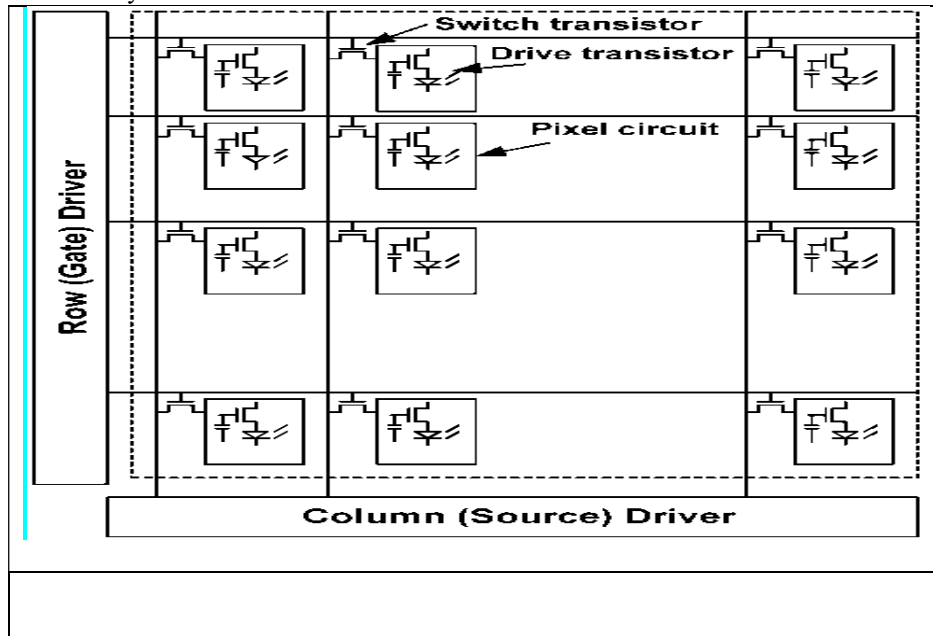
- 1) Use multiple surfaces or elements to minimise that path deviations and angles
- 2) Use different glasses with different dispersions and refractive indices.
- 3) Use aspherical surface profiles to avoid paraxial limits.

d) [20%] Not using refractive systems to design the optics. Other models such as diffraction allows a different approach to the design of elements in the system. This means that effects such as wave guiding and gratings can be used to design the optical elements based on different rules not limited by paraxiality.

Approaches such as diffraction use a different model for the light source and require well defined wave properties of the light in order to control its propagation. A good example would be diffractive optics which require some form of coherence within the optical source. Note that coherence is a relative term which means that the physical size of the diffractive elements need to be smaller than the coherence length of the light source. This does not always mean that lasers are required, but rather that the diffractive elements require very fine features.

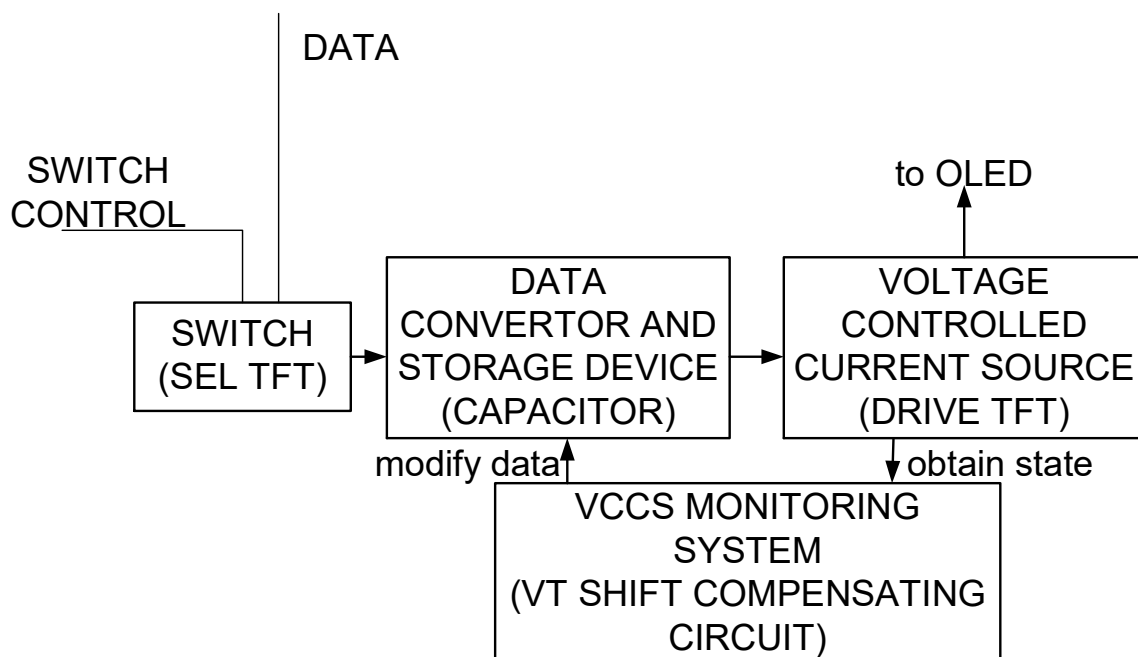
[A popular and straightforward question. A few answered part a) with a wavefront analogy. b) was ok except a few missed the paraxial role of the apex angle and the thin lens approximation. d) was well answered but no-one mentioned diffraction, most mentioned dispersion.]

2 a) [25%] OLED Array



The driver diagram for the LCD should be similar to that of the OLED array except that the drive transistor and diode is now replaced by the liquid crystal capacitance. There should be a clear definition between the current drivers needed for the OLED array as opposed to the electric field (or voltage drivers required for the LCD)

b) [25%] In addition to the switching transistor used in the AMLCD pixel, the AMOLED pixel requires a current driving stage (since the OLED is a current-driven device) and a stage to compensate for V_T -shift in the TFT.



Rightmost and lower blocks highlight the fundamental differences in terms of backplane requirements between the AMOLED and AMLCD.

c) [30%] The drive current of a TFT is proportional to $(V_{GS}-V_T)^2$ where V_T degrades as a function of time.

So we can express the OLED drive current as

$$I_{\text{OLED}} = K ((V_{\text{GS}} - V_{\text{Ti}}) - [(V_{\text{GS}} - V_{\text{Ti}}) (1 - \exp(-t/\tau))])^2 = K (V_{\text{GS}} - V_{\text{Ti}}) \exp(-2t/\tau)$$

Hence the relative degradation from initial ($t=0$) to $t=1000\text{s}$ is

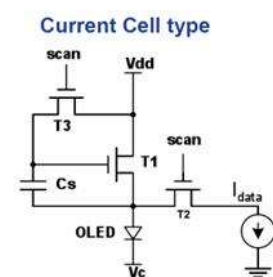
$$I_{\text{OLED1}} / I_{\text{OLED2}} = \text{Luminance}_{(t=0)} / \text{Luminance}_{(t=1000)}$$

$$= K (V_{\text{GS}} - V_{\text{Ti}}) / [K (V_{\text{GS}} - V_{\text{Ti}}) \exp(-2000/2000)] = e = 2.7$$

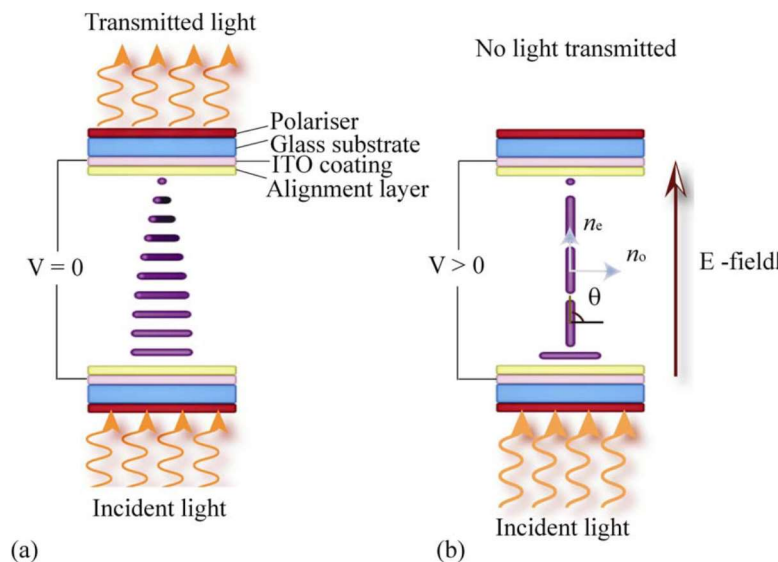
Hence the luminance after 1000s is 0.368 of the initial luminance or 37% of initial L.

[A less popular question, mostly book work. Quite a few were thrown by the lifetime calculations and a few forgot the square law dependency on current. d) was very well answered in general with good definitions of the threshold compensation.]

d) [20%] The degradation is solved through some form of compensation such as e.g. through electrical feedback as shown. The main problem with compensation in general is the extra complexity of the backplane and the extra area taken up by the compensation TFTs which reduces can reduce the aperture ratio depending on the pixel architecture.



3 a) [30%] A schematic of the twisted nematic device is shown in the figure. Illustrations should show, correctly, the arrangement of the polarizers, glass substrates, ITO coating, alignment layer, and colour filters.



The important features are that the rubbing directions are twisted relative to each other and that, as a result, the liquid crystal has a helical configuration. The propagation through helical birefringent medium can be described by two elliptically polarized eigenmodes. The twisted nematic device can operate in one of two modes: the first is the normally white mode whereby the transmission direction of the polarizer is parallel to the rubbing direction on the respective glass substrate. Without an electric field, the twisted structure cause the plane of polarization of the incident light that has passed through the first polarizer to rotate so that it passes through the second polarizer (analyzer). However, when an electric field is applied across the cell, the LC molecules reorient to form a

predominantly homeotropic alignment (with the exception of the molecules in the vicinity of the substrates), which is optically inactive. The device can also function in the normally black mode whereby the transmission directions of the polarizers are parallel with respect to each other. In the absence of an electric field no light is transmitted but the device becomes optically active with the application of an electric field. For the TN-LCDs, liquid crystals with a positive dielectric anisotropy are required to ensure that a homeotropic alignment is formed with an applied electric field. A sketch of the transmission as a function of voltage to show grayscale capability should accompany the diagrams explaining the operating principle of the TN device.

The redistribution of the director in the TN device occurs above a particular value of the voltage, known as the threshold voltage, and is related to the liquid crystal properties such as elastic constants and dielectric anisotropy.

The Mauguin limit is the condition for waveguiding along a twisted structure. This can be expressed as,

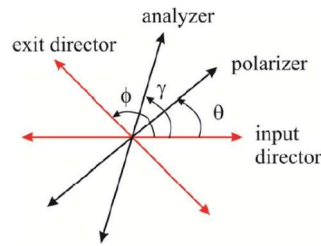
$$\phi \ll \frac{2\pi}{\lambda} \Delta n d$$

where ϕ is the twist angle, Δn is the birefringence, and d is the film thickness. For a TN-LCD with

$\phi = 90^\circ$ the condition reduces to $\frac{\lambda}{4} \ll \Delta n d$. When this condition is not satisfied the output beam is elliptically polarized

The isocontrast curve is a means with which to visualise the viewing angle dependency of a device. It is a polar plot consisting of both the polar angle, defined as the angle between the observation direction and the direction normal to the device, and the azimuth angle, which is defined as the angle between the transmission axis of the first polarizer and the projection of the observation direction onto the cell.

b) i) [15%]



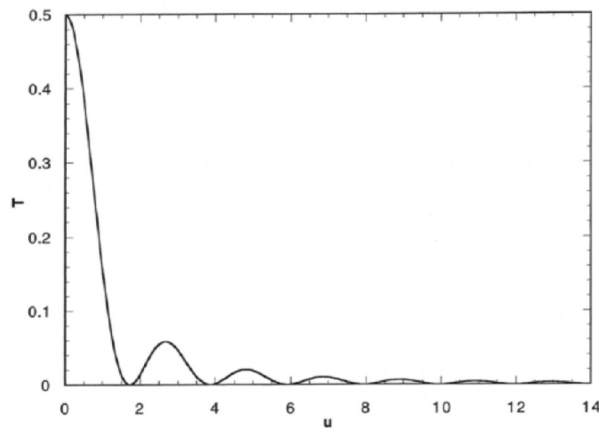
If we set, $\phi = 90^\circ$ and $\theta = \gamma = 0$, then the expression for the transmission can be reduced to:

$$T = \frac{1}{2} \frac{\sin^2\left(\frac{\pi}{2} \sqrt{1+u^2}\right)}{1+u^2},$$

where the retardation parameter, u , is

$$u = \frac{\Gamma}{2\phi} = 2d \frac{\Delta n}{\lambda}.$$

ii) [35%] The Gooch Tarry curve:



To find the values of the optical retardation, u , at the 1st three minima, the transmission T above.

For the minima, $T = 0$, therefore,

$$\sin^2\left(\frac{\pi}{2}\sqrt{1+u^2}\right) = 0 \text{ and hence we can say } \left(\frac{\pi}{2}\sqrt{1+u^2}\right) = n\pi \text{ where } n \text{ is an integer. It is then}$$

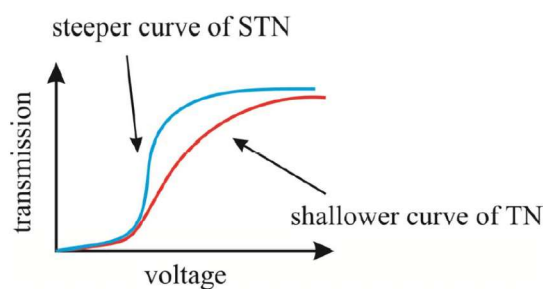
straightforward to show that, 1st min, $u = \sqrt{3}$, 2nd min, $u = \sqrt{15}$, 3rd min, $u = \sqrt{35}$.

To find film thickness, it is necessary to use the expression for the optical retardation, u , in terms of the birefringence

$$\text{1st min, } u = \sqrt{3} = \frac{2d\Delta n}{\lambda}, d = 3.9\mu\text{m}, \quad \text{2nd min, } u = \sqrt{15} = \frac{2d\Delta n}{\lambda}, d = 7.8\mu\text{m}$$

The first minimum is generally preferred due to the smaller viewing angle dependence and the faster response time

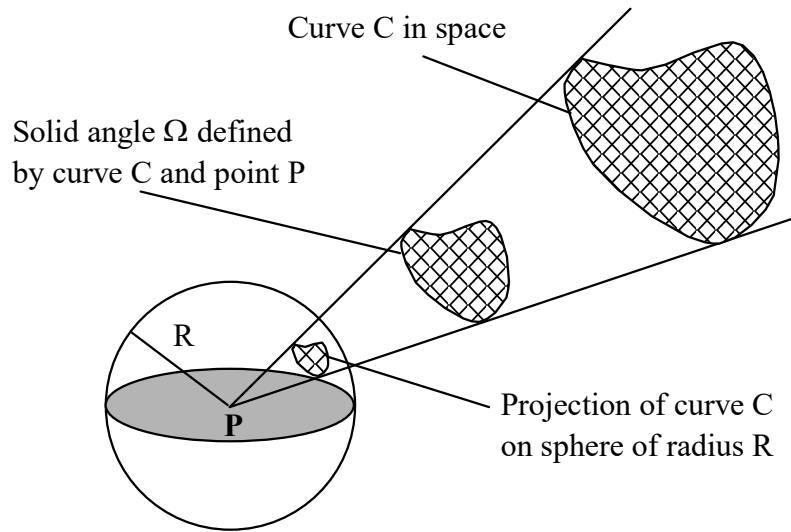
c) [20%] When used in a passive matrix addressed display, the supertwist nematic (STN) device has a greater twist angle ($>90^\circ$) which results in a steeper electro-optic curve. It is more suitable for multiplexing in comparison to the TN device. However, grayscale can be more difficult for the STN mode because of the very steep T-V curve.



The STN device was preferred to the TN mode in passive matrix addressed LCDs, however the steep transmission curve means that the control of grayscale is difficult. STN failed for two main reasons: 1) The viewing characteristics were much worse than the TN due to this higher twist. 2) As active matrix backplanes became better and cheaper it became unnecessary to rely on passive matrix displays and so the better viewing characteristics of the TN could be used at high levels of multiplex.

[Most could do the standard bookwork of part (a), but (b) not so well answered, mostly because quite few missed the difference between retardation and retardation factor. c) was very well answered.]

Q4 a) [30%] The definition of plane angle can be extended into 3 dimensions to define the solid angle Ω . The solid angle is defined by a closed curve and a point in space. Its magnitude is the area of a closed curve projected onto a sphere of unit radius as shown below.



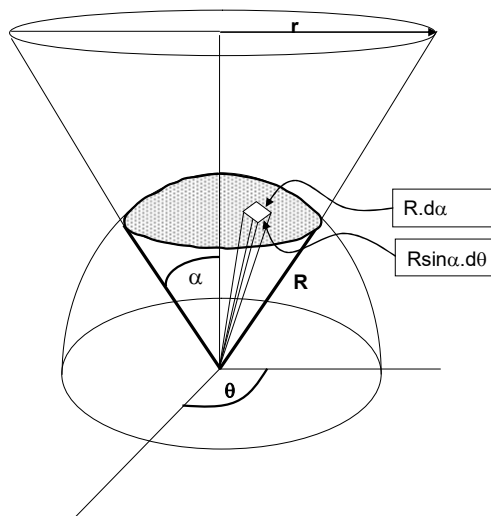
Equivalently, the solid angle can be defined as the quotient of the area A of the projected curve onto a sphere of radius R and the radius squared.

$$\Omega = \frac{A}{R^2}$$

The solid angle Ω has no units, so the units **steradians** (abbreviated to **sr**) is used much like radians.

The **Luminance** of a projection display, L_v is the photometric equivalent of Radiance and is often thought of photometric brightness, as it comes relatively close to the human perception of brightness. The units of luminance are lumen per meter squared per steradian ($\text{lm}/\text{m}^2\text{sr}$), hence the brightness of the projector will depend heavily on the solid angle of the projection optics.

b) [20%] The solid angle generated by a circular plane of radius r , subtended at an angle α from the centre of the angle. Project circle of radius, r , onto a sphere of radius R . From spherical co-ordinate geometry. Surface area element, $dS = R \sin \alpha \cdot d\theta \cdot R \cdot d\alpha = R^2 \sin \alpha \cdot d\theta \cdot d\alpha$



Then, for $d\alpha = 0$ to α , and $d\theta = 0$ to 2π , then area of sphere segment, $S = \int_0^{2\pi} \int_0^\alpha R^2 \sin \alpha \, d\theta d\alpha$

$= 2\pi R^2(1-\cos\alpha)$. Solid angle $\Omega = A/R^2$, therefore $\Omega = 2\pi R^2(1-\cos\alpha)/R^2 = 2\pi(1-\cos\alpha)$.

c) [20%] Possibly one of the most vital concepts in any projection system is *etendue*, however it is also one of the most difficult to visualise in simple terms as it relates the properties of the light source and the area and solid angle that can be covered by that source element. Etendue is important because it *never decreases* in any optical system. A perfect optical system produces an image with the same etendue as the source. The etendue is related to the Lagrange invariant and the optical invariant, which share the property of being constant in an ideal optical system. The radiance of an optical system is equal to the derivative of the radiant flux with respect to the etendue.

From the source point of view, it is the area of the source times the solid angle that the system's entrance pupil subtends as seen from the source. From the system point of view, the etendue is the area of the entrance pupil times the solid angle the source subtends as seen from the pupil. These definitions must be applied for infinitesimally small "elements" of area and solid angle, which must then be summed (integrated) over both the source and the pupil as in the diagrams below.

In reality mathematical expressions have to be resolved over the geometry of the source and the projection optics (ie the path to the display area). The analysis is the same as used in thermodynamics, for relating the radiation from heat sources and the different geometries are pre-calculated as 'view factors' in many different thermodynamic tables. We can also relate the etendue in the system to the concept defined as radiance and it is also often quoted that etendue relates to the concept of numerical aperture.

The fact that the etendue of a source cannot be decreased is the key concept as the source and its optics set the overall projection limitations of the display. This is why the area of the optical source is a critical parameter in defining the overall throw in a projection display and hence forms an overall limit for the projection system as a whole.

d) [30%] All of the elements in a system will have an effect on the etendue. The most critical part is the area of the light source and the exit pupil of the condensing lens. All of the other elements will also have the effect of increasing the etendue and therefore making the projector less efficient and less able to project a bright image over a short distance

[Well answered in general. Especially the bookwork sections and the derivation of the sold angle. Very good descriptions of etendue and its implications. The final part d) was a little disappointing with few spotting that all elements in the projector will affect the etendue of the system.]

