4D7 Concrete Structures. Solutions

1. (a). Limit States

Ultimate Limit states include

- strength of components and systems to avoid collapse. These usually govern concrete structures
- stability limits. Buckling not usually an issue for concrete structures but overall stability must always be considered, especially, but not limited to, the structure state during construction. Can the weight be properly carried; will the beams topple from bearings; are the structural elements capable of being lifted?
- underwater structures can be limited by external pressure causing rupture.

Serviceability Limit states include

- crack width to avoid water penetration and ingress of materials to cause corrosion.
- dynamic response/vibrations - response from wave and wind loading but also from footfall that can cause oscillations that are uncomfortable, even if the associated deflection is small. Fatigue is not usually a problem for concrete structures but should be considered if high and variable loads are applied.
- durability - adequate detailing of concrete mix, concrete placement and reinforcing protection.
- deflection - must be small enough that the structure is not perceived to be unsatisfactory, either from a visual or a comfort point of view.

Highway Bridge

- Strength is clearly a limiting factor, as is its long-term durability. Fatigue may be an issue under concentrated wheel loads. Deflection not usually an issue but cracking and durability are important. Protection from corrosion due to road salt is essential. Resistance to impact from vehicles, especially to parapets and supporting columns. Fire resistance, especially for wide bridges where vehicles could catch fire underneath the structure.

Hospital Wards and Labs

- Strength must be considered but deflection criteria may well govern, and conditions of noise transmission from one part of the building to the next would be critical. Resistance of the building structure to fire would be essential to ensure that the fire brigade had access to evacuate possibly immobile patients. Laboratories may well have sensitive equipment such as MRI scanners that require non-magnetic reinforcement and which would be sensitive to shocks transmitted from elsewhere. Corrosion of the principle structure less likely to be an issue because it would be clad, but durability of secondary elements like window frames and the cladding itself might be important.

4D7

1. (a) Bookwork.

(b) Characteristic strength $f_{ck} = 50$ MPa

$\sigma_c = 8$ MPa

(i) Mean strength $= 50 + 1.645 \times 8 = \underline{63.2 \text{ MPa}}$

Design strength $= \dfrac{f_{ck}}{\gamma_{mc}} = \dfrac{50}{1.5} = \underline{33.3 \text{ MPa}}$

So this is also the design stress in the concrete

(ii) How many S.D below mean is 33.3 MPa?

$\dfrac{63.2 - 33.3}{8} = \underline{-3.73}$ S.D. $= z$

From prob. dist data sheet Area $-\infty$ to z =

$= 0.99990426$

$\therefore$ Area below $z = 1 - 0.99990426$

$= \underline{\underline{95.7 \times 10^{-6}}}$

(iii) So what is β?

Characteristic stress $= \dfrac{\text{design stress}}{1.4} = \dfrac{33.3}{1.4} = 23.8$ MPa

Mean stress $= 23.8 - 1.645 \cdot 6 = 13.9$ MPa

$\therefore \beta = \dfrac{(\mu_r - \mu_s)}{\sqrt{\sigma_r^2 + \sigma_s^2}} = \dfrac{(63.2 - 13.9)}{\sqrt{8^2 + 6^2}}$

$= 4.93$

$p_f = 1 - 0.999999\,5889 = 0.4 \cdot 10^{-6}$

(iv)

Require $\beta = 3.5$. What mean strength needed

$\dfrac{\mu_r - 13.9}{\sqrt{8^2 + 6^2}} = 3.5 \Rightarrow \mu_r = 48.9$ MPa

Characteristic strength $= 48.9 - 1.645 \cdot 8 = 35.7$ MPa

(v) Test machine must be at top end of range of concrete strengths

$99.99\% \approx 3.72$ SD above mean.

$\therefore$ Strength required

$= 48.9 + 3.72 \times 8 = 78.6$ MPa

2. (a) Four "C"s and water cement ratio

- Cement Content to ensure strength and sufficient binding matrix to hydrate, react and fill voids. Also needed to provide alkalinity to passivate steel and prevent its corrosion. A surplus of cement can encourage autogenous healing of fine cracks.
- Cover provides protective barrier to prevent ingress of deleterious materials (like chlorides) and to slow the effects of carbonation. Ensures adequate bond but too much can effectively leave an unreinforced layer that can spall.
- Compaction removes air bubbles from concrete which assists in reducing porosity and thus penetration of nasty materials. Voids effectively form flaws that govern the tensile strength of concrete and indirectly also its compressive strength but we must avoid over-compaction that can allow the different sizes of aggregate to segregate, thus leading to weak layers.
- Curing is essential to retain moisture long enough for the chemical reaction between cement and water to take place.
- Water-cement ratio should be high enough to ensure the cement can fully hydrate but not too high that excess water remains and will evaporate to form voids.

(b) (i) Chloride-induced corrosion is a catalytic reaction at the surface of the steel that allows very rapid corrosion of steel if sufficient oxygen is present. Chlorides come from road salt and/or sea-spray in coastal regions. Dense concrete slows the penetration of the chlorides, reduces the amount of oxygen present and can increase the electrical resistance that slows the effect.

Carbonation is the ingress of water and CO_2 which together form a weak acid. This reduces the pH of the concrete to below 10 at which time the steel can corrode if water and oxygen are present.

(ii) Corrosion can be detected by spalling concrete caused by the fact that rust is larger than the steel it comes from, plus staining from rust. Half-cell potential gives an indication of the likelihood of corrosion but is not a direct measure of its presence. Resistivity measurement and hammer-surveys to detect delamination are useful. Radar and X-ray measurements can be used but are of doubtful effectiveness in beams with complex patterns of reinforcement. Once detected, silane coating to prevent ingress of water and chlorides, or the use of cathodic protection to stop further corrosion. Corrosion cannot be reversed.

2 (a)
(b) } Bookwork

(c) Need to decide between two options

Option 1. Repair every 25 years @ £250,000

$$\text{Net PV} = \frac{\sum C_i}{(1+r)^{i-1}}$$

$$= \frac{250.10^3}{(1.03)^{24}} + \frac{250.10^3}{(1.03)^{49}}$$

$$= 122.98.10^3 + 58.74.10^3 = \underline{£181.7K}$$

Option 2. Cathodic protection

$$1.03 \text{ annually} = 1 + 0.03 = \exp(r_c)$$

$$\therefore r_c = \ln(1.03) = 0.0296$$

NPV of continuous cost

$$= \int_0^{75} \frac{3000}{\exp(r_c t)} . dt = 3000 \left[\frac{\exp(-r_c t)}{-r_c} \right]_0^{75}$$

$$= \frac{3000}{0.0296} \left(1 - \exp(-75 . 0.0296) \right) = £90.4K$$

Add £60k start up $= £150.4K$

∴ Cathodic protection is the recommended option.

(d) Repeat but with 6% annual

Option 1 ⇒ £76.1K

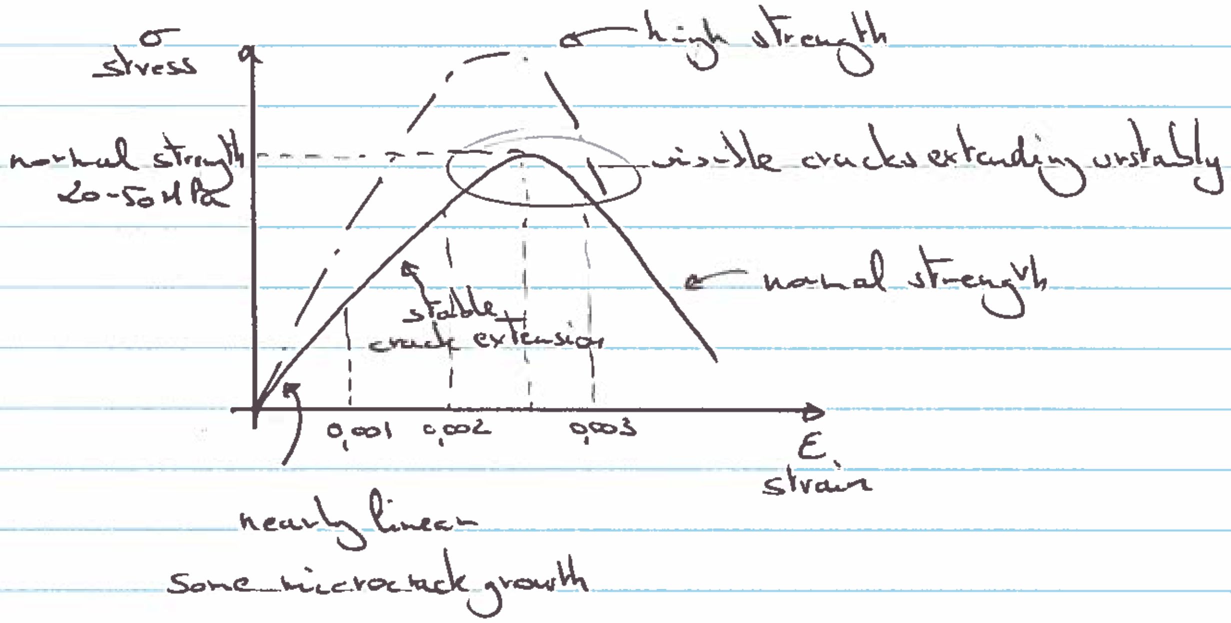
Option 2 ⇒ £110.8K

Higher discount rates reduce the value of later costs ∴ Repair options become significantly cheaper.

Question 3

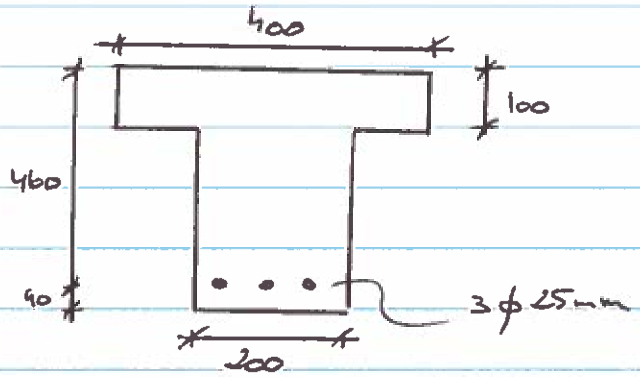
4D7/23/1

a) bookwork



- high strength concrete tends to be stiffer and more brittle.

b) (i)



$E_s = 210\ GPa \quad f_{yd} = 440\ MPa$

$E_c = 30\ GPa \quad f_{cd} = 30\ MPa$

$$A_s = 3 \cdot \frac{\pi \cdot 25^2}{4} = 1472,6\ mm^2$$

$$F_s = A_s \cdot f_{yd} = 1472,6 \times 440 = 647,95\ kN$$

- assume n.a. is in flange:

$\rightarrow$ max $F_{cF} = 0,6 \cdot f_{cd} \cdot 100 \cdot 400$

$= 0{,}6 \cdot 30 \cdot 100 \cdot 400$

$= 720{,}0\,kN \quad > \quad 647{,}95\,kN$

$\rightarrow$ n.a. is indeed in flange

$F_s = F_{cF}$

$647{,}95 \cdot 10^3 = 0{,}6\, f_{cd} \quad x \cdot 400$

$\rightarrow x = 89{,}99\,mm$

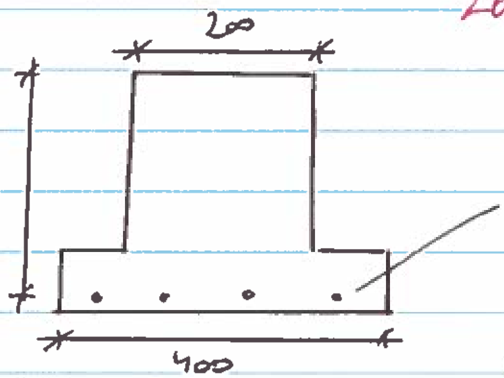
Take moment about centroid of steel:

$M_u = F_{cF} \cdot (460 - \frac{89{,}99}{2})$

$=$

$268{\cdot}9\ KNm$

(ii)



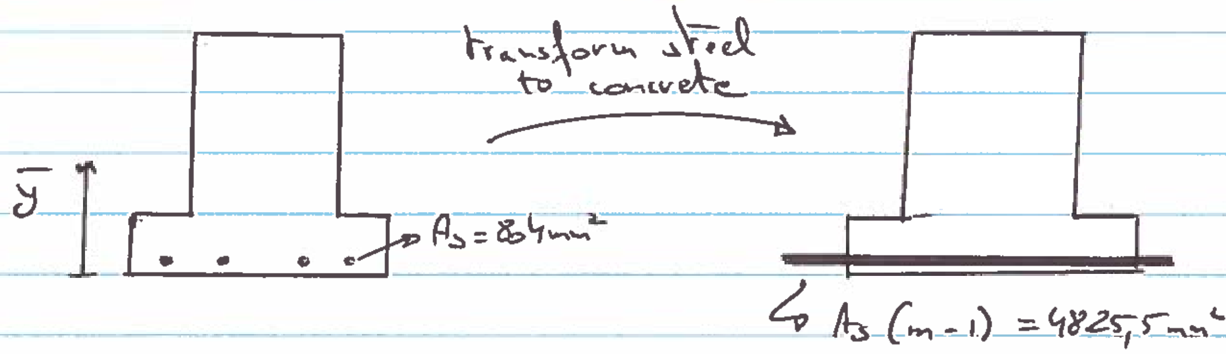
$4\phi\,16\,mm$

$A_s = 4 \cdot \pi \cdot \frac{16^2}{4} = 804\,mm^2$

$f_{yd} = 440\,MPa \qquad E_s = 210\,GPa$

$f_{cd} = 30\,MPa \qquad E_c = 30\,GPa$

· assume n.a. is in the web.



$m = E_s / E_c = 210/30 = 7$

Find $\bar{y}$: $\quad \bar{y} = \frac{\sum A_y}{A} = \frac{100 \cdot 400 \cdot 50 + 200 \cdot 400 \cdot 300 + 4825{,}5 \cdot 40}{100 \cdot 400 + 200 \cdot 400 + 4825{,}5}$

$= 209{,}84\,mm \rightarrow$ indeed in web.

$$I_{UN} = \frac{400.100^3}{12} + 400.100.(209,84-50)^2$$

$$+ \frac{200.400^3}{12} + 200.400.(209,84-300)^2$$

$$+ 4825,5.(209,84-40)^2$$

$$= 3,33.10^7 + 1,02.10^9 + 1,07.10^9 + 6,50.10^8 + 1,39.10^8\ mm^4$$

$$= 2,91.10^9\ mm^4$$

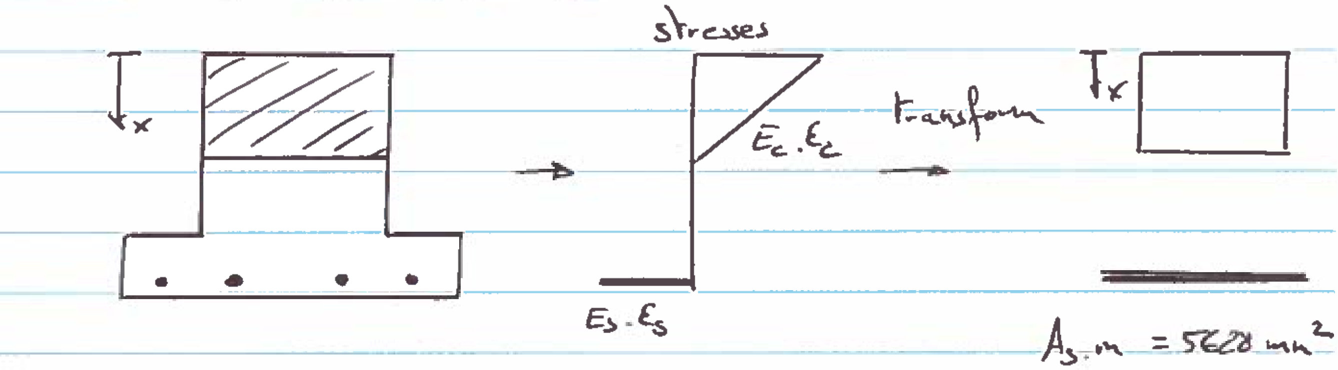
$\sigma = \frac{M.y}{I}$ first cracking:

$$M_{cr} = \frac{I.f_{ct}}{y} = \frac{2,91.10^9.3,5}{209,84}\ Nmm$$

$$M_{cr} = \underline{\underline{48,5 kNm}}$$

(iii) cracked elastic behaviour

. assume x is in the web:



find x:

$$200.x.\frac{x}{2} = 5628.(460-x)$$

$$\frac{200x^2}{2} = -5628x + 2588880$$

$$x = 135.2\ mm \leftarrow \text{is indeed in web}$$

4D7/Q3/4

$$I_{cr} = \frac{200 \cdot 135.2^3}{12} + 200 \cdot 135.2 \left(\frac{135.2}{2}\right)^2 + 5628\,(460 - 135.2)^2$$

$$= \underline{\underline{758.4 \cdot 10^6\ mm^4}}$$

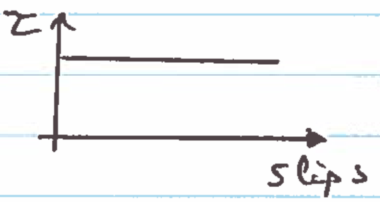
Question 4

(a) bookwork

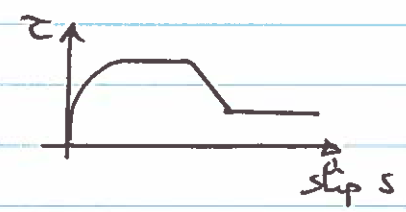
The bond stress between the reinforcement and the concrete causes tension to build up in the concrete.
From equilibrium for the reinforcement:

$$\frac{d\sigma_s}{dx} = \tau . 2\pi . r$$

so a high bond stress causes a more rapid build up of stress in the concrete. However, once the fully developed crack pattern has formed, there is insufficient length over which sufficient tensile forces can be transferred to the concrete to cause cracking.
An example of a constant bond stress-slip relationship:



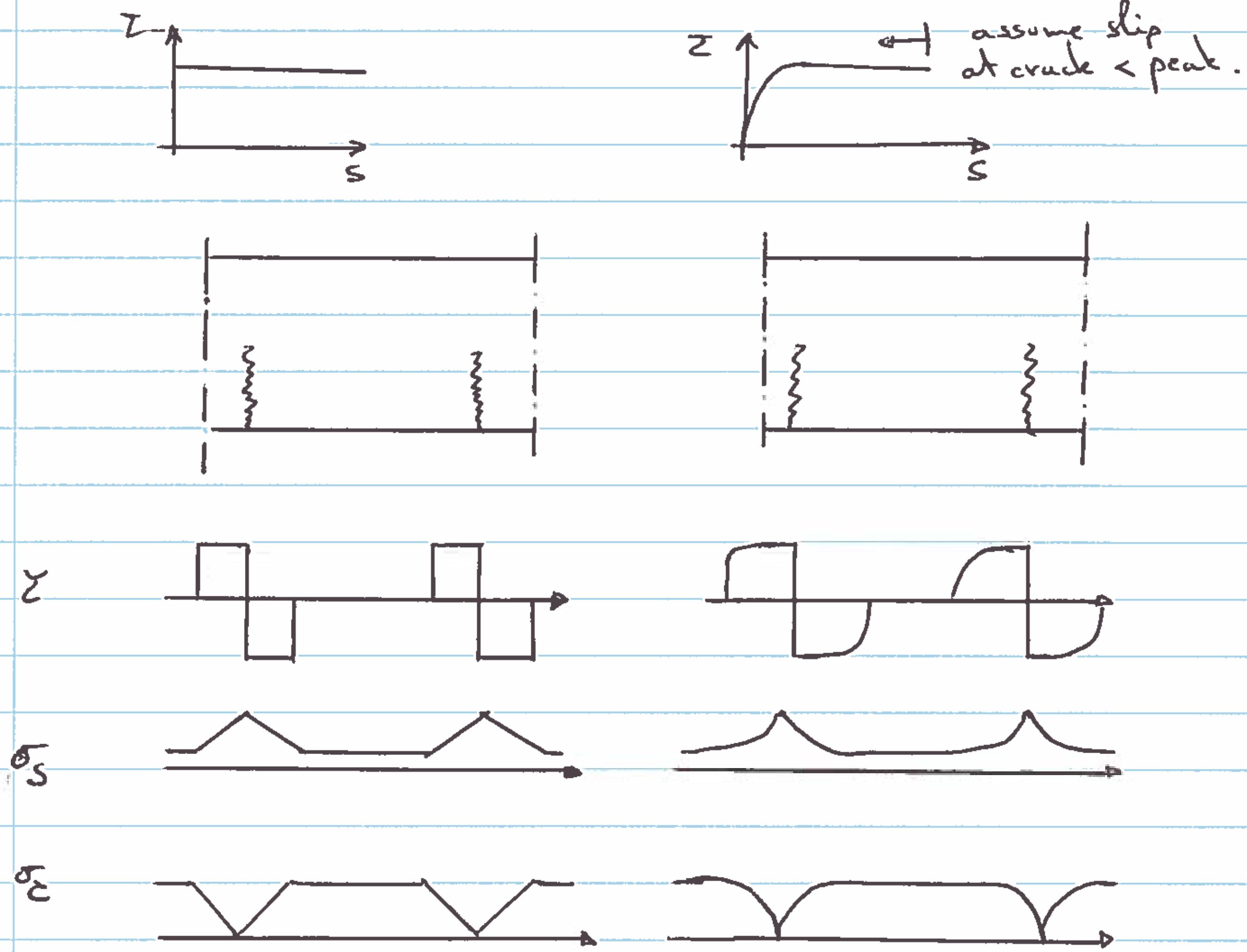
An example of a non-linear stress-slip relationship:



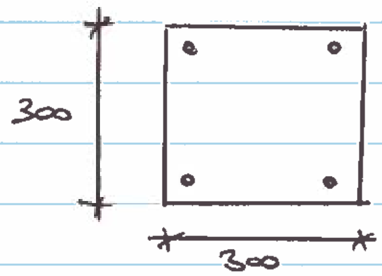
The integration of the slip over the crack spacing leads to the crack width.

4D7/24/2

(i) (ii)



(b) (i)



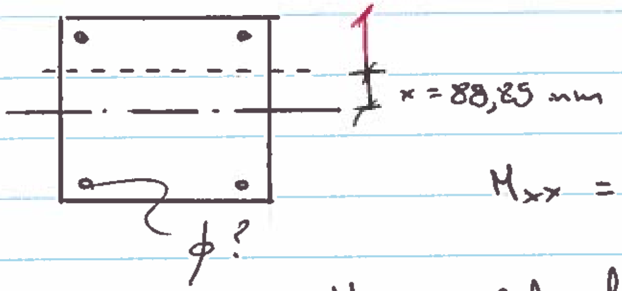
$N = 480\,kN$ $\quad f_{yd} = 440\,MPa$

$M_{xx} = 150\,kNm$ $\quad f_{cd} = 30\,MPa$

When in bending, at ult. load, two bars yield in tension and two in compression.

∴ depth of compressive zone (x) not affected by steel

$$x = \frac{480\,000}{0{,}6 \cdot 30 \cdot 300} = 88{,}89 \text{ mm}$$



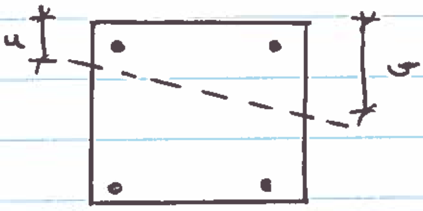
$M_{xx} = 150$ kNm

$$M_{xx} = 2A_s \cdot f_{yd} \cdot (d - d') + 0{,}6 f_{cd} \cdot b \cdot x \cdot \left(\frac{h}{2} - \frac{x}{2}\right)$$

$\Rightarrow A_s = 490{,}78 \text{ mm}^2$

$\phi \simeq 25$ mm

(ii)



Assume n.a. such that 2 bars in tension and 2 in compression

$$\therefore \left(\frac{u+v}{2}\right) \cdot 0{,}6 \cdot 30 \cdot 300 = 480\,000$$

$\Rightarrow u + v = 177{,}78$ mm (= 2.88,89 mm as expected) (1)

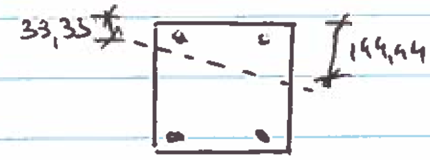
$$M_{yy} = \frac{(v-u)\,300}{2} \cdot 0{,}6 \cdot 30 \cdot \left(\frac{300}{2} - \frac{300}{3}\right)$$

$$= 135 \cdot 10^3\,(v-u) = 15 \cdot 10^6 \text{ Nmm} \qquad (2)$$

$\therefore$ Solve for u & v: $u = 33{,}33$ mm

$v = 144{,}44$ mm

→ check assumption above is valid:



$A_s = 490{,}87 \text{ mm}^2$

4D7/Q4/4

$$M_x = 2 \cdot 490{,}87 \cdot 440\,(265 - 35) + 0{,}6 \cdot 30 \cdot \left[300 \cdot 33{,}33\left(\frac{300}{2} - \frac{33{,}33}{2}\right)\right] + 0{,}6 \cdot 30 \cdot \frac{1}{2} \cdot 300 \cdot (144{,}44 - 33{,}33) \cdot \left[\frac{300}{2} - 33{,}33 - \frac{1}{3}(144{,}44 - 33{,}33)\right]$$

$$= 147{,}24 \text{ kNm.}$$

$\therefore$ Just less then before.

Q1. Done well by most candidates. The biggest problem was the inability to recognise that when designing the test machine one would be interested in the upper end of the strength distribution rather than the lower end.

Q2. Done well by almost all candidates.

Q3. Done well.

Q4. Least popular question and clearly the last one attempted, so several were out of time. The final section required them to show more understanding with less reliance on their bookwork and it showed.