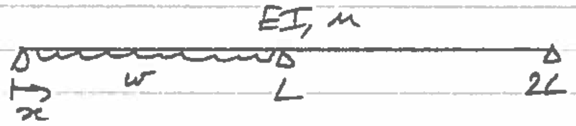
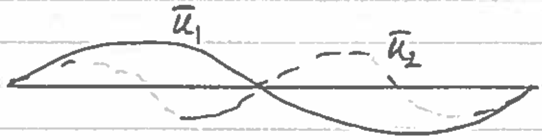
Part IIB 4D6 : Dynamics in Civil Engineering 2018

(1) a). The SRSS method involves taking the square-root of the sum of the squares of the response amplitude of each mode. It generally gives good estimates for well separated modes. 1 1

Alternatives are to superpose the maximum amplitudes of each mode – which is quick but overpredicts response – or to superpose the complete time-histories of each mode – which is correct but time-consuming. 1 1

b).





Simply-supported spans ∴ modes are simple sinusoids :

$\bar{u}_1 = \sin\frac{\pi x}{L}$; $\bar{u}_2 = \sin\frac{2\pi x}{L}$ 2

$$K_{eq} = \int_0^{2L} EI\left(\frac{d^2\bar{u}}{dx^2}\right)^2 dx$$

$$\therefore K_{eq1} = EI\int_0^{2L}\left(-\frac{\pi^2}{L^2}\sin\frac{\pi x}{L}\right)^2 dx = EI\int_0^{2L}\frac{\pi^4}{L^4}\cdot\frac{1}{2}\left(1-\cos\frac{2\pi x}{L}\right)dx$$ 2

$$= \frac{\pi^4 EI}{2L^4}\cdot 2L = \frac{\pi^4 EI}{L^3}$$

Similarly, $K_{eq2} = \frac{16\pi^4 EI}{L^3}$ $\Rightarrow T_2 = \frac{T_1}{4}$ 2

$$M_{eq} = \int_0^{2L} m\bar{u}^2 dx \quad \therefore M_{eq1} = m\int_0^{2L}\frac{1}{2}\left(1-\cos\frac{2\pi x}{L}\right)dx = mL = M_{eq2}.$$ 2

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{M}} \Rightarrow T = 2\pi\sqrt{\frac{M}{K}} \quad \therefore T_1 = 2\pi\sqrt{mL\cdot\frac{L^3}{\pi^4 EI}} = \frac{2L^2}{\pi}\sqrt{\frac{m}{EI}}$$

$$T_2 = \frac{L^2}{2\pi}\sqrt{\frac{m}{EI}}$$ 2

c) $F_{eq1} = \int_0^{2L} f\bar{u}\,dx = \int_0^{L} w\sin\frac{\pi x}{L}dx = w\left[-\frac{L}{\pi}\cos\frac{\pi x}{L}\right]_0^L = \frac{2wL}{\pi}$ = 9.55 kN

2

$F_{eq2} = 0$, ie. only mode 1 contributes

① cont.

$$T_1 = \frac{2L^2}{\pi}\sqrt{\frac{m}{EI}} = \frac{2\times15^2}{\pi}\sqrt{\frac{28{,}000}{1.44\times10^{10}}} = 0.2$$

$$\frac{t_d}{T_1} = \frac{0.01}{0.2} = 0.05 \Rightarrow DAF_1 \simeq 0.15$$ 2

$$\therefore U_{max} \simeq 0.15\,\frac{F_{eq1}}{K_{eq1}} = 0.15 \cdot \frac{2wL}{\pi} \cdot \frac{L^3}{\pi^4 EI} = 0.3\,\frac{wL^4}{\pi^5 EI}$$

$$= 0.3\times\frac{1000\times10^5\times15^4}{\pi^5\times1.44\times10^{10}} = 3.4\times10^{-3}$$ i.e. $\simeq 3.4$ mm at $x = L/2$ 2

② (a) Possible design strategies

(i)- Add strength = increase size of members / reinforcing

(ii)- Add ductility = eccentric braced frame, buckling restrained braces
special moment resisting frame (weak beam / strong column),
yielding "fuses"

(iii)- Add damping = viscous dampers, hysteretic dampers, tuned mass damper

(iv)- Base isolation = base isolators (lead rubber), allow rocking (controlled)

(b) Irregularities:

(i) - soft storey = assumed linear mode shape is inaccurate.

(ii) - weak storey = ductility not distributed uniformly due to concentration of inelastic behaviour.

(iii) vertical mass irregularity = assumed linear mode shape inaccurate.

(iv) Plan irregularity (re-entrant corners, Large diaphragm openings, torsional irregularities)
= 3D effects must be considered (i.e. assumed 2D behaviour violated)

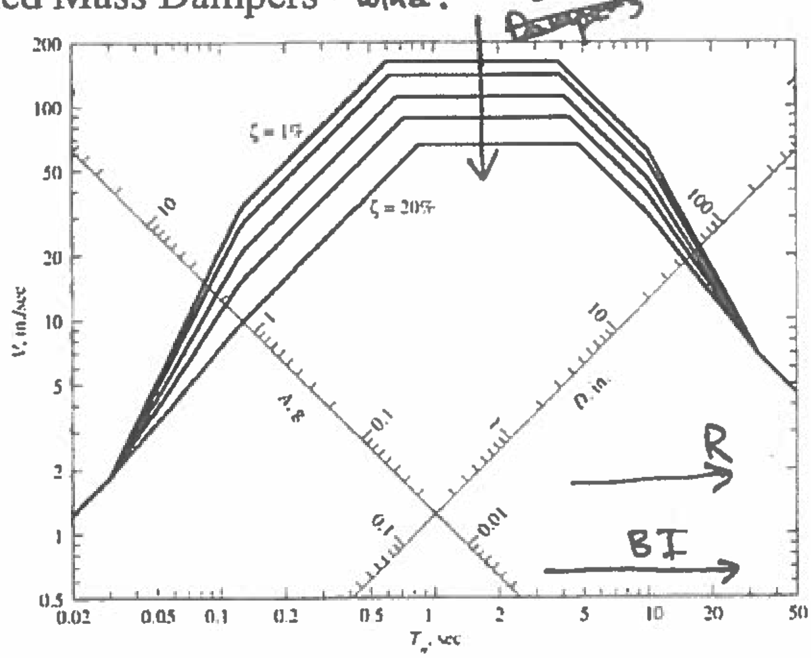
0) Add Strength

5.5 Alternate Design Strategies

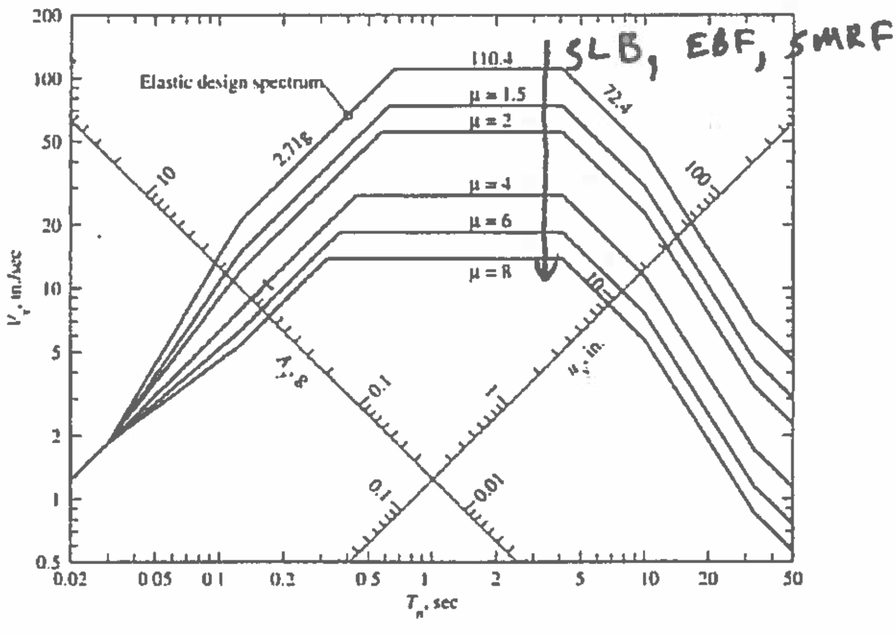
1)- Dampers (viscous, hysteretic) in lateral bracing (D)
2) - Base isolation (BI)
4) - Allow uplift/rocking (similar to base isolation) (R)
3) - ~~Shear Link Beams (SLB)~~ Add Ductility (controlled) = EBF, SLB, SMRF
4) Tuned Mass Dampers - Wind!

EBF

Elastic



Inelastic



② (c) $k_1 = k_2 = 2\left(\frac{12EI}{L^3}\right) = \frac{3}{8}EI = k$

$$\underline{k} - \omega^2 \underline{m} = k\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} - \omega^2 m \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$ 2

$$\lambda = \frac{\omega^2 m}{k} \Rightarrow \det\begin{bmatrix} 2-2\lambda & -1 \\ -1 & 1-\lambda \end{bmatrix} = 0$$

$$(2-2\lambda)(1-\lambda) - 1 = 1 - 4\lambda + 2\lambda^2 = 0$$

$$\lambda = 1 \pm \frac{\sqrt{2}}{2}$$

$$\omega = \sqrt{\left(1 \pm \frac{\sqrt{2}}{2}\right)\frac{k}{m}} = \underline{\underline{\sqrt{\left(1 \pm \frac{\sqrt{2}}{2}\right)\frac{3}{8}\frac{EI}{m}}}} \quad \left(\rightarrow T_{n_1} = \frac{2\pi}{\omega_1} = \frac{1895}{\sqrt{EI}}\right)$$ for part (d) 2

$$= \sqrt{0.640\frac{EI}{m}}$$

$$0.110$$

(d) First mode $\rightarrow (2-2\lambda)\phi_1 + (-\phi_2) = 0 \rightarrow \phi = \begin{bmatrix} \frac{1}{\sqrt{2}} \\ 1 \end{bmatrix} = \begin{bmatrix} 0.707 \\ 1 \end{bmatrix}$ 2

$$\Gamma = \frac{\phi_1 m_1 + \phi_2 m_2}{\phi_1^2 m_1 + \phi_2^2 m_2} = \frac{1+\sqrt{2}}{2} = 1.207$$ 2

$$S_{a,design} = \underbrace{2.5\frac{0.6}{T_{n_1}}}_{Sa}\underbrace{(0.3g)}_{PGA} = \frac{0.45g}{T_{n_1}} = 2.063 \text{ m/s}^2$$

$$V_{base} = \begin{bmatrix} m_1 & m_2 \end{bmatrix}\begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}\Gamma S_{a,design} \Rightarrow 60 \text{ kN} = 24{,}140\,(1.207)\,\frac{0.45g}{T_{n_1}}$$

$T_{n_1} = 2.14$ s 2

$$EI = \left[\frac{1895}{T_{n_1}}\right]^2 \approx 780 \text{ kN}\cdot\text{m}^2 \rightarrow \underline{\underline{EI > 780 \text{ kN}\cdot\text{m}^2}}$$ 2

4D6

Q 3) a) Soil stiffness depends on the effective stresses and the strain amplitudes the soil body is subjected to. During earthquake loading the soil body is subjected to large strains. This causes the stiffness and the shear modulus of the soil to degrade. Several models use hyperbolic relations to account for this degradation in soil stiffness.

The second issue is the generation of ~~ef~~ excess pore pressures in the saturated soil body. Generation of excess pore pressures causes the effective stresses to reduce causing a degradation in soil stiffness. In extreme case of full liquefaction the effective stresses reduce to near zero values causing a total loss of stiffness. [10 %]

3b) When using finite element method for two phase material like saturated soil, one solid mesh and one fluid mesh are generated and overlaid. These meshes when overlaid occupy the same space. Such a formulation is normally called a 'u-p' formulation. Solution involves working out the solid node displacements, giving strains, from which pore pressures are worked out. These change the effective stresses and the soil body undergoes more displacements. This cycle is continued until convergence is achieved at each time step.

In addition the semi-infinite extent can be simulated using

a) absorbing boundaries → use viscous dashpots on boundary nodes.

b) use Smith-Cundall boundary → overlay meshes at the edge with fixed-free boundary conditions

c) use CPC → Compound Parabolic Collector type boundaries to trap stress waves until the analyses is complete. [20 %]

3c) Mass of the roof = 10000 kg.

Stiffness of left wall $k_L = 2\times10^{6}$ N/m.

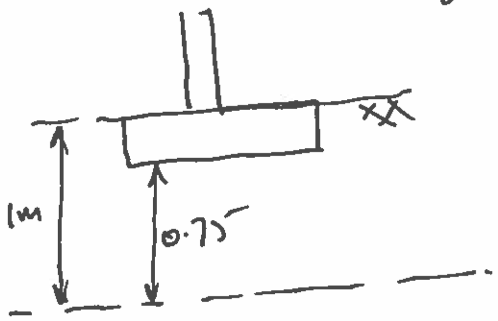
$$\therefore\ \omega = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2\times2\times10^{6}}{10000}} = 20 \text{ rad/s} \quad \text{or} \quad \underline{\underline{3.18 \text{ Hz}}}$$

[15 %]

3d) First work out the shear modulus 'G' of the soil.

Vertical stress due to slab on each wall $= \dfrac{10000 \times 9.81/2}{0.5 \times 1} = 98100 \ N/m^2$

Bearing pressure $= 98.1$ kPa.



Over burden stress $= \sigma_v = \gamma z = 17.5 \times 0.75$

$= 13.125$ kPa.

Total vertical stress $= \sigma_v = \sigma_v' = \underline{111.225 \text{ kPa}}$

$$K_0 = \frac{\nu}{1-\nu} = \frac{0.3}{1-0.3} = 0.428.$$

$$p' = \left(\frac{1+2K_0}{3}\right)\sigma_v' = \underline{68.854 \text{ kPa}}$$

From Data book

$$G = 100 \frac{[3-e]^2}{1+e} \sqrt{p'} = 100 \frac{[3-0.65]^2}{1.65} \times \sqrt{\frac{68.854}{1000}}$$

$$= \underline{\underline{87.824 \text{ MPa}}}$$

Use Wolf formulae.

$2l = 0.5$ m $\quad 2b = 1$ m $\quad e = 0.25$ m.

$l/b = 0.5 \quad e/b = 0.5$

$$K_{hx} = \frac{Gb}{2-\nu}\left[6.8\left(l/b\right)^{0.65} + 2.4\right]\left[1 + 0.33 + \frac{1.34}{1+l/b}\left(\frac{e}{b}\right)^{0.8}\right]$$

$$= G \times \frac{0.5}{1.3}\left[6.8 \times 0.5^{0.65} + 2.4\right]\left[1 + 0.33 + \frac{1.34}{1+0.5} \times 0.5^{0.8}\right]$$

$$= 3.37194\,G = \underline{296.138 \text{ MPa/m}}.$$

$$K_{ry} = \frac{Gb^3}{1-\nu}\left[3.73\left(l/b\right)^{2.4} + 0.7\right]\left[1 + \frac{e}{b} + \frac{1.6}{0.35 + (l/b)^4}\left(\frac{e}{b}\right)^2\right]$$

$$= 0.4307\,G = \underline{37.825 \text{ MN-m/rad}}.$$

3e) Here we can use a simple SDOF and compare the nat. frequency with the 2DOF case in 3c) above. If these are close, we can then do the

3 e) Contd.

$$f_h = \frac{1}{2\pi}\sqrt{\frac{2\,K_h}{M}} = \frac{1}{2\pi}\sqrt{\frac{2 \times 296.138 \times 10^{6}}{10000}}$$

$$= \underline{38.73\ Hz}$$

This is ignoring any soil participation in horizontal vibrations. Even if we considered '$10 \times m$' for soil participation mass, we still get $\underline{12.25\ Hz}$. This is still much higher than 3c) ans of 3.18 Hz. Hence there will be very little benefit of using a 2-DOF system.

For rocking vibrations :-

K_{ry} = is known.

$I = Mr^2 = 10000 \times 4.5^2 = 202500\ kg\text{-}m^2$.

$$\therefore \omega_r = \sqrt{\frac{2K_{ry}}{I}} = \sqrt{\frac{28.282 \times 10^6 \times 2}{202500}} = 16.71 \text{ rad.}$$

$f_r = \underline{\underline{2.65\ Hz}}$.

This is quite serious, the portal frame can undergo rocking vibrations

[25 %]

3 f) From above the portal frame has a tendency to undergo rocking vibrations. This ~~will~~ can cause

a) Buckling of the walls, if they are sufficiently slender.

b) uplift of the strip foundations in alternate cycles of earthquake loading, causing gradual settlement.

c) can lead to differential settlement or even bearing capacity failure

[10 %]

4D6 2018 Q4 . Answers

a) Basic marks will be awarded for a description of the eight flutter derivatives as the (dimensionless) coefficients relating lift forces and moments to the angle of attack and the heave displacement and their first derivatives. Extra marks will be awarded for noting that these are only defined for periodic solutions due to fluid memory effects. Descriptions of wind tunnel testing should focus on the need for prescribed harmonic motions of deck section models. [20%]

b) Marks will be awarded for a description of how buffetting analysis – unlike earthquake Response Spectrum Analysis - takes account of **spatial** decorrelations of forces. [10%]

c) Marks will be obtained for a through description of wind buffeting analysis. It should include descriptions of mechanical and aerodynamic admittances. The description should contain a statement explaining how the peak response for design of any parameter (such as a displacement or bending moment) can be estimated as the (static) mean hourly response plus some gust factor (typically in the range 3 to 3.5) multiple of the standard deviation of that response. The standard deviation, in turn, is the square root of the area under the spectral density of the response, which was obtained via the admittances from the spectral density of the incident velocities. Finally, there should be a brief description that for large structures, the resulting equations for spectral densities need to be mode generalised, although this leads to some rather large integral equations for the cross-correlations of the various mode-generalised random variables. [50%]

d) Marks will be awarded for descriptions of rivulets affecting the aerodynamic cross-section of the cables, and why this can lead to a large-amplitude crosswind galloping response associated with the switching of boundary layer separation points from the various sharp corners of the cross-section. [10%]

e) First, reflections of blast waves from the ground can lead to an almost doubling of pressures. Further, the thermodynamics of wave reflection from a surface such as a wall or window, as described by the Rankine-Hugoniot equations, can lead to magnification of high incident shock pressures by factors as great as ten. [10%]

Q1
23 candidates attempted Q1. In part (a) most candidates demonstrated a good understanding of modal superposition. Parts (b) and (c) were generally answered well but a few candidates were thrown by the fact that the modal force associated with the second mode is zero.

Q2
16 candidates attempted Q2. Part (a) was done poorly, with only a few candidates attempting proper sketches of response spectra. In part (b), candidates demonstrated a reasonable understanding of the 'irregularities' but often failed to link these to the associated assumptions. In part (c), natural frequencies and mode shapes were calculated well. Most understood the need to calculate the modal participation factor but many failed to incorporate this into the calculation of flexural stiffness.

Q3
Another popular question, with 23 attempts. Most candidates could explain the origins for strength degradation in soils subjected to cyclic loading. The attempts to explain how the finite-element method can be used to solve a dynamic problem in two phase media were varied. Some candidates did an excellent job in describing the procedure for overlapping solid and fluid phases, and could identify the semi-infinite boundary as an issue as well as suggesting methods to simulate a non-reflecting boundary. The numerical parts of the questions were reasonably well done, with candidates able to make reasonable estimates of horizontal and rotational stiffness offered by the soil, and then use simple discrete models to find the natural frequency. There were a significant number of candidates who had numerical errors in their solutions

Q4
There were more attempts than usual at this question (wind engineering and blast resistant design), albeit still only 10 candidates. Nevertheless, the answers submitted showed a pleasingly high level of understanding of fundamental principles and of many nuances.

JPT/MSPG/MJD/FAM