80 marks in total

4D8 2018 Solutions

Tim Ibell

(1)

1(a) In "sagging" regions, $p = \frac{8hP}{L^2} = \frac{8 \times 500 \times 5 \times 10^6}{30{,}000^2}$

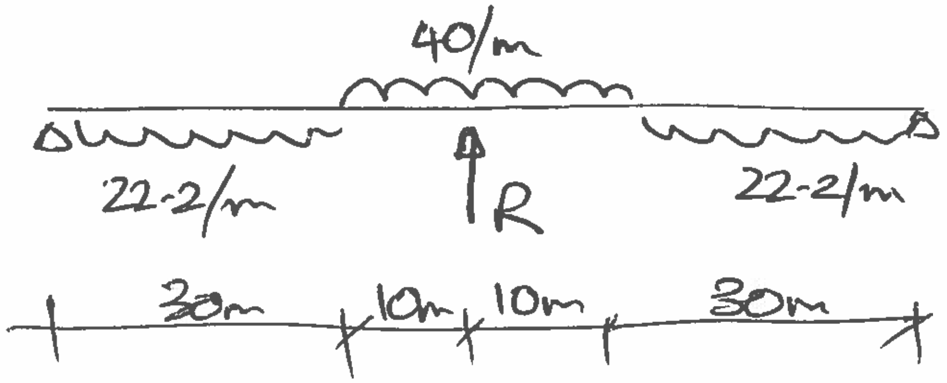
$= \underline{22.2 \text{ kN/m}}$

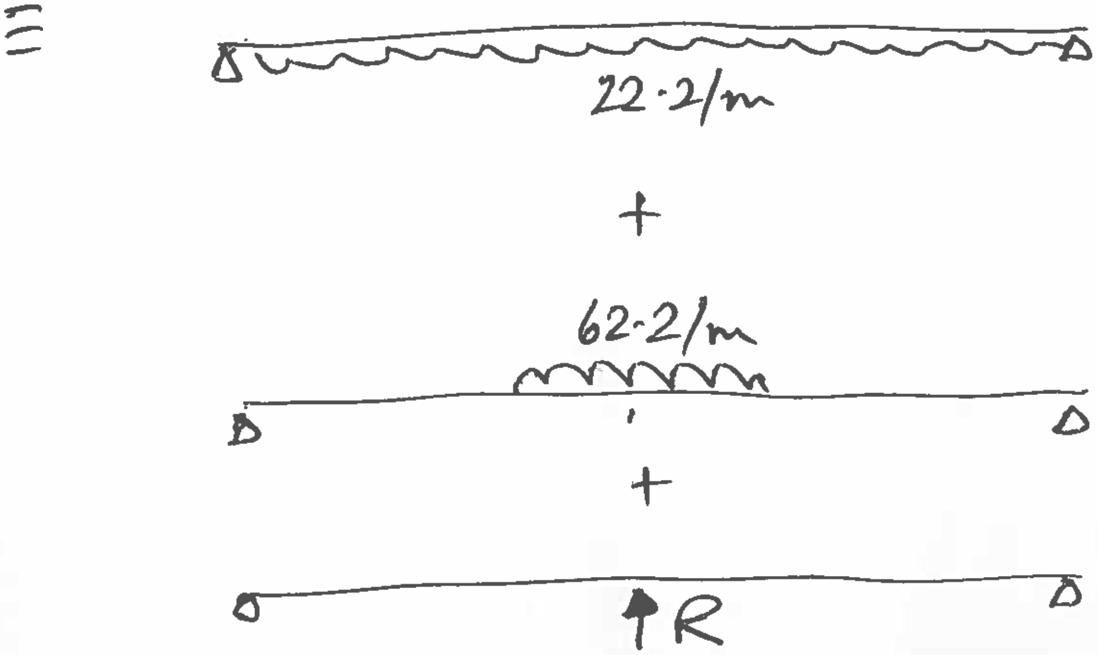
In "hogging" region, $p = \frac{8 \times 400 \times 5 \times 10^6}{20{,}000^2}$

$= \underline{40 \text{ kN/m}}$

[4 marks]

So we have:





(1)

(2)

[6 marks]

(3)

(2)

Deflection up due to (1) at midspan

is $\delta_1 = \frac{5 \times 22.2 \times 80{,}000^4}{384EI} = \frac{11.84 \times 10^{18}}{EI}$

Deflection up due to (3) at midspan

is $\delta_3 = \frac{R \cdot 80{,}000^3}{48EI} = \frac{10.67 \times 10^{12} R}{EI}$

Deflection down due to (2) at midspan

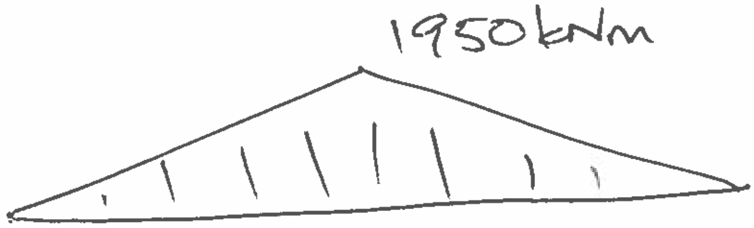
is $\delta_2 = \frac{497 \times 62.2 \times 10{,}000^4}{24EI} = \frac{12.88 \times 10^{18}}{EI}$

$\delta_1 + \delta_3 = \delta_2$ [6 marks]

$\Rightarrow R = 97.5\,kN$

$\therefore M_2 = \frac{97.5}{2} \times 40 = 1950\,kNm$

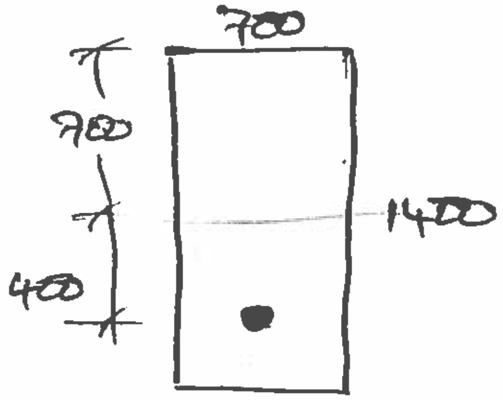
M_2:



[4 marks]

(3)

(b) It's in hogging but a touch easier to consider as upside down:



$f_{cu} = 60\text{MPa}$

$\varepsilon_{cu} = 0.0035$

$k_1 = 0.4$

$k_2 = 0.5$

$E_c = 30\text{GPa}$

$E_s = 200\text{GPa}$

$$\varepsilon_{pe} = \frac{5\times10^6}{6250\times200{,}000}$$

[2 marks]

$$= 0.004$$

$$\varepsilon_e = \left(\frac{P}{A} + \frac{Pey}{I}\right)/E = \frac{\left(\frac{5\times10^6}{700\times1400} + \frac{5\times10^6\times400\times400}{\left(\frac{700\times1400^3}{12}\right)}\right)}{30{,}000}$$

(fine to ignore but say so.)

$$= \underline{0.00034} \text{ (small)}$$

[4 marks]

Choose $x = 500$ mm:

$$C = 700\times500\times0.4\times60 = \underline{8.4\text{MN}}$$ [2 marks]

$$\varepsilon_{pb} = 0.004 + 0.00034 + 0.0035\times\left(\frac{1100-500}{500}\right)$$

$$= \underline{0.00854} \Rightarrow \sigma = 1369\text{MPa}$$ [4 marks]

$$T = 1369\times6250 = \underline{8.56\text{ MN}}$$

Rough estimate of M :

$$M = \frac{(8400 + 8560)}{2} \times \frac{(1100 - 0.5 \times 500)}{1000}$$

$$= \underline{7200 \text{ kNm}}$$

[4 marks]

c) If a structure is ductile then there is a compelling reason to ignore M_2 at the ULS. However, in this case $x \simeq 0.5d$, so this is not a ductile section, so M_2 should be retained in the calculation process unless a more detailed (cracked) analysis is conducted to demonstrate moment redistribution. Additionally, test results seem to suggest that it is wise to include M_2 in the check at ULS.

[4 marks]

Q1 Secondary moments and ultimate limit state
Part (a). Generally answered adequately, with the hint being used appropriately. The majority of candidates were unable to work through to the fully correct secondary moment outcome, but the approach taken by most students was correct.
Part (b). Generally answered very well. An easy component in this examination.
Part (c). Answered poorly on the whole, despite this being discussed during the lectures.

(2)

$$Z_1 = \frac{-0.106}{(1250-720)} = -200 \times 10^6 \text{ mm}^3 \quad [1 \text{ mark}]$$

$$Z_2 = \frac{+0.106}{720} = 147 \times 10^6 \text{ mm}^3 \quad [1 \text{ mark}]$$

$$e \geq \frac{200 \times 10^6}{500 \times 10^3} + \frac{18 \times (-200 \times 10^6)}{P} + \frac{3000 \times 10^6}{P}$$

$$e \geq 400 - \frac{600 \times 10^6}{P} \quad \text{— (1)} \qquad (280)$$

$$e \geq -\frac{147 \times 10^6}{500 \times 10^3} + \frac{(-1).(147 \times 10^6)}{P} + \frac{3000 \times 10^6}{P}$$

$$e \geq -294 + \frac{2853 \times 10^6}{P} \quad \text{— (2)} \qquad (277)$$

$$e \leq 400 + \frac{(-1)(-200 \times 10^6)}{P} + \frac{600 \times 10^6}{P}$$

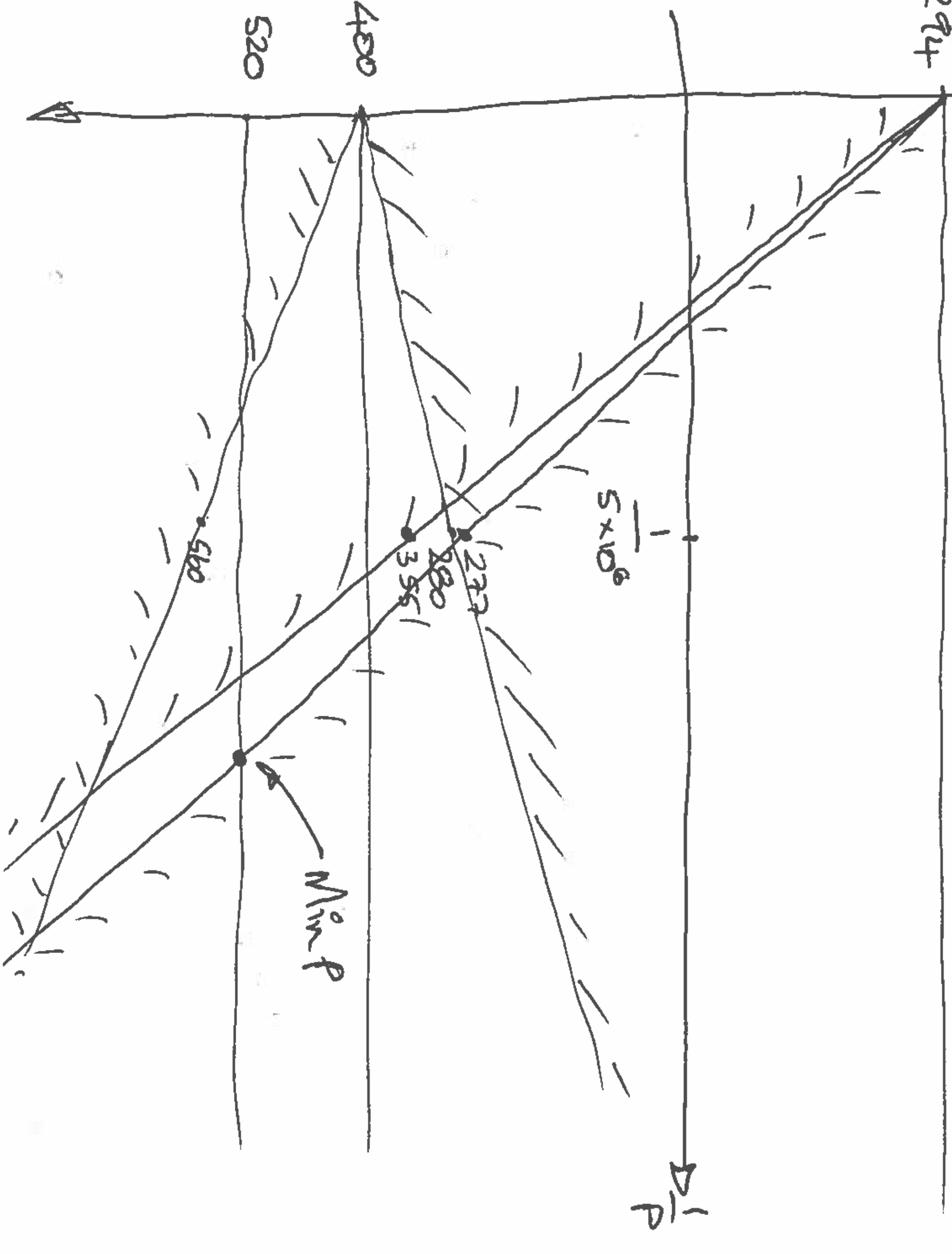
$$e \leq 400 + \frac{800 \times 10^6}{P} \quad \text{— (3)} \qquad (560)$$

$$e \leq -294 + \frac{18 \times 147 \times 10^6}{P} + \frac{600 \times 10^6}{P}$$

$$e \leq -294 + \frac{3246 \times 10^6}{P} \quad \text{— (4)} \qquad (355)$$

[6 marks]

Use $P = 0.5 \times 20 \times$ ~~1250~~ $500{,}000 = 5$ MN



[6 marks for diagram]

[6 marks for part (b)]

(b) $520 = -294 + \frac{2853 \times 10^6}{P} \Rightarrow \underline{P = 3.5\,MN}$

(Min P)

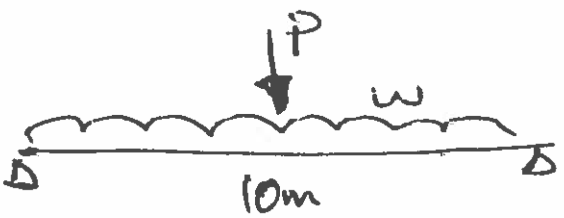
Q2 Magnel diagram

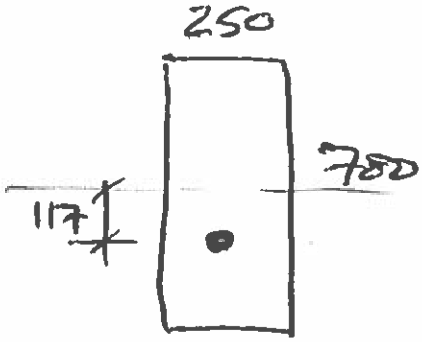
Part (a). By far the easiest question on the examination paper, and answered very well indeed.

Part (b). Answered very well by most candidates.

(7)

3(a)





(a) $w = 0.25 \times 0.7 \times 24 = \underline{4.2\,kN/m}$ [2 marks]

We know that when the beam is cracked and loaded the second time, it loses stiffness when $\delta_t = 0$ at the soffit. [4 marks]

At this point, $M_{max} = \frac{120 \times 10}{4} + \frac{4.2 \times 10^2}{8}$

$= \underline{353\,kNm}$ [2 marks]

$$\frac{P}{A} + \frac{Pey}{I} - \frac{M_{max}\,y}{I} = 0$$

$$\frac{P}{(250 \times 700)} + \frac{P \cdot 117 \cdot 350}{\left(\frac{250 \times 700^3}{12}\right)} - \frac{353 \times 10^6 \times 350}{\left(\frac{250 \times 700^3}{12}\right)} = 0$$

17.265

$\Rightarrow P = \underline{1.51\,MN}$ [4 marks]

To find the tensile strength, we know that during the first loading, the beam cracked when P = 170 kN. [2 marks]

So, $M_{max} = \frac{170 \times 10}{4} + \frac{4.2 \times 10^2}{8} = 478\,kNm$

We know that $P = 1.51\,MN$.

So stress at soffit

$$\sigma_t = f_t = \frac{1.51 \times 10^6}{250 \times 700} + \frac{1.51 \times 10^6 \times 117 \times 350}{\left(\frac{250 \times 700^3}{12}\right)}$$

$$- \frac{478 \times 10^6 \times 350}{\left(\frac{250 \times 700^3}{12}\right)} = -6.12\,MPa$$

[2 marks]

(b) During the first loading, the tensile strength of the concrete would have been LOWER than 6.12 MPa, so we might have expected a lower load-to-loss-of-stiffness than 170 kN. However, creep will not yet have occurred so P would be higher than 1.51 MN. Thus, it is hard to tell whether the first test would have yielded > or < 170 kN.

However, in the second loading phase, given that P would have been higher before creep, we would have expected a higher load-to-loss-of-stiffness than 170 kN. [4 marks]

Q3 Load-deflection response
My suspicion in this question is that those students who attended the voluntary laboratory session performed far better than those who did not, given that the laboratory emphasised the relationship between prestressing force and load-deflection response.
Part (a). Students either found this easy or struggled to even start the question.
Part (b). In general, a poorly answered question, with only very few students picking up all the core issues.