Version ICL/4

EGT3
ENGINEERING TRIPOS PART IIB

Friday 4 May 2018   2.00 to 3.40

Module 4F1

CONTROL SYSTEM DESIGN - SOLUTIONS

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4F1 Formulae sheet (3 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Block diagram of two degree of freedom controller

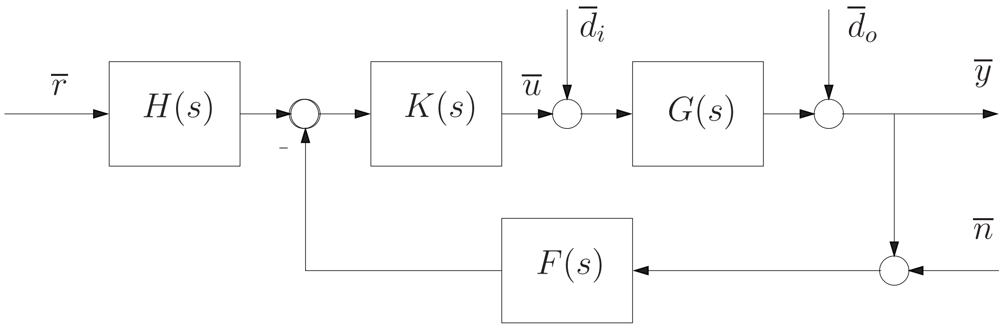

Fig. 1

Transfer function $T_{r\to y}$ should satisfy

- Any RHP zero of G must remain in $T_{r\to y}$.
- $T_{r\to y}$ must roll off at high frequencies at least as fast as G.

(b) (i) To find the closed loop poles we need to solve $1+kG_1(s)G_2(s)=0$. So we have

$$k(s^2+\omega_1^2)+(s^2+\omega_2^2)(\tau s+1)=0$$

or

$$\tau s^3+(1+k)s^2+\omega_2^2\tau s+\omega_2^2+k\omega_1^2=0$$

From the Routh-Hurwitz criterion in the data book, we have

$$(1+k)\omega_2^2\tau > \tau(\omega_2^2+k\omega_1^2) \quad (1)$$
$$\omega_2^2+k\omega_1^2>0 \quad (2)$$

From (1) we have $k(\omega_2^2-\omega_1^2)>0$ which implies $k<0$. From (2) we have

$$-\left(\frac{\omega_2}{\omega_1}\right)^2<k. \quad (3)$$

Hence we get

$$-\left(\frac{\omega_2}{\omega_1}\right)^2<k<0. \quad (4)$$

(ii) Root locus diagrams

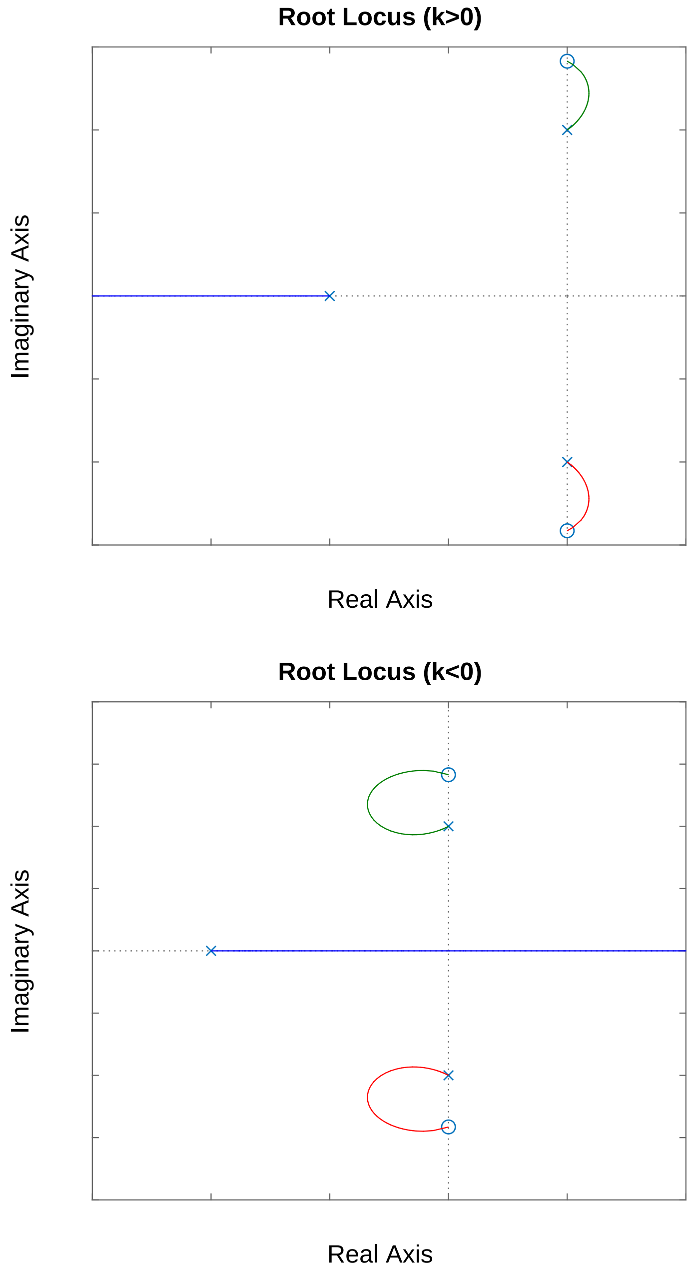

Fig. 2

(iii)

$$S = \frac{1}{1+G(s)K(s)}$$

$S(j\omega_2) = 0,\ S(j\omega_1) = 1$

$$T = \frac{G(s)K(s)}{1+G(s)K(s)}$$

$T(j\omega_2) = 1,\ T(j\omega_1) = 0$

(iv) In this range the sensitivity is close to zero, so there is reduced sensitivity to output disturbances and modeling uncertainty. Around ω_2 T is close to 1, so sensor noise not attenuated.

(v)

$$T_{r\to y} = H(s)\frac{G(s)K(s)}{1+G(s)K(s)} = \frac{s^2+1}{(s+1)^3}$$

So

$$\begin{aligned} H(s) = \frac{1+G(s)K(s)}{G(s)K(s)}\frac{s^2+1}{(s+1)^3} &= \frac{[(s^2+\omega_2^2)(s+1)+k(s^2+1)]}{k(s^2+1)}\frac{(s^2+1)}{(s+1)^3} \\ &= \frac{(s^2+\omega_2^2)(s+1)+k(s^2+1)}{k(s+1)^3} \end{aligned}$$

Also k needs to satisfy the condition in (b)(i).

Comments: The question was attempted by most students and was generally well answered. Mistakes were often made in the root-locus diagrams.

2 (a) Small sensitivity reduces the effect of disturbances on the plant output and the effect of plant uncertainty on the command transfer function.

Small complementary sensitivity provides improved robustness to multiplicative uncertainty (deduced via the small gain theorem).

(b) (i) Let $L(s) = G(s)K(s)$. $S(s)$ is analytic and has no zeros in the RHP so $\log S(s)$ is analytic in $Re(s) > 0$. Let $\sigma = \sigma_0$ and $\omega_0 = 0$ in the Poisson formula (provided in the data sheet). Then:

$$\sigma \ln|S(\sigma)| = \frac{2}{\pi}\int_0^\infty \frac{\sigma^2}{\sigma^2+\omega^2}\ln|S(j\omega)|d\omega$$

As $\sigma \to \infty$ the RHS converges to:

$$\frac{2}{\pi}\int_0^\infty \ln|S(j\omega)|d\omega$$

This follows because $\frac{\sigma^2}{\sigma^2+\omega^2}$ is close to one except for large ω, and $\ln|S(j\omega)| \to 0$ as $\omega \to \infty$ because $|S(j\omega)| \to 1$ as $\omega \to \infty$.

By assumption

$$L(\sigma) \sim \frac{c}{\sigma^k}$$

for large σ where $k \geq 2$, and c is a real constant. Thus

$$\sigma \ln|S(\sigma)| \sim -\sigma \ln(1 + c\sigma^{-k}) \tag{5}$$
$$= -\sigma(c\sigma^{-k} + \ldots) \tag{6}$$
$$= -c\sigma^{-k+1} + \ldots \tag{7}$$

Thus

$$\sigma \ln|S(\sigma)| = 0$$

as $\sigma \to \infty$. We therefore obtain:

$$\int_0^\infty \ln|S(j\omega)|d\omega = 0$$

(ii)

$$|1 + G(j\omega)K(j\omega)| \geq 1 - |G(j\omega)K(j\omega)|$$

Hence

$$|S(j\omega)| \leq \frac{1}{1 - |G(j\omega)K(j\omega)|}$$

provided $|G(j\omega)K(j\omega)| \leq 1$. Therefore

$$\ln|S(j\omega)| \leq -\ln(1 - \omega^{-2})$$

We now have

$$\int_{10}^{\infty} \ln(1-\omega^{-2})d\omega = \left[\omega \ln(1-\omega^{-2}) + \ln\left(\frac{\omega+1}{\omega-1}\right)\right]_{10}^{\infty} = -0.100$$

So

$$\int_{10}^{\infty} \ln|S(j\omega)|d\omega \leq 0.100$$

(iii)

$$\begin{aligned} 0 &= \int_{0}^{\infty} \ln|S(j\omega)|d\omega \\ &\leq \int_{0}^{1} \ln(\varepsilon)d\omega + \int_{1}^{10} \ln(1.5)d\omega + 0.100 \\ &= \ln(\varepsilon) + 9\ln(1.5) + 0.100 \end{aligned}$$

So $\varepsilon \geq 0.0235$.

Comments: Relatively few students attempted this question but generally good answers were provided. Relatively few provided a complete answer for the derivation in b(i).

3 (a) (i) Figure 3 below shows the bode plot of $G(s)$, $K_1(s)$ and $G(s)K_1(s)$ respectively.

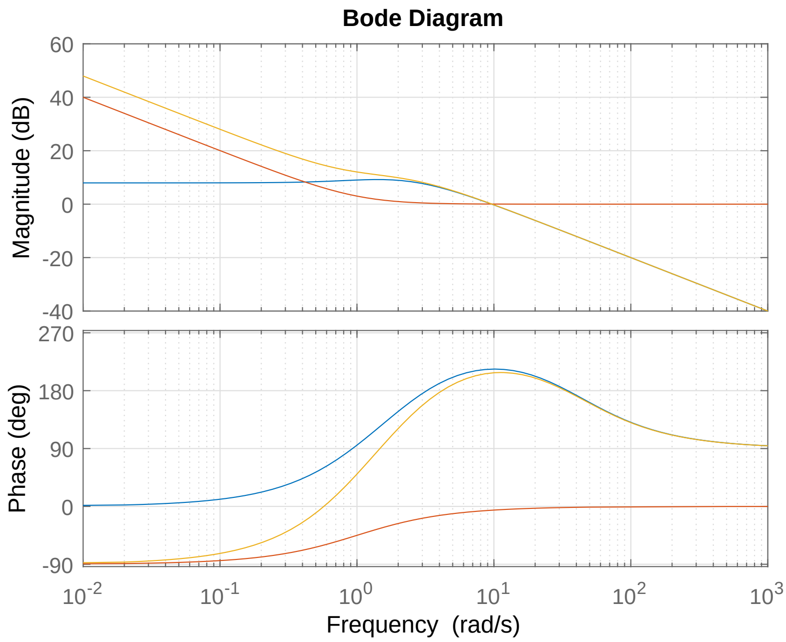

Fig. 3

(ii) Figure 4 shows the Nyquist plot of $G(s)$.

(iii) There are two counterclockwise encirclements of the point -1 in the Nyquist plot of $G(s)K_1(s)$ so there are two RHP poles.

(b) (i) The minimum phase plot of $G(s)$ is shown in Figure 5 (superimposed on its original bode plot).

(ii) There is one RHP zero. The two RHP poles provide a phase lead of 180° and their location is at $s = 2$. The RHP zero provides a phase lag and its location is at $s = 40$.

(iii) The crossover frequency should be above that of the RHP poles and below that of the RHP zero.

(c) (i) To achieve specification A with a single compensator this will need to have a gain at $s = 0$ of $10/G(0) = 4$. A phase lead compensator of the form

$$K_2(s) = \alpha \frac{(s + \omega_c/\alpha)}{(s + \omega_c \alpha)}$$

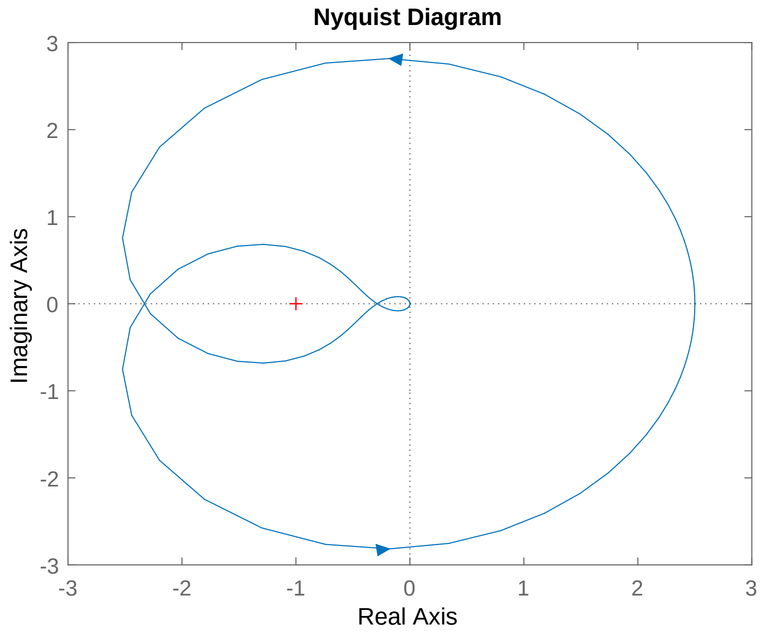

Fig. 4

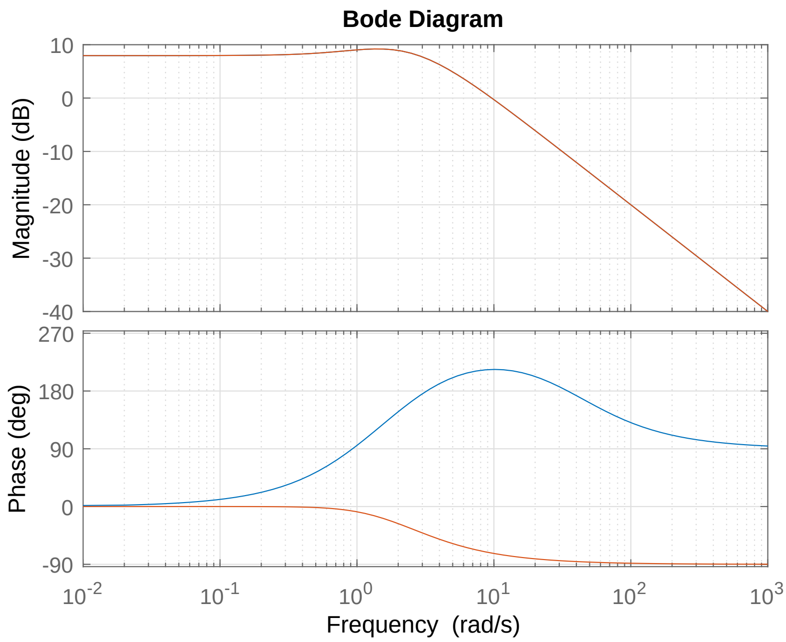

Fig. 5

needs to be multiplied by a constant 4α to achieve this. From the expression for the maximum phase advance in the data book, it follows that an increase in α, which increases the maximum phase advance, increases the crossover frequency due to the increased gain, and hence the requirement for phase advance increases. The maximum phase advance provided is then not adequate to satisfy the specification.

(ii) Chose $K_2(s)$ as a lead lag compensator. The phase lag compensator provides the required increased gain at low frequencies, and the lead compensator provides the phase lead needed for specification B. So choose

$$K_2(s) = \alpha \frac{(s+\omega_c/\alpha)}{(s+\omega_c\alpha)} \frac{(s+0.4\alpha)}{(s+0.1)}$$

where $\alpha = 2.1$, $\omega_c = 10$. The bode plot plot of $L(s)$ with this compensator is shown in Figure 6, superimposed on the plot without $K_2(s)$.

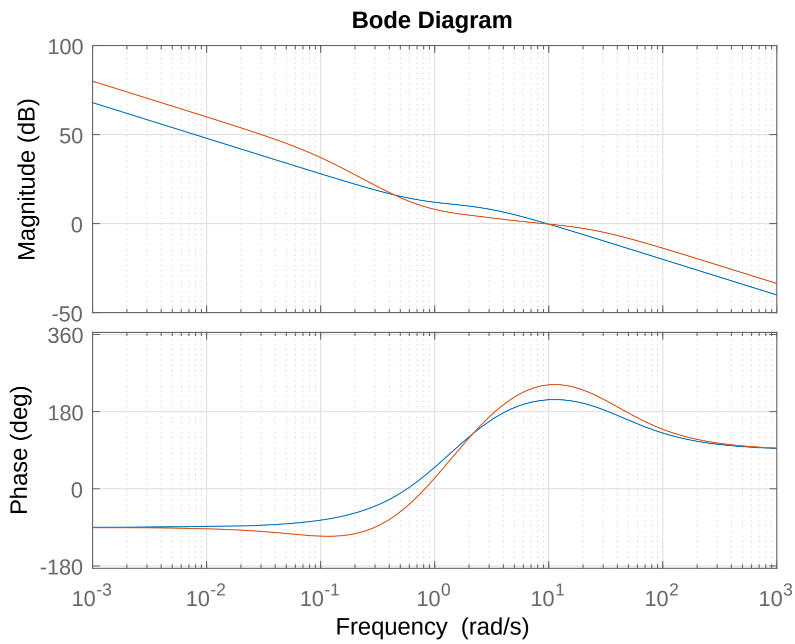

Fig. 6

Comments: A popular question attempted by most students. Generally good designs of lead/lag compensators were proposed. A number of students provided incomplete answers for c(i).

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