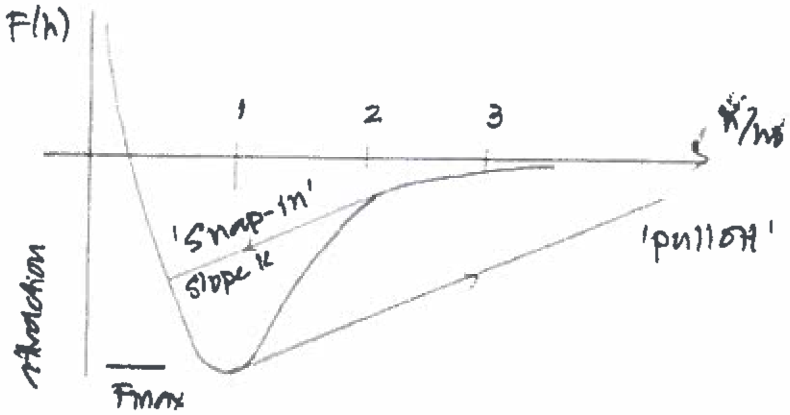
1

(a) Expression for F(h) contains both attractive + repulsive terms

$$F(h) = \frac{8\pi R w}{3}\left\{ \underbrace{\left(\frac{h}{h_0}\right)^{-2}}_{\text{attraction}} - \underbrace{\frac{1}{4}\left(\frac{h}{h_0}\right)^{-8}}_{\text{repulsion.}} \right\}$$

Thus plot of F(h) vs h/h_0 will have form shown plotting attraction ↓



at Fmax $\frac{dF(h)}{dh} = 0$ ie. $\frac{8\pi R w}{3}\left\{ -\frac{2}{h_0}\left(\frac{h}{h_0}\right)^{-3} + \frac{2}{h_0}\left(\frac{h}{h_0}\right)^{-9}\right\} = 0$

$$\therefore \quad h/h_0 = 1$$

By virtue of Derjaguin approximation h_0 is also the equilibrium separation of two parallel surfaces of same material (see lecture notes section 1.3)

(b) As z increases then separation of surfaces h reduces. When slope of non-linear relation between force of attraction F(h) becomes equal to stiffness of linear spring the two surfaces will snap together.

ie. $\frac{dF(h)}{dh} = \frac{8\pi R w}{3}\frac{2}{h_0}\left\{ -\left(\frac{h}{h_0}\right)^{-3} + \left(\frac{h}{h_0}\right)^{-9}\right\} = -k$

If we can assume that $h \gg h_0$ so that $(h/h_0)^{-9} \ll (h/h_0)^{-3}$

then $\frac{8\pi R w}{3} \cdot \frac{2}{h_0}\left(\frac{h_0}{h}\right)^3 = k$

So that $h = \left(\frac{16\pi R w h_0^2}{3k}\right)^{1/3}$

1

(c) When direction of motion is reversed the force exerted by the spring on the junction will grow. The surfaces will ~~sep~~ separate when once again $\frac{dF(h)}{dh} = |k|$. But this must be very close to conditions of F_{max} when $h/h_0 = 1$

$$\therefore P = F_{max} = \frac{8\pi R w}{3}\left\{1^{-2} - \frac{1}{4}1^{-4}\right\} = \underline{2\pi R w}$$

(d) If at this point spring extension is 12×10^{-9} m and $k = 20$ N m^{-1}

then $F_{max} = 12\times10^{-9}\times 20 = 2.4\times10^{-7}$ N

from relation in (c) $w = \frac{2.4\times10^{-7}}{2\pi\times R}$

ie. $w = \frac{2.4\times10^{-7}}{2\pi\times 65\times10^{-8}} \Rightarrow \underline{59 \text{ mN m}^{-1} \text{ or mJ m}^{-2}}$

(e) If indenter and specimen of same material

$w = 2\gamma$ so $\underline{\gamma = P/4\pi R}$

(f) If ceramic indenter replaced by soft polymer then there is likely to be significant elastic deformation so forming a Hertzian contact. If surfaces are clean, ie. relatively uncontaminated, then there will be additional adhesion effects. These could be analysed by JKR (Johnson Kendall Roberts) formulation. See Lecture notes section 1.6).

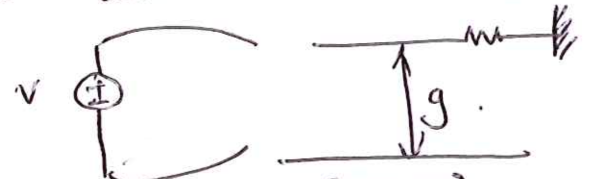
The use of an AFM to evaluate surface energies is described in:

Awala, Castelein & Brogly "Surface and Interface Analysis" 37 (2005) 755-64 and

Tevis, Jacobs et al "Tribology Letters" 59 (2015) 39-48

Q2 (a)

For a comb drive unit cell



t – structural thickness, $l-x$ (overlap length)

$$W = \frac{\frac{1}{2}\epsilon_o (l-x)\, t\, V^2}{g}$$

$$F = -\frac{\partial W}{\partial x} = \frac{\epsilon_o t V^2}{2g}$$ and for N gaps we have

$$F_N = \frac{N\epsilon_o t V^2}{2g} = \frac{N\epsilon_o t}{2g}(V_p - V_{ac})^2$$

(b) $$i = \frac{dQ}{dt} = V_p \frac{d}{dt} C_{sense}$$

$$= V_p \frac{N\epsilon_o t}{g}\left(\frac{-\partial x}{\partial t}\right) = -\frac{V_p N \epsilon_o t}{g}\dot{x}$$

(c) $$|i| = \left(\frac{N\epsilon_o t V_p}{g}\right)^2 \cdot \frac{Q}{k} \cdot V_{ac} \cdot \omega_{res}$$ for symmetric drive and sense.

$$\simeq 1.16 \times 10^{-10}\ \text{A}$$

(d) Equivalent motional parameters obtained by drawing analogies between mechanical and electrical domains

mechanical transfer function

$$\frac{\dot{x}}{F} = \frac{1}{sm + b + k/s}$$

$$\frac{i/\eta}{\eta V_{ac}} = \frac{1}{sm + b + k/s}$$

$$\therefore \frac{i}{V_{ac}} = \frac{\eta^2}{sm + b + k/s}$$

$\Downarrow$

$$\frac{i}{V_{ac}} = \frac{1}{sL_m + R_m + \frac{1}{sC_m}}$$

where η = electromechanical transduction coefficient

$$\eta = \frac{N \epsilon_0 t V_p}{g} \quad \text{for this case.}$$

$$\Rightarrow C_m = \frac{\eta^2}{K}, \quad L_m = \frac{m}{\eta^2}, \quad R_m = \frac{b}{\eta^2}$$

$$\Rightarrow C_m = 3.65 \times 10^{-15} F, \quad L_m = 0.27 \text{ MH}, \quad R_m = 0.866 \Omega.$$

(e) Process scaling options include

→ reduced gaps (limited by lithography and etch constraints

→ increase thickness-to-gap ratio (limited by etch aspect ratio, thickness of structural films).

→ increase η by incorporating other transduction approaches e.g. employing piezoelectric films such as AlN.

→ increase Q (change gap to substrate for example).

Design scaling options include.

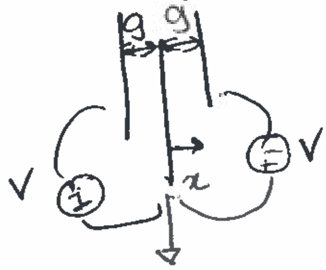
→ increase N (number of gaps, also increases mass of resonator, damping in air)

→ increase Quality factor (approaches to vacuum encapsulation).

→ increase V_p (limited by pull-in effects e.g. side-ways instability for this device)

→ parallel-plate electrode schemes (higher electromechanical coupling but limited by pull-in).

3 (a) For this capacitive unit cell.



$$W = \frac{1}{2}\frac{\epsilon_0 A}{(g-x)} V^2 + \frac{1}{2}\frac{\epsilon_0 A}{g+x} V^2$$

$$W_{net} = \frac{N \epsilon_0 A V^2}{2}\left[\frac{1}{g-x} + \frac{1}{g+x}\right].$$

$$F = \frac{-\partial W_{net}}{\partial x} = \frac{-N \epsilon_0 A V^2}{2}\left[\frac{1}{(g-x)^2} - \frac{1}{(g+x)^2}\right]$$

$$\frac{\partial F}{\partial x} = N \epsilon_0 A V^2 \left[\frac{1}{(g-x)^3} + \frac{1}{(g+x)^3}\right].$$

The pull-in condition is $F_{NET} = 0$ and $\frac{\partial F}{\partial x} = k$

$$F_{NET} = \frac{-N \epsilon_0 A V^2}{2}\left[\frac{1}{(g-x)^2} - \frac{1}{(g+x)^2}\right] + kx = 0$$

Trivial solution $x = 0$

$$\therefore \frac{2 N \epsilon_0 A V_{PI}^2}{g^3} = k$$

or $$V_{PI} = \sqrt{\frac{k g^3}{2N \epsilon_0 A}}$$

(b) $$C_{NET}(x) = N\left[\frac{\epsilon_0 A}{g-x} + \frac{\epsilon_0 A}{g+x}\right]$$

$$\frac{\Delta C}{C_{NET}(x=0)} = \frac{N\left[\frac{\epsilon_0 A}{g-x} - \frac{\epsilon_0 A}{g+x}\right]}{\frac{2 N \epsilon_0 A}{g}} = \frac{2xg}{2(g^2 - x^2)}$$

For $x \ll g \Rightarrow \frac{\Delta C}{C_{NET}(x=0)} \approx \frac{x}{g} = \frac{ma}{kg} \approx 10^{-2}$.

(c) $$a_n \approx \frac{\sqrt{4 k_B T b}}{\cancel{k} m} \quad m/s^2/\sqrt{Hz}.$$

$$a_n \simeq 1.28 \times 10^{-4} \; m/s^2/\sqrt{Hz}$$

(d) From (b) we have

accelerometer response $$\frac{\Delta C}{C_{NET}(x=0)} \simeq \frac{a}{\omega_n^2 \, g}$$

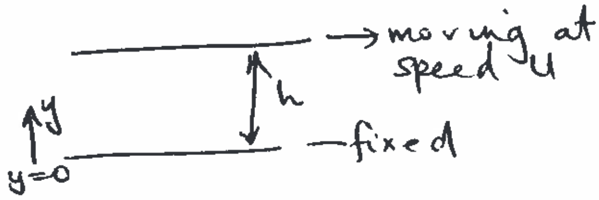
$$\frac{\partial}{\partial T}\left(\frac{\Delta C}{C_{NET}(x=0)}\right) = \frac{-2a}{\omega_n^3 \, g}\left(\frac{\partial \omega_n}{\partial T}\right).$$

$\therefore$ shift in device response $= -\dfrac{2 \times 10}{10^{-6}} \times \dfrac{(-30 \times 10^{-6}) \times 1K}{(10^9)}$

$= 6 \times 10^{-7}$

relative shift in response $\simeq \dfrac{6 \times 10^{-7}}{10^{-2}} \simeq 60\,ppm.$

Q4 (a) $\frac{\partial^2 V_x}{\partial y^2} = 0$



$\therefore \quad U_x = \frac{y}{h} U$

$\tau_w = -\eta \frac{\partial U_x}{\partial y} = -\frac{\eta U}{h}$

and retarding force $= \tau_w A = \frac{\eta U A}{h} = c \cdot U$

$\therefore \quad c = \frac{\eta A}{h}$.

(b)

$$Q \approx \frac{m\,\omega_0}{c} = \frac{10^{-11} \times 2330 \times 2\pi \times 10^4}{1.8 \times 10^{-5} \left[\frac{10^{-6}}{2\times 10^{-6}} + \frac{30\times 10^{-6} \times 10^{-5}}{10^{-6}}\right]}$$

$$= 162$$

(c)

$$x \approx \frac{F}{k} \cdot Q$$

$$= N \epsilon_0 \frac{t}{g} V_p \cdot V_{ac} \cdot \frac{Q}{m \cdot \omega_d^2}$$

$$= \frac{30 \times 8.85 \times 10^{-12} \times \frac{10}{1} \times 10^2 \times 0.1 \times 162}{(2330 \times 10^{-11}) \times (2\pi \times 10^4)^2}$$

$$= 46.8 \text{ nm}$$

(d) By using case (a) in CUED databook.

$$\left|\frac{kY}{F_c}\right| = \frac{1}{\left\{\left[1-\left(\omega/\omega_n\right)^2\right]^2 + \left(\omega/\omega_n Q\right)^2\right\}^{1/2}}$$

$$= \frac{1}{\left\{\left[1-\left(1/1.1\right)^2\right]^2 + \left(\frac{1}{1.1 \times 162}\right)^2\right\}^{1/2}}$$

$$Y_c/k = \frac{2m\Omega_z(2\pi f_d \cdot x_d)}{k_s} = \frac{\Omega_z f_d x_d}{\pi f_s^2}$$

$$\therefore Y = \frac{5.76 \times 1 \times 10^4 \times 46.8 \times 10^{-9}}{\pi \times (1.1)^2 \times 10^8}$$

$$= 7.09 \text{ pm}$$

In practice Q_y is lower due to squeeze film drag between adjacent comb fingers.

(e) An estimate for thermo-mechanical noise is obtained by equating the Coriolis force to the thermal force generator (in magnitude)

$$2m\Omega_{zn}\omega_d x_d = \sqrt{4k_B T b_y}$$

$$\Omega_{zn} = \sqrt{\frac{k_B T b_y}{m^2\omega_d^2 x_d^2}}$$

$$= \sqrt{\frac{k_B T \omega_s}{Q_y m \omega_d^2 x_s^2}}$$

$$\simeq 2.96 \times 10^{-3} \text{ rad/s}/\sqrt{\text{Hz}}$$

(f) Quadrature error arises from undesired elastic coupling between drive and sense axes. There are several physical origins for this effect e.g. due to misalignment of drive and sense axes with the principal axes of elasticity and this coupling can result in a response orders of magnitude higher than that due to the Coriolis force in a practical gyroscope implementation. Feedforward cancellation approaches can be employed by noting that the Coriolis and quadrature signals are 90° out of phase

Q1 Nano-indenter (average 14.75/15)

This question was very well done as indicated by the high average mark for the Part IIB students displaying an excellent understanding of the underlying concepts.

Q2 Comb drive resonator (average 12.5/15)

This question was generally well done as indicated by the high average mark. Some students had difficulty deriving the expression for motional current in (b) and calculating the numerical value in (c). Others were not able to derive expressions for the motional parameters of the resonator in (c).

Q3 Capacitive accelerometer (average 10.7/15)

The electrostatic “pull-in” analysis was generally well done in (a). Parts (b) and (c) were also generally well done but some students had difficulties with estimating the temperature sensitivity of the accelerometer scale factor in (d).

Q4 Gyroscope (average 9.7/15)

Most students were able to do parts (a)-(c) correctly. Some students had difficulty with applying databook equations for calculations in (d), and some students did not do part (f) correctly with responses indicating some misunderstanding of the term “quadrature error”.