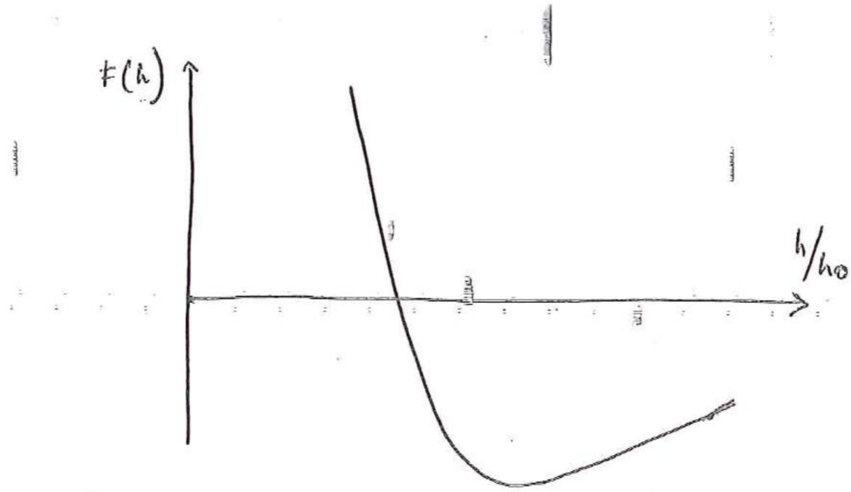
1 (a)



For the maximum force of attraction –

$$\frac{dF}{d(h/h_o)} = 0$$

$$\therefore -3\left(\frac{h}{h_o}\right)^{-4} + 9\left(\frac{h}{h_o}\right)^{-10} = 0$$

$$\therefore \left(\frac{h}{h_o}\right)^{6} = 3 \quad \text{or} \quad h = 3^{1/6} h_o$$

$\therefore$ maximum force of attraction

$$= \frac{8wA}{3h_o}\left(\frac{h}{h_o}\right)^{-3}\left[1 - \frac{1}{3}\right] = \frac{16\,wA}{9\sqrt{3}\,h_o}$$

(b)

$$V_{PI} = \sqrt{\frac{8}{27}\frac{k g^3}{\varepsilon_o A}}$$

$$V_{PI} = \sqrt{\frac{8}{27} \times \frac{3E \times w^3 h}{12 l^3} \times \frac{g^3}{\varepsilon_o A}}$$

$$V_{PI} = \sqrt{\frac{8}{27} \frac{\times 3 \times 160 \times 10^9 \times 4^3 \times 10 \times 2^3 \times 10^{-12}}{12 \times 200^3 \times 20 \times 10 \times 8.85 \times 10^{-12} \times 10^3}}$$

$$V_{PI} \simeq 2.07V$$

(c) $$F_{max}|_{adh} = \frac{16 \times 100\times10^{-3} \times 20\times10 \times 10^{-6} \times 10^{-6}}{9\sqrt{3} \times 0.5\times10^{-9}}$$

$$= 41.05\ mN$$

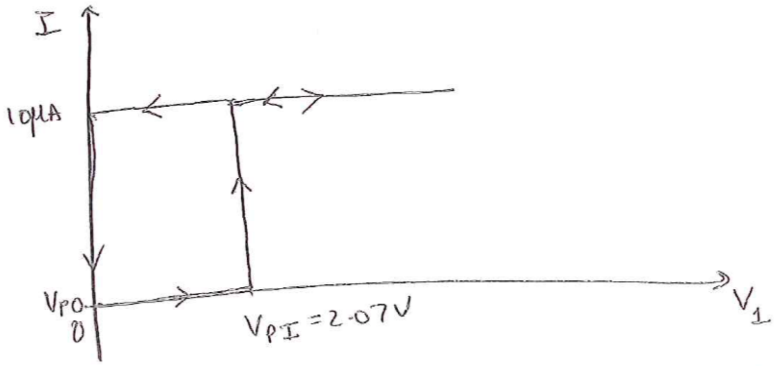
$$F_{estatic} = k \times g/3 = 2.13\ \mu N$$

$$\therefore \text{Ratio} = 19242$$

(d) $$\frac{\varepsilon_0 A V_{PO}^2}{2g^2} = -k \cdot g + \frac{F_{max}|_{adh}}{100} \approx \cancel{47.44}\ 34.86\ \mu N$$

$$\therefore V_{PO} = 39.7\ V$$

(e)



If $V_{PO} = 0\ V$ then w is such that

$kg > F_{max}|_{adh}$.

$\therefore$ w is reduced by a further factor of 6.4x relative to (d).

This question was less well done relative to the other questions. While most students were able to sketch the force distribution in (a), only a few students were able to apply the straightforward pull-in analysis required in (b) or visualise the pull-off condition as the work of adhesion of the surfaces in contact was varied.

Q2 (a)

$$i = \frac{d(CV_p)}{dt}$$

$$\simeq V_p\, \omega_0 x \left.\frac{dC}{dx}\right|_{x=0} \qquad \text{where } C = \frac{4\epsilon_0 A}{g_0 - x}$$

$$i = V_p\, \omega_0 x \frac{4\epsilon_0 A}{g_0^2}$$

(b)

$$x \simeq \frac{F}{k} \cdot Q$$

$$\simeq \frac{4\epsilon_0 A\,(2V_p V_{ac})}{2 g_0^2 k} \cdot Q = 1.99 \times 10^{-10}\,\text{m}.$$

$$i = 50 \times \sqrt{\frac{44.4\times10^{6}}{233\times10^{-9}}} \times \frac{1.99\times10^{-10} \times 4 \times 8.85\times10^{-12} \times 1800 \times 25}{9}$$

$$= 24.3 \text{ nA}$$

(c)

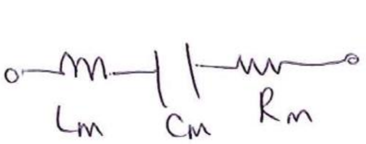
$$L_m = \frac{M}{\eta^2}, \quad C_m = \frac{\eta^2}{k}, \quad R_m = \frac{m\omega_0}{Q\eta^2}$$

$$\eta = \frac{4\epsilon_0 A V_p}{g_0^2} = 8.85\times10^{-6}$$

$$L_m = 2.97 \text{ KH}$$

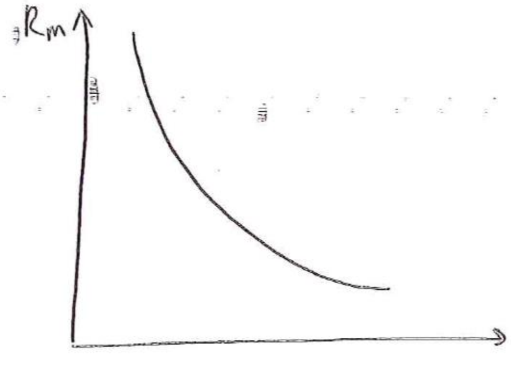
$$C_m = 1.76 \text{ aF}$$

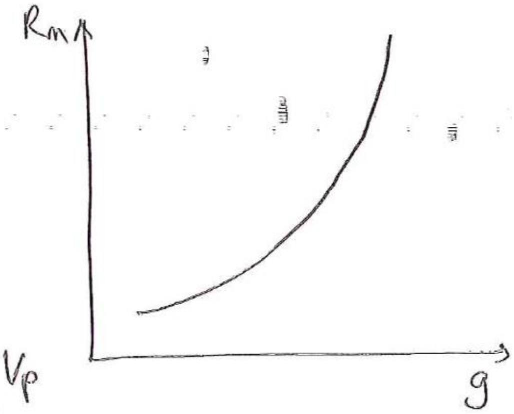
$$R_m = 4.10 \text{ M}\Omega$$



≡ equivalent circuit model

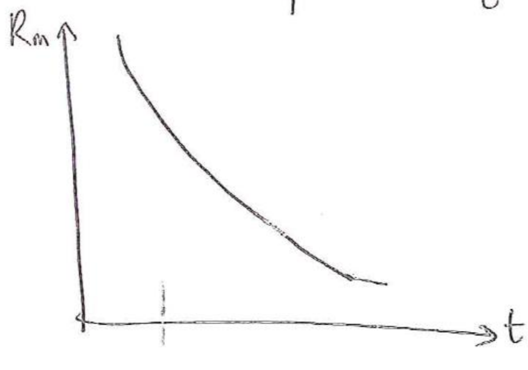
(d) $R_m \propto \frac{1}{\eta^2}$ ∴ $R_m \propto \frac{1}{V_p^2}$ and $R_m \propto g^4$





∴ Minimise R_m by increasing V_p and reducing g. Practically limited by lithography limits and etch aspect ratio for g and pull-in considerations /operating voltage limits for V_p.

$$R_m \propto \frac{\sqrt{km}}{\eta^2} \propto \frac{t}{t^2} \propto \frac{1}{t}$$



reduces with thickness
∴ ↑ thickness for reduced motional resistance
again limited by etch aspect ratio (t/g)

Part (a) was generally well done but student had difficulties estimating the displacement in (b) and deriving expressions for the motional parameters in (c) though these concepts had been reviewed in the lectures and examples class.

Q3 (a)

$$F^{NET}_{electrostatic} = \left[\frac{\epsilon_0 A}{(g-x)^2} - \frac{\epsilon_0 A}{(\alpha g_0 + x)^2}\right]\frac{N V_p^2}{2}$$

$\alpha = 10/1$ in this case

For pull-in $\frac{\partial F^{NET}}{\partial x} = k$

$$N \epsilon_0 A V_{PI}^2 \left[\frac{1}{g_0^3} + \frac{1}{(\alpha g_0)^3}\right] = k$$

$$V_{PI} = \sqrt{\frac{k g_0^3}{N \epsilon_0 A}\left(1 + \frac{1}{\alpha^3}\right)} = 10.6\,V$$

lower bound $V_{PI} = \sqrt{\frac{8}{27}\frac{k g_0^3}{\epsilon_0 A}} = 5.79\,V$ (from datasheet)

(b)

$$\Delta C_{NET} = N\left(\frac{\epsilon_0 A}{g_0 - x}\right) - N\left(\frac{\epsilon_0 A}{\alpha g_0 + x}\right)$$

$$= \frac{N\epsilon_0 A}{g_0}\left[1 + \frac{x}{g_0} - \frac{1}{\alpha}\left(1 - \frac{x}{\alpha g_0}\right)\right]$$

$$\therefore \frac{\Delta C_{NET}}{C_{NET}} \simeq \frac{N\epsilon_0 A / g_0}{\frac{N\epsilon_0 A}{g_0}} \frac{\left(\frac{x}{g_0}\right)\left(1 + \frac{1}{\alpha^2}\right)}{\left(1 - \frac{1}{\alpha}\right)}$$

$$\therefore x \simeq \frac{ma}{k} = \frac{a}{\omega_0^2}$$

$$\therefore \frac{\Delta C_{NET}}{C_{NET}} = \frac{a}{g\,\omega_0^2}\left(\frac{\alpha^2 + 1}{\alpha^2 - \alpha}\right) \simeq 0.011 \text{ (plugging in values)}.$$

(c)
$$\bar{a}_n = \sqrt{\frac{4k_B T b}{m}}$$
$$= 1.286 \times 10^{-6} \quad m/s^2/\sqrt{Hz}$$

(d)
$$x \simeq 0.1 g_0$$
$$\therefore \quad a = \frac{100}{10^{-6}} \times 0.1 \times 10^{-6} = 10\, m/s^2$$ (at the upper end of dynamic range)

$\therefore$ dynamic range is $10\, m/s^2$

$$\frac{\Delta C_{true}}{C_{true}} \simeq \left(\frac{N\epsilon_0 A}{g_0 - g_0/10} - \frac{N\epsilon_0 A}{\alpha g_0 + g_0/10} \right) \frac{1}{\frac{N\epsilon_0 A}{g_0}\left(\frac{\alpha-1}{\alpha}\right)} - \frac{\frac{N\epsilon_0 A}{g_0}\left(\frac{\alpha-1}{\alpha}\right)}{\frac{N\epsilon_0 A}{g_0}\left(\frac{\alpha-1}{\alpha}\right)}$$

$$\simeq \frac{\frac{N\epsilon_0 A}{g_0}}{\frac{N\epsilon_0 A}{g_0}\left(\frac{\alpha-1}{\alpha}\right)} \left[\frac{1}{1-\frac{1}{\alpha}} - \frac{1}{\alpha + \frac{1}{\alpha}} - \left(\frac{\alpha-1}{\alpha}\right) \right]$$

$$\therefore \frac{\Delta C_{true}}{C_{true}} \simeq 0.125$$

Linear approximation gives
$$\frac{\Delta C_{est}}{C_{est}} = 0.11$$

$\therefore$ difference $= 0.125 - 0.11 = 0.015$ or 13.2 % variation due to transducer nonlinearity

Very few students were able to obtain both lower and upper bounds for the pull-in voltage in (a). Parts (b) and (c) were generally well done but students had difficulties estimating the dynamic range in (d) and estimating deviation from linear response.

Q4 (a)

$$U_0 = -\frac{\epsilon \times \zeta}{\eta} \times E_x$$

$$= \frac{-80 \times 8.85 \times 10^{-12} \times 0.1}{10^{-3}} \times \frac{100}{0.01}$$

$$= 7.08 \times 10^{-4} \ m/s.$$

$$\therefore AU_0 = 10^{-8} \times 7.08 \times 10^{-4} = 7.08 \times 10^{-12} \ m^3/s.$$

(b)

$$Q = \frac{h w^3 k}{12\eta} = 7.08 \times 10^{-12}$$

$$\therefore k = \frac{7.08 \times 10^{-12} \times 12 \times 10^{-3}}{10^{-16} \times}$$

$$\therefore k = 849.6$$

$$\therefore \Delta P \simeq k \cdot x \simeq 8.49 \ Pa.$$

(c)

$$U_0 = -\frac{\epsilon \times \zeta}{\eta} \times \frac{200}{0.006}$$

$$= 2.36 \times 10^{-3} \ m/s.$$

(d) At 4mm : $t = \frac{4 \times 10^{-3}}{2.36 \times 10^{-3}} = 1.69s.$

$\Delta\mu \, E.t$ = separation distance

$$10^{-8} \times \frac{200}{0.006} \times 1.69 = 563 \ \mu m$$

(e) The species diffuse in solution such that the band size = $\sqrt{Dt}$, $t \simeq L/u_0$

∴ use short columns, large electric fields and low ionic strength buffers to achieve large L_D and good separation while ensuring plug flow behaviour for transporting bulk electrolyte.

This question was generally well done. Some students had difficulties with part (e) that required discussing optimisation of device design and geometry.