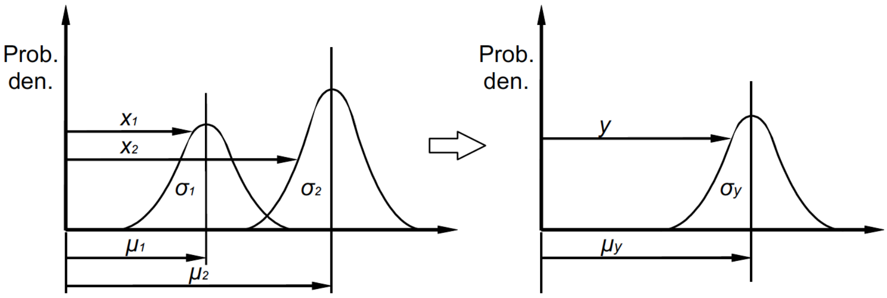
Question 1

a) From the notes, the Margin of Safety is the difference between a measured distributed variable and a target distributed variable. For example, the clearance between the stamped blank diameter (140 mm $\pm$ 0.1 mm) and the spacing between blank centres (150 mm $\pm$ 0.1 mm) could be a Margin of Safety for ensuring each blank is stamped cleanly from the sheet.

To determine the probability of failure, the probability density for the margin of safety can be determined using the rules for combining distributed variables, as shown in the diagram below.



$y = a_1 x_1 + a_2 x_2$ $\mu_y = a_1 \mu_1 + a_2 \mu_2$ $\sigma_y^2 = a_1^2 \sigma_1^2 + a_2^2 \sigma_2^2$

b) The clearance is expressed as a minimum, therefore we are only interested in the linear distance between the centres of two blanks. The space between adjacent blanks depends on both the distance between centres and the diameter of the blanks. Therefore this space must be calculated first, and then compared to the minimum clearance required (9.9 mm $\pm$ 0.1 mm), to determine that fraction of damaged blanks, which is 4.2%. Details are given below:

	symbol	units	mean	+ / −	range	no. dev	dev
Blank diameter	ϕ_b	mm	140	0.1	0.2	6	0.0333
Spacing btwn centres	ϕ_s	mm	150	0.1	0.2	6	0.0333

	symbol	units	mean	+ / −	range	no. dev	dev
Min spacing btwn blanks	b	mm	9.9	0.1	0.2	6	0.0333
Spacing btwn blanks	$c = \phi_s - \phi_b$	mm	10.00				0.0471
Clearance	$d = c - b$	mm	0.1				0.0577

Probability (Pc<0)=1-P(c>0)	$z = -\mu_c / \sigma_c$	-1.73	
	$P(z)$	0.9584	*from chart*
	$1 - P(z)$	4.2%	*about 4% of the blanks will be damaged*

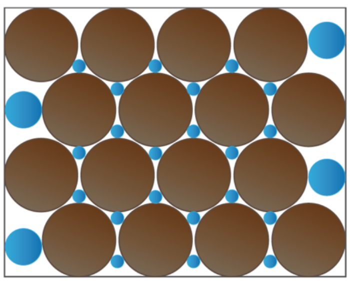
c) The target of 25% maximum aluminium scrappage rate translates to a minimum 75% material yield through the stamping process. This yield ratio is the area of the blanks produced (i.e. 10 cans at 140 mm ± 0.1 mm) divided by the area of sheet required for 10 cans. The sheet area is calculated by finding the sheet width and multiplying by the *repeating length*, with standard deviations carried through as per Table 1. The sheet width equals the spacing between blank centres (150 mm ± 0.1 mm) multiplied by 10.5 (the additional half spacing comes from the alternating left/right space at the sheet edge caused by the grid pattern). The repeating length is equal to 150 x sin(60°)=129.9 mm. This gives a packing ratio of 75.2% and the fraction of stamping runs above the 75% yield rate equals 98.4%. Details are given below:

	symbol	units	mean	+ / –	range	no. dev	dev
Cans		no.	10				
Extra space at end (half circle)		no.	0.5				
Sheet width	w	mm	1575				0.350
Repeating length	l	mm	129.90				0.029
Area of blank	A_b	mm2	2.05E+05				64.3
Area of 10 cans	A_c	mm2	1.54E+05				222.1
Calculated packing density	ρ	%	75.2%				0.11%
Target packing density	ρ_t	%	75%				0.00%
Difference	$c = \rho - \rho_t$	%	0.2%				0.11%

Probability (Pc<0)=1-P(c>0)	$z = -\mu_c / \sigma_c$	-2.15	
	$P(z)$	98.4%	*from chart, 98.4% of the sheets < 25% scrap*

d) A variety of answers is possible, but a mix of both process and design changes is required for full marks. Possible answers include:

- reducing the clearance between the blanks, by improving the stamping process or moving to an alternative cutting method such as laser cutting. Eliminating the clearance altogether would improve the yield to 80.6%.
- increasing the width of the sheet means the additional half spacing is spread over more blanks. For example, having 20 blanks across the sheet would improve the yield to 77.7%.
- changing the beverage can design to use hexagon blanks (see example, left).
- making a rectangular cuboid shaped can, with tessellating parts.
- cutting small items out of the skeleton left behind (see example, right).

This was a popular question, but more difficult than previous probability questions. Many students struggled to define the Safety Margin and Probability of Failure/Density accurately in part a). Part b) was answered well by most students, as it followed the form of previous questions. However, students struggled in part c) to define the repeating area of sheet aluminium needed for the blanking process (simple geometry), and therefore found giving a probability of failure difficult. Suggestions for reducing yield loss were innovative and answered well.

Question 2

A product currently consists of three components C_1, C_2 and C_3 with corresponding individual probabilities of failure P_1, P_2 and P_3 and reliabilities $R_1 = (1 - P_1)$, $R_2 = (1 - P_2)$ and $R_3 = (1 - P_3)$.

a) Question: Derive an expression for the *reliability* of the product if the components are wired in *series*, which means all of the components must operate satisfactorily for the product to function correctly. (10%)

Answer: R_{series} is the probability of three independent events of non-failure. Therefore $R_{\text{series}} = R_1 R_2 R_3$.

b) Question: Derive an expression for the *reliability* of the product if the components are wired in *parallel*, which means all of the components must fail for the product to fail. (10%)

Answer: The probability of all three components failing is the product of the individual probabilities of each component failing: $(1 - R_1)(1 - R_2)(1 - R_3)$. Therefore $R_{\text{parallel}} = 1 - (1 - R_1)(1 - R_2)(1 - R_3)$.

c) Question: The hazard function is defined as $\lambda(t) = \frac{f(t)}{R(t)}$, where $f(t)$ is the probability density function and $R(t)$ is the reliability function. Show that the hazard function is constant when the time t before a failure is exponentially distributed. Comment on the significance of this result when determining the failure rate of a product. (30%)

Answer: The exponential probability density function is $f(t) = \lambda e^{-\lambda t}$. The reliability function is $R(t) = 1 - F(t) = 1 - (1 - e^{-\lambda t}) = e^{-\lambda t}$ (where $F(t)$ is the cumulative distribution function of an exponential distribution). The hazard function is therefore λ and thus constant when the time t before a failure is exponentially distributed.

A constant hazard function arises because of the memoryless property of the exponential distribution. The distribution of a product's remaining time until a failure, given that the product has not failed in time t, does not depend on t.

d) Question: Assume each component has a constant failure rate, the probability of a failure of an individual component is independent of the other components and $P_1 = P_2 = P_3$. The mean time between failures ($MTBF$) for an individual component is 100,000 hours. Calculate the probability of the product failing within a 5-year warranty period when i) all three components wired in series and ii) all three components are wired in parallel. (40%)

Answer: As the failure rate is constant $\lambda(t) = \lambda$. Therefore the reliability function is $R(t) = 1 - F(t) = 1 - (1 - e^{-\lambda t}) = e^{-\lambda t}$ (where $F(t)$ is the cumulative distribution function of an exponential distribution).

The failure rate is $\frac{1}{100000} = 0.00001$ failures per hour. The reliability of an individual component in the given time interval is therefore $e^{-\lambda t} = e^{-0.00001 \times 24 \times 365 \times 5} \approx 0.65$.

The reliability of the product in the given time interval when all three components are wired in series is $R_{\text{series}} = R_1 R_2 R_3 = (0.65)(0.65)(0.65) \approx 0.27$. The probability of failure of the product when all components are in series is therefore approximately $1 - 0.27 = 0.73$.

The reliability of the product in the given time interval when all three components are wired in parallel is $R_{\text{parallel}} = 1 - (1 - R_1)(1 - R_2)(1 - R_3) = 1 - (1 - 0.65)(1 - 0.65)(1 - 0.65) \approx 0.96$. The probability of failure of the product when all components are in parallel is therefore approximately $1 - 0.96 = 0.04$.

e) Question: State the definition of *value* in value engineering and calculate the gain in *value* for the parallel wiring configuration of the product, assuming identical component and assembly costs. (10%)

Answer: Value is the ratio of function to cost. The only function provided is the reliability of the product. As the denominator is unity for both the series and the parallel wiring configurations the value is 0.96 for parallel wiring and 0.27 for series wiring. The gain in value is approximately 356%.

This was a new question, and the least popular. A few candidates did excellent modelling and calculations and received very high marks. Many candidates made a series of mistakes when calculating the probabilities of the serial and parallel configurations and/or found it difficult to calculate failure probabilities for the scenario posed in the question. Candidates did well at the value engineering part of the question.

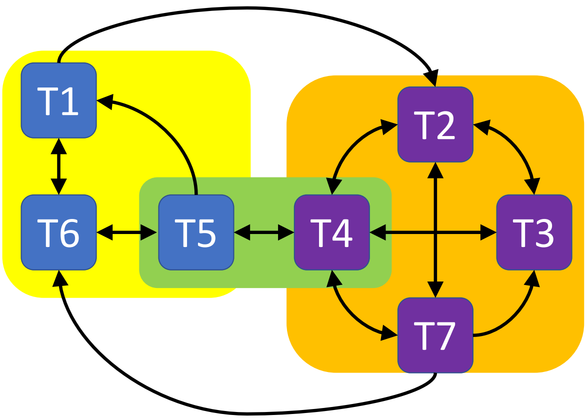
Question 3

a) Clustering involves reordering the tasks (rows and columns) to find subsets of DSM tasks, known as clusters, that are mutually exclusive or minimally interacting subsets. Clusters absorb most, if not all, of the interactions internally and the interactions or links between separate clusters are eliminated or at least minimised.

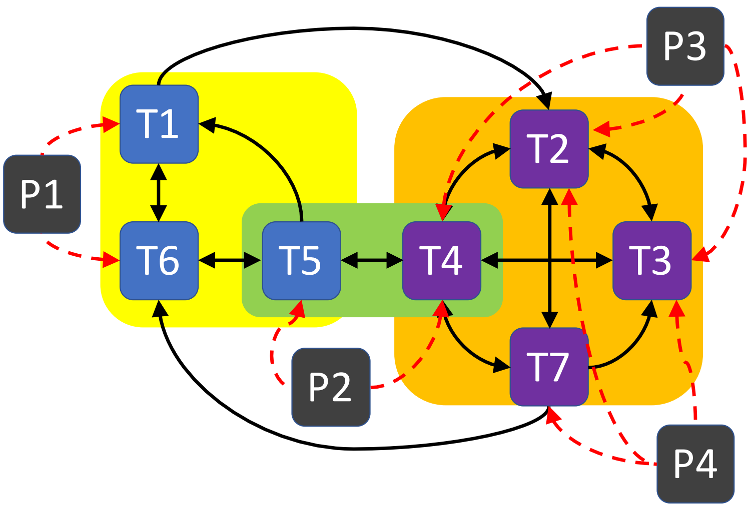
Several different clustering options are possible, but in each case three clusters should emerge and be clearly highlighted. Note the interactions in the task to person Domain Mapping Matrix (DMM) also need to be reordered. The reordered MDM could look like:

	task 1	task 6	task 5	task 4	task 2	task 3	task 7	person 1	person 2	person 3	person 4
task 1		X	X								
task 6	X		X				X				
task 5		X		X							
task 4			X		X	X	X				
task 2	X			X		X	X				
task 3				X	X		X				
taks 7				X	X						
person 1	X	X									
person 2			X	X							
person 3				X	X	X					
person 4					X	X	X				

b) The DSM is firstly mapped in a task diagram. The arrows showing interaction should be one-way or two-way as indicated by the DSM. The clusters from part a) should be clearly shown.



The task to person Domain Mapping Matrix (DMM) interactions are added to the diagram. The task to person interaction arrows should be clearly different from the task to task arrows.



c) The DSM (bottom right) should be filled in, taking note of the links between tasks and people from part b). In particular, person 1 performs tasks 1 and 6 without interaction with any other person. Persons 2, 3, and 4 interact on tasks 2-5 and 7. This is easier to see if part a) is completed correctly, but could also be spotted from the original MDM. A possible DSM configuration is:

	task 1	task 6	task 5	task 4	task 2	task 3	task 7	person 1	person 2	person 3	person 4
task 1		X	X								
task 6	X		X				X				
task 5		X		X							
task 4			X		X	X	X				
task 2	X			X		X	X				
task 3				X	X		X				
taks 7				X	X						
person 1	X	X									
person 2			X	X						X	X
person 3				X	X	X			X		X
person 4					X	X	X		X	X	

d) From the lecture notes: structural diagramming methods are particularly useful for defining the system architecture while behavioural diagramming methods assist in the definition of the detail and may then be used to directly support risk assessment.

- structural systems mappings diagrams include task diagrams, information diagrams and organisational diagrams.
- behavioural systems mappings diagrams include system diagrams, flow diagrams and communication diagrams.

This was not a popular question, however students that persevered scored well in this question. Many students found two clusters, but not the third cluster between tasks 4 and 5. Students struggled to draw coherent task diagrams, that clearly showed the task interactions, clustering and person DMM (Domain Mapping Matrix). The difference between structural and behavioral system mapping was poorly understood.

Question 4

People operating in certain areas run the risk of severe brain damage due to blast waves caused by high explosives. A particular problem in triage is to quickly and accurately diagnose whether a person without visible physical injuries requires immediate medical attention due to the risk of unobservable brain trauma. Due to resource constraints it is not viable to give full medical attention to all people in the vicinity of an explosion.Your company has been contracted to design a wearable system worn by users in these high risk areas. The wearable system will sense blast waves and based on this data estimate the risk of brain trauma on the wearer and communicate this status to medical personnel on site.

a) Question: Suggest one type of wearable sensor suitable for estimating the damage caused by a blast wave on the user. Motivate your answer. (10%)

Answer: As a blast wave changes pressure a pressure sensor is suitable.

b) Question: State the overall function of the wearable device using a solution-neutral problem statement. (10%)

Answer: Design a wearable pressure sensor system capable of indicating the likelihood of a brain trauma on the user in the event of a blast wave.

c) Question: List five critical requirements for the wearable device. (25%)

Answer: Examples of critical requirements include (all essential):

1. The system is effective in diagnosing brain trauma (e.g. $> 95\%$ precision). Source: For instance, medical professional opinion.
2. A small form factor that ensures the system is worn at all times (e.g. no larger than 20 mm (w) $\times$ 20 mm (h) $\times$ 10 mm (d)). Source: For instance, human factors engineers or via on-site observations and/or interviews with domain experts.
3. Long battery life (e.g. a device should work continuously for four weeks without charging). Source: For instance, procurement officer.
4. Cost-effective (e.g. maximum 100 unit cost per device). Source: For instance, procurement officer.
5. The system is easy-to-use in triage (e.g. clear visibility / legibility of brain trauma status in presence of background noise (up to 90 dB) and up to 1 m smoke visibility). Source: For instance, on-site observations and/or interviews with domain experts.
6. The system is securely fastened to the user. Source: For instance interviews with domain experts.

d) Question: Discuss how the requirements set out in (c) can be verified. (25%)

Answer: Verification is the process of ensuring requirements are met. Requirements on effectiveness in diagnosis need to be verified in medical testing procedures. Requirements on weight, size, battery life and other similar characteristics can be measured or lab-tested. Unit cost can be verified by taking into account volume, manufacturer and bill of materials. Ease of use for triage can be verified in simulation (computer simulation and simulation of physical environment). Ergonomics, such as whether the system is securely fastened can be verified by calculation and lab-based stress testing.

e) Question: Identify solution principles that address the requirements in (c) and describe the design of a system that addresses the design brief set out in the question. (30%)

Answer: Solution principles are likely focussed on achieving a *light* device, which is *effective* in indicating likelihood of brain trauma, *inexpensive*, *comfortable* to wear (so users do not take them off) and can run for long periods of time without charging. A pressure sensor can be made small but will need to be attached to several parts of the body to be effective, at the very least to the front and to the back of the body. Attaching a sensor to headwear (such as a helmet) is unlikely to be a robust solution. Effectiveness means careful calibration needs to be carried out to map pressure to likelihood of brain trauma.

This was a popular question. Historically candidates have struggled with design-oriented questions at the exam. This year, however, candidates were much stronger and did excellent analyses of the sensor configuration required, arrived at well thought-out solution-neutral problem statements, requirements, verification steps for the requirements, and identification of viable solution principles.