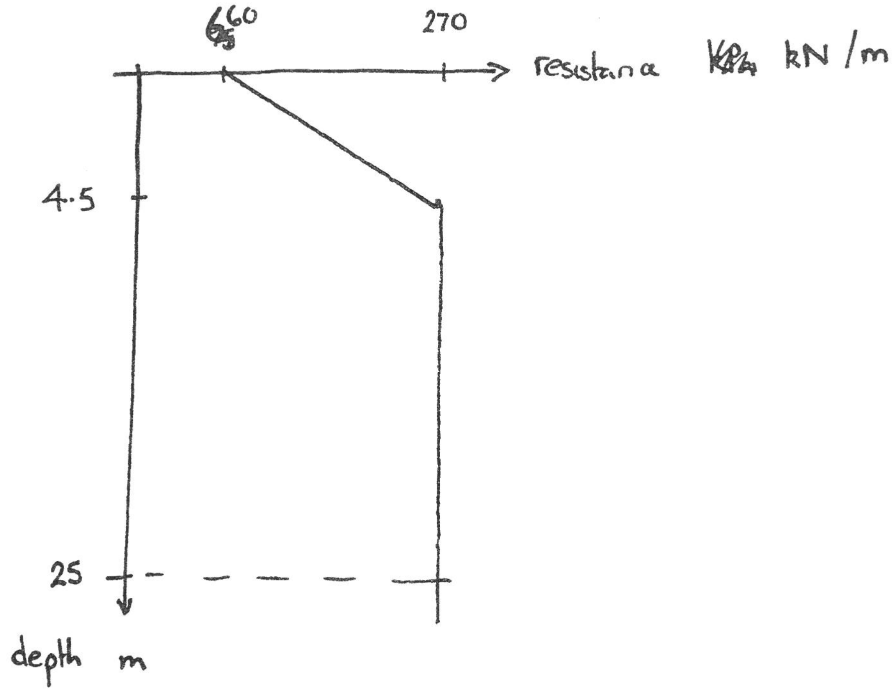
1. From databook



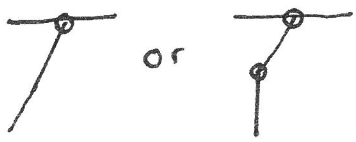
b) Pile restrained at head no bending failure so pile translates through soil

Resistance is area to left of graph..

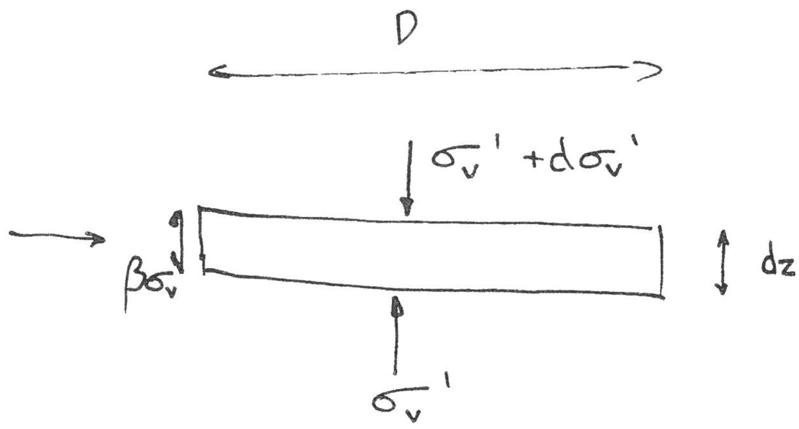
$$F = 60 \times 25 + 210 \times 20.5 + \tfrac{1}{2} \times 210 \times 4.5$$

$$= 6.2775 \text{ MN}$$

c) Failure by bending is either due to a single hinge at top or two at top and at depth.



c)



$$\uparrow \quad -\frac{\cancel{\pi} D^{\cancel{2}}}{4} d\sigma_v' = \cancel{\pi}\cancel{D}\beta\sigma_v'\, dz + \frac{\cancel{\pi} D^{\cancel{2}}}{4}\gamma' dz$$

$$\frac{d\sigma_v'}{\cancel{dz}\, dz} = -\underbrace{\left(\frac{4}{D}\beta\sigma_v' + \gamma'\right)}_{\cancel{\alpha}} \cancel{dz}$$

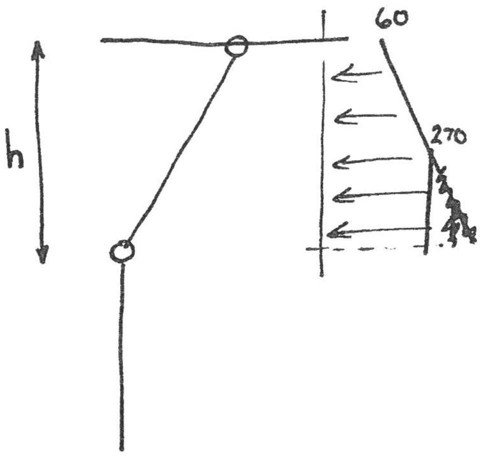
$$-\int_{q'_{plug}}^{0} \frac{d\sigma_v'}{\frac{4}{D}\beta\sigma_v' + \gamma'} = \int_0^h dz$$

$$\left[\frac{D}{4\beta}\ln\left[\frac{4\beta}{D}\sigma_v' + \gamma'\right]\right]_{q_{plug}}^{0} = -h$$

$$\ln\gamma' - \ln\left(\frac{4\beta}{D} q_{plug} + \gamma'\right) = -\frac{4\beta h}{D} = -\lambda$$

$$\frac{\frac{4\beta}{D} q_{plug} + \gamma'}{\gamma'} = e^{+\lambda}$$

Consider 2 hinges If $h > 4.5\,m$



Moments	Shear forces
$-M_p$	H
M_p	0

Taking moments about soil surface:

$$2M_p = \frac{60h^2}{2} + \frac{210 \times 4.5}{2} \times 3 + 210 \times (h-4.5) \times \frac{(h+4.5)}{2}$$

$$= 30h^2 + 1417.5 + 105h^2 - \cancel{10125}\ 2126.25$$

$$M_p = 67.5h^2 - 709$$

[$h > 4.5\,m \Rightarrow M_p > 658\,kNm$

Horizontal eqbm

$$H = 60h + 210(h-4.5) + \frac{210 \times 4.5}{2}$$

$$= 270h - 472.5$$

$$\therefore\ H = 270\sqrt{\frac{M_p + 709}{67.5}} - 472.5$$

d)

no hinge vs 1 hinge

$$540\sqrt{\frac{M_p + 85084}{270}} - 7222.5 = 6277.5$$

$$M_p = 83666 \text{ kNm}$$

no hinge vs 2 hinges

$$270\sqrt{\frac{M_p + 70[illegible]9}{67.5}} - 472.5 = 6277.5$$

$$M_p = 41478.5 \text{ kNm}$$

So $M_p > 83666$ kNm

$$t = \frac{M_p}{D^2 \sigma_y} = \frac{83.666}{1.5^2 \times 200} = 0.186 \text{ m} = 186 \text{ mm}$$

Most realistic piles will fail in bending with 1 hinge.

Q1

Not a very popular question looking at lateral loading on piles, but reasonably well handled by most students who attempted it fully. Several students were obviously running out of time and submitted incomplete solutions. Some students left the pile cap stationary and moved the pile, rather than vice-versa, leading to an incorrect assumption of relative pile-soil displacements.

2 a) As a pile is driven into sand:

As pile tip approaches, sand beneath tip experiences very high vertical stresses + moderate horizontal stresses

As sand is forced around tip, vertical stresses reduce but horizontal ones increase

As tip moves past, horizontal stresses relax.

Cyclic shearing of sand reduces horizontal stresses due to densification

b) Friction fatigue is caused by the cyclic contraction of sand around the pile shaft during repeated blows of the pile hammer.

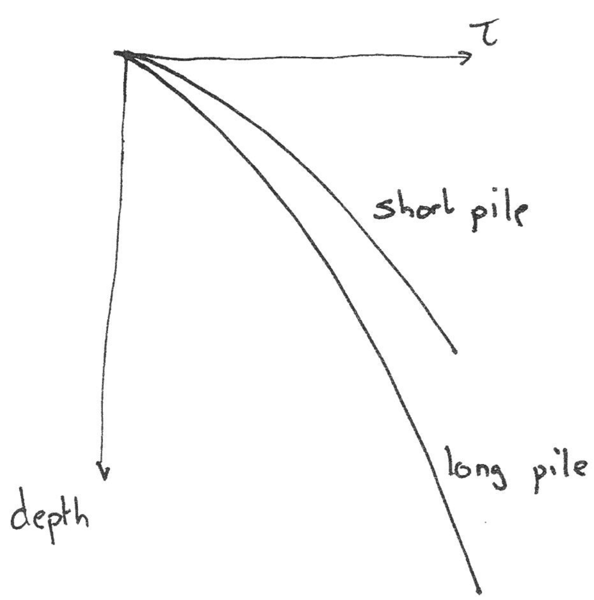
Due to axial shortening of the pile during hammering, the pile surface moves cyclically against the sand causing shear.

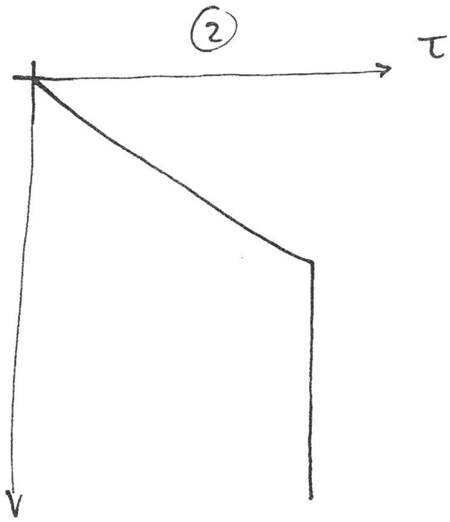
The sand subjected to cyclic shear densifies

Horizontal stresses reduce so limiting friction reduces.

API does not explicitly allow for this, however it applies limiting values to max skin friction.

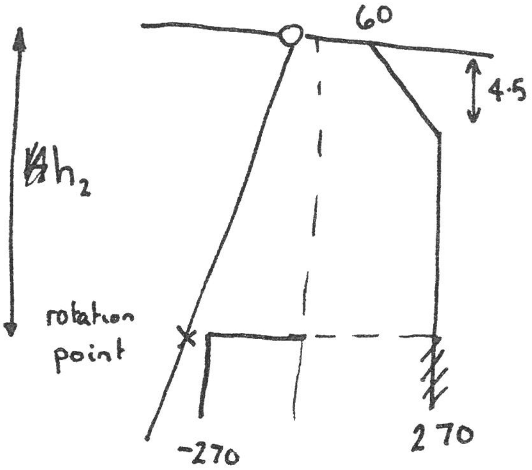
Real skin friction looks like 1 below, API like 2.





Integration gives similar effects.

Consider 1 hinge



Moment	Shear force
$-M_p$	H
0	0

Horizontal eqbm

$$H + 270\left(25 - h_2\right) = 270\left(h_2 - 4.5\right) + 60 \times 4.5 + \tfrac{1}{2} \times 210 \times 4.5$$

$$H = 540\, h_2 - 7222.5$$

Moment eqbm about head.

$$M_p = 270 \times 25 \times 12.5 - 540 \times (25 - h_2) \times \frac{(25 + h_2)}{2} - \tfrac{1}{2} \times 210 \times 4.5 \times 1.5$$

$$= 270\, h_2^2 - 85084$$

$$H = 540 \sqrt{\frac{M_p + 85084}{270}} - 7222.5$$

$$\tau_{plug} = \frac{\gamma' e^{+\lambda} - \gamma'}{\frac{4\beta}{D}}$$

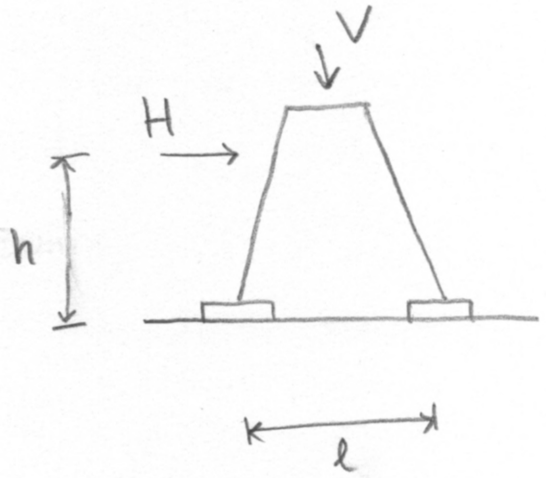
$$\frac{\tau_{plug}}{\gamma' h} = \frac{e^{+\lambda} - 1}{\lambda}$$

QED

Q2

A very popular question, answered by all candidates on friction fatigue and stresses during pile driving. Almost all candidates had a good grasp of the processes involved but varied in the completeness of their descriptions. The derivation of stresses on a plug was well handled by most candidates.

PROBLEM 3

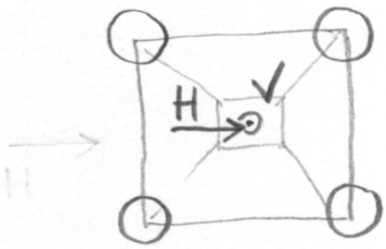


$h = 30\,m$

$l = 10\,m$

$S_u = 25\,kPa$

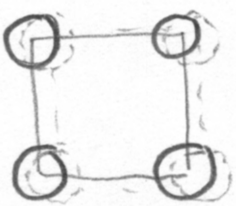
$V = 1\,MN$



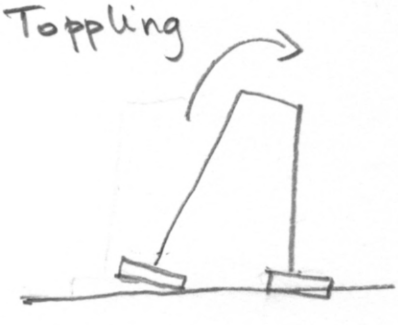
It is more conservative to assume H acts normal to one of the sides.

(a) Failure modes

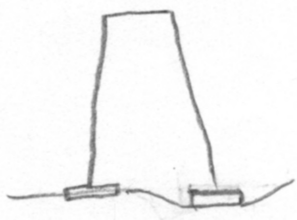
(i) Sliding



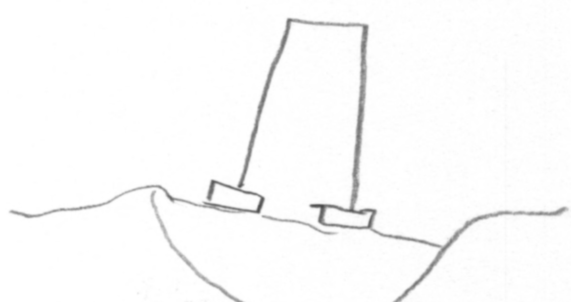
(ii) Toppling



(iii) Bearing capacity (one set of footings)



(iv) Global failure



(b) Need to assess failure modes (i) to (iii) -

(i) Sliding

For one footing

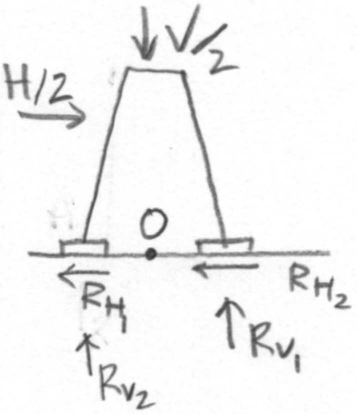
$$H_{ult,f} = A S_u = \frac{\pi d^2}{4} S_u = \frac{\pi (4m)^2}{4} S_u = 4\pi S_u$$

$$= 0.314 \text{ MN}$$

$H_{ult} \leq (4)(0.314 \text{ MN}) = 1.26 \text{ MN}$ for the structure

(ii) Toppling - foundation is about to lift off.

(consider one set of footings)



In general

$R_{H_1} = R_{H_2} = H/4$

$\Sigma M_o = 0$ $\quad \frac{H}{2} h + R_{V_1} \frac{l}{2} - R_{V_2} \frac{l}{2} = 0$

$$R_{V_2} - R_{V_1} = \frac{Hh}{l} \quad (1)$$

$\Sigma F_V = 0$ $\quad R_{V_1} + R_{V_2} = V/2 \quad (2)$

Solve (1) + (2) to obtain

$$R_{V_1} = \frac{V}{4} - \frac{Hh}{2l}$$

$$R_{V_2} = \frac{V}{4} + \frac{Hh}{2l}$$

At toppling, the foundation is about to lift off.

Therefore $R_{V_1} = 0$ OR $H \leq \frac{Vl}{2h} = \frac{(1MN)(10m)}{2(30m)} =$

$H_{ult} \leq 0.16 \text{ MN}$ for the whole structure

We also need to check if the load on the front legs is lower than bearing capacity. From data book

$$\frac{H_f}{H_{ult,f}} \le 1 - \left(2\frac{V_f}{V_{ult,f}} - 1\right)^2$$

$$V_{ult,f} = \xi_c(2+\pi) A S_u = (1.18)(2+\pi)(4\pi)(25\text{kPa}) = 1.91\text{ MN}$$

$$V_f = \frac{V}{4} + \frac{Hh}{2L} \qquad \text{with } H \le 0.16\text{MN}$$

Check for $H = 0.16\text{MN} \Rightarrow H_f = \frac{0.16\text{MN}}{4} = 0.04\text{MN}$

$$\frac{0.04\text{MN}}{0.314\text{MN}} \le 1 - \left[2\,\frac{0.25\text{MN} + \frac{(0.04)(30\text{m})}{2(10\text{m})}}{1.91\text{MN}} - 1\right]^2$$

$0.127 \le 0.773$ ✓ ok — No bearing capacity failure

(ii) For tension allowed in back footing, find maximum bearing load in front footing

$$\frac{H_f}{H_{ult,f}} \le 1 - \left(2\frac{V_f}{V_{ult,f}} - 1\right)^2$$

where H is not constrained

$$\frac{H/4}{AS_u} \le 1 - \left[2\,\frac{\frac{V}{4} + \frac{Hh}{2\ell}}{V_{ult,f}} - 1\right]^2$$

$$\frac{H}{H_{ult}} \le 1 - \left[\frac{V/2 + \frac{Hh}{2\ell}}{V_{ult,f}} - 1\right]^2$$

$$\frac{H}{(1.26\text{MN})} \le 1 - \left[\frac{0.5\text{MN} + \frac{H(30\text{m})}{(2)(10\text{m})}}{1.91\text{MN}} - 1\right]^2$$

$$\frac{H}{1.26\,MN} \le 1 - \left[\frac{0.785}{MN}H - 0.738\right]^2$$

$$\frac{0.794}{MN} H \le -\frac{0.616}{MN^2} H^2 + \frac{0.579}{MN} H - 0.455$$

$$\frac{0.616}{MN^2} H^2 + \frac{0.215}{MN} H - 0.455 \le 0$$

$$H \le 1{,}051\ MN$$

Much larger horizontal load allowed (compare to 0.16 MN in part (b)(ii))

(c) The vertical load in the back leg at failure would be

$$V_f = \frac{V}{4} - \frac{Hh}{2l} = \frac{1\,MN}{4} - \frac{(1{,}051\ MN)(30\,m)}{(2)(10\,m)} = -1.33\ MN$$

Tension can be sustained in the soil thanks to the development of negative pore pressures. However, these negative excess pore pressures start to dissipate as soon as they are generated and are unlikely to be acting for long periods given that the drainage paths are rather short. Adding skirts can lengthen the drainage path and prolong the permanence of negative excess pore pressures.

The tension calculated above is very large and it is unlikely the soil would be able to generate it

(d) When the connection is fixed the foundation footings are subjected to moments and the V-H-M capacity needs to be checked. Maximum horizontal load capacity will decrease.

Q3

This was a popular question on lateral loading of multi-legged structure. Students displayed good grasp of the concepts related to the bearing capacity of shallow foundations. Most students identified the need to assess V-H interaction. There was some sloppiness in labelling loads for one foundation vs total structural loads which caused

PROBLEM 4

S_u @ characteristic depth $z = 0.3B = (0.3)(2.5m) = 0.75m$

$S_u(0.6m) = 40 + (20 \frac{kPa}{m})(0.75m) = 55\ kPa$

$q_{ult} = (2+\pi)\ S_u\ S_c = (5.14)(55\ kPa)(1.18) = 334\ kPa$

Design approach 1

$\gamma_F = 1.35$ on loads (Comb. 1)

$\gamma_F \frac{Q_1}{A} < q_{ult}$

$\frac{(1.35)(1,000\ KN)}{(2.5m)^2} = 216\ kPa < 334\ kPa$ ok ✓

$\gamma_M = 1.4$ (Comb. 2)

$q_{all} = \frac{q_{ult}}{\gamma_M} = 238\ kPa$

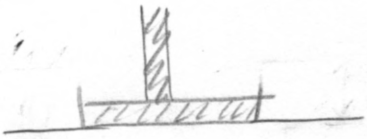
$\frac{1,000\ KPa}{(2.5m)^2} = 160\ kPa < 238\ kPa$ ✓ ok

Q4

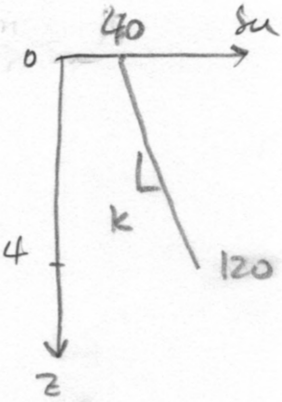
The question was attempted by most students. In general, students displayed good understanding of the various methods to assess settlement of foundations, but had more difficulty in identifying soil non-linearity as a cause for discrepancies in the results. Students also performed well in terms of applying the design factors.

PROBLEM 4

$S_u(0m) = 40kPa$
$S_u(4m) = 120kPa$ $\Rightarrow k = 20\frac{kPa}{m}$



$\gamma_T = 18\frac{kN}{m^3}$



$Q_1 = 1{,}000 kN$

$Q_2 = 500 kN$

*NOTE: using method from notes:

(a) $B = 2.5m$ $\frac{kB}{S_{u0}} = \frac{(20kPa/m)(2m)}{(40kPa)} = 1.0$ $\Rightarrow F \approx 1.1$

shape factor $S_c = 1.05$ for square

Design approach 1, combination 1

$\gamma_F = 1.35$ on loads

$q_{ult} = F\left[(2+\pi)S_{u0} + \frac{kB}{4}\right]S_c = (1.1)\left[(5.14)(40kPa) + \frac{(20kPa/m)(\overset{5}{\cancel{2}}m)}{4}\right](1.05) =$

$= 252 kPa$

$\gamma_F \frac{Q_1}{A} < q_{ult}$

$\frac{(1.35)(1{,}000kN)}{(2.5m)^2} = 216 kPa < 252 kPa$ ✓ok

combination 2

$\gamma_M = 1.4$

$q_{all} = \frac{q_{ult}}{1.4} = 180kPa$

$\frac{1{,}000kN}{(2.5m)^2} < 180kPa$

✓ok

(b) $\gamma_{M=2} = 0.02$

$\tau_{mob} = \frac{Q_1}{AN_c} = \frac{(1{,}000kN)}{(2.5m)^2(5.14)(1.18)} = 26.4kPa$

Characteristic depth $z = 0.3B = (0.3)(2.5m) = 0.75m$

Su @ characteristic depth $S_{u ch} = 40kPa + (20kPa/m)(0.75m) =$
$= 55kPa$

$$\frac{\tau_{mob}}{S_u} = \frac{26.4}{55} = 0.48$$

$$\frac{\tau_{mob}}{S_u} = 0.5\left(\frac{\gamma}{\gamma_{M=2}}\right)^{0.6} \Rightarrow \gamma = \gamma_{M=2}\left(\frac{\tau_{mob}}{0.5\,S_u}\right)^{1/0.6}$$

$$\gamma = (0.02)\left(\frac{0.48}{0.5}\right)^{1/0.6}$$

$$\gamma = 0.0187$$

$$W_u = \frac{\gamma_{mob}}{1.35}B = \frac{(1.87\times10^{-2})}{1.35}(2.5m) = 35\ mm$$

(c) Need to estimate G

From data book $G_{50} \approx 0.5\,\frac{S_u}{\gamma_{M=2}} = \frac{(0.5)(40kPa)}{0.02} = 1\,MPa$ at surface

stiffness gradient $m = \frac{(0.5)(20kPa/m)}{(0.02)} = 0.5\,\frac{MPa}{m}$

$$W \approx \frac{qa}{2(G_0 + ma)} = \frac{\left[\frac{(1{,}000\ kN)}{(2.5m)^2}\frac{(2.5m)}{2}\right]}{2\left[1\,MPa + (0.5\,\frac{MPa}{m})(1.25m)\right]} =$$

a = radius $\approx \frac{B}{2}$

$W = 61\ mm$

The estimate of stiffness is based on G_{50}, which may be lower than what it would actually be in the field.

(d) In order to determine the size of footing that will minimise differential settlement, find B that results in the same settlement

$D \propto V^{1/(2-b)}$

Therefore when V is halved D should be multiplied by $\left(\frac{1}{2}\right)^{1/(2-b)} \approx 0.6$

$D = (2.5m)(0.6) = 1.52 \text{ m} \quad \Rightarrow D = 1.6m$

$z_c = (1.6 \text{ m})(0.3) = 0.48m$

$S_u = 40 + (20 \text{ kPa/m})(0.45) = 50 \text{ kPa}$

$\tau_{mob} = \frac{500 \text{ kPa}}{(1.5m)^2 (5.14)(1.18)} = 33 \text{ kPa}$

$\frac{W}{D} = \frac{(0.02)}{1.35}\left(\frac{33}{(0.5)(50)}\right)^{1.667} = 23 \times 10^{-2}$

$W = 36 \text{ mm}$

$\Delta W = 1 \text{ mm}$

$\delta = \frac{1 \text{ mm}}{6000 \text{ mm}} = 1.6 \times 10^{-4}$ much less than required for damage to appear.

Check capacity:

$q_{ult} = (2+\pi)(1.18)(50 \text{ kPa}) = 303 \text{ kPa}$

Design approach 1, comb. 2

$q_{all} = \frac{303 \text{ kPa}}{1.4} = 217 \text{ kPa}$ $\quad \frac{500 \text{ kN}}{(1.6m)^2} = 195 \text{ kPa} < q_{all}$ ✓ ok