- Q1
- (a)

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{2.315 \times 10^6 \times 365 \times 24 \times 3600} = 9.492 \times 10^{-15} \,\mathrm{s}^{-1}$$

 $A = \lambda N$

$$N = \frac{ML}{m}$$
 where *M* is the mass (1 kg here), *m* is the molar mass and *L* is Avogadro's number.

$$\therefore N = \frac{1 \times 6.022 \times 10^{26}}{134.91} = 4.4637 \times 10^{24}$$

:
$$A = 9.492 \times 10^{-15} \times 4.4637 \times 10^{24} = 4.237 \times 10^{10} \text{ Bq}$$
 [15%]

(b) The mass of a neutron is 1.00867 u (4M16 Data Sheet page 2) The mass defect Δm = 134.90598 + 1.00867 - 135.90731 = 0.00734 u 1 u = 931.5 MeV (4M16 Data Sheet page 1)
∴ energy released = 0.00734 × 931.5 = 6.837 MeV [10%]

(c) As the atomic number increases by one and the mass number is unchanged, this is β^{-} decay. [5%]

(d) For
$${}^{136}_{55}$$
Cs $\lambda' = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{13.04 \times 24 \times 3600} = 6.151 \times 10^{-7} \text{ s}^{-1}$
 $A' = A'_0 \exp(-\lambda' t)$ (1.1)
 $A'_0 = \lambda' N_0$

From (a) $N_0 = 4.4637 \times 10^{24}$

$$\therefore A_0' = 6.151 \times 10^{-7} \times 4.4637 \times 10^{24} = 2.7456 \times 10^{18} \text{ Bq}$$

Rearranging equation 1.1, the time for the activity to fall below that found in (a) $(4.237 \times 10^{10} \text{ Bq})$ is given by

$$t = \frac{1}{\lambda'} \ln\left(\frac{A'_0}{A'}\right)$$

$$\therefore \quad t = \frac{1}{6.151 \times 10^{-7}} \ln\left(\frac{2.7456 \times 10^{18}}{4.237 \times 10^{10}}\right) = 2.9242 \times 10^7 \text{ s} \approx 338 \text{ days}$$

i.e. less than a year. Thus, transmutation will be effective in reducing the duration of the hazard associated with ${}^{135}_{55}$ Cs, if the rapid (and cost-effective) transmutation of ${}^{135}_{55}$ Cs into ${}^{136}_{55}$ Cs can be achieved. [20%]

(e)
$$\eta = \frac{v\Sigma_f}{\Sigma_a}$$

 $\Sigma_f = N_{\rm U5}\sigma_{f\rm U5} = 580N_{\rm U5}$ (working in barns)

$$\Sigma_a = \sum_i N_i (\sigma_{fi} + \sigma_{ci})$$

....

$$\therefore \quad \Sigma_a = N_{\rm U5}(\sigma_{f\rm U5} + \sigma_{c\rm U5}) + N_{\rm U8}(\sigma_{f\rm U8} + \sigma_{c\rm U8}) + N_{\rm Cs}(\sigma_{f\rm Cs} + \sigma_{c\rm Cs})$$
$$\Sigma_a = N_{\rm U5}(580 + 107) + 4 \times 10^{28}(0 + 2.75) + 1 \times 10^{27}(0 + 8.3) = 687N_{\rm U5} + 11.83 \times 10^{28}$$

Note that the fission cross-section for ${}^{135}_{55}$ Cs is not given in the question because any self-respecting 4M16 student should know that it is zero.

$$\therefore \quad \eta = \frac{v\Sigma_f}{\Sigma_a} = \frac{2.43 \times 580 N_{U5}}{687 N_{U5} + 11.83 \times 10^{28}} = \frac{1409.4 N_{U5}}{687 N_{U5} + 11.83 \times 10^{28}}$$

If $\eta < 1$
$$\therefore \quad \frac{1409.4 N_{U5}}{687 N_{U5} + 11.83 \times 10^{28}} < 1$$

$$\therefore \quad N_{U5} < \frac{11.83 \times 10^{28}}{1409.4 - 687} = 1.638 \times 10^{26} \text{ m}^{-3}$$
 [20%]

(f) It should be noted that, as stated in the preamble to the question, ${}^{135}_{55}$ Cs decays by the emission of a low-energy β particle. β particles are easily shielded against. The fact that ${}^{135}_{55}$ Cs has a very long half-life means its specific activity (activity per unit mass) is low and this, combined with the fact that its radiation is low-energy, means it will not present a significant decay heat burden to a repository. There is therefore no compelling need to handle ${}^{135}_{55}$ Cs by any means other than long-term storage. To compete any alternative must be very efficient and cost-effective.

The calculation in (d) shows that transmutation would be an effective way of eliminating the 'hazard' presented by ${}^{135}_{55}$ Cs if the transmutation process itself is efficient. However, the nuclear data given in (e) shows that the capture cross-section of ${}^{135}_{55}$ Cs is quite small. Therefore, high fluxes will be needed to achieve significant transmutation rates. In an accelerator-driven system, as suggested in the question, this would imply a high power/high current (and therefore very expensive) accelerator.

A further, highly significant consideration is that ${}^{135}_{55}$ Cs is a daughter product of the common fission product ${}^{135}_{54}$ Xe – data on page 4 of the 4M16 Data Sheet. Thus, ${}^{135}_{54}$ Xe (and thence ${}^{135}_{55}$ Cs) will be being produced through the fission of ${}^{235}_{92}$ U as ${}^{135}_{55}$ Cs is being transmuted.

The rate of transmutation of ${}^{135}_{55}$ Cs is proportional to $\phi N_{Cs} \sigma_{cCs}$ where ϕ is the flux.

The rate of production of ${}^{135}_{54}$ Xe is proportional to $\gamma_{Xe}\phi N_{U5}\sigma_{fU5}$ where γ_{Xe} is the fission yield of ${}^{135}_{54}$ Xe (0.064 according to the 4M16 Data Sheet).

For the numbers in the question, the ratio of these rates is

$$\frac{N_{\rm Cs}\sigma_{c\,\rm Cs}}{\gamma_{\rm Xe}N_{\rm U5}\sigma_{f\rm U5}} = \frac{1\times10^{27}\times8.3}{0.064\times1.638\times10^{26}\times580} = 1.365$$

Thus, the rate of transmutation of ${}^{135}_{55}$ Cs is similar in magnitude to the rate of creation of ${}^{135}_{54}$ Xe. Of course, by no means all the ${}^{135}_{54}$ Xe will decay to ${}^{135}_{55}$ Cs (it is, after all, an important reactor poison because of its high capture cross-section) and the amounts of ${}^{235}_{92}$ U and ${}^{135}_{55}$ Cs in the fuel could be varied, but nevertheless all of the above calls into question the viability and appropriateness of the scheme under consideration. [30%]

4M16 2018

Assessor's Comments:

All candidates: 61 attempts, Average mark 13.4/20, Maximum 19, Minimum 3. A popular question attempted by 84% of candidates and generally done quite well. A surprising number of candidates overlooked the mass of the neutron in calculating the energy released in (b).

Several candidates failed to distinguish between β^- decay and β^+ decay in answering (c). Quite a few candidates lost an easy mark by failing to comment on the significance of the result found in (d).

Calculations in (e) most often went astray because of a failure to distinguish appropriately between absorption cross-section and capture cross-section.

Discussion in (f) was often very brief despite the significant number of marks available. Few candidates recognised that Cs-135 would be produced from Xe-135 as well as being transmuted in the accelerator-driven transmutation system proposed.

Some candidates incorrectly assumed that the fact that the U-235 number density calculated in (e) implied an 'enrichment' below the natural composition of uranium meant that the reactor was not feasible.

Substituting in

- (a) $\frac{dn}{dt}$ is the rate of change of the neutron population
 - $\frac{\rho \beta}{\Lambda}n$ is the net rate of production of prompt neutrons through fission

 λc is the rate of production of delayed neutrons and also the rate of decay of delayed neutron precursors

- $\frac{dc}{dt}$ is the rate of change of the precursor population $\frac{\beta}{\Lambda}n$ is the rate of production of precursors through fission
- (b) From $\frac{dc}{dt} = 0$, the equilibrium relationship between *n* and *c* is

$$c_0 = \frac{\beta}{\Lambda\lambda} n_0$$

Taking Laplace transforms (p is the transform variable and overbars indicate transformed variables)

$$p\overline{n} - n_0 = \frac{\rho - \beta}{\Lambda}\overline{n} + \lambda\overline{c} \qquad (1)$$

$$p\overline{c} - c_0 = \frac{\beta}{\Lambda}\overline{n} - \lambda\overline{c}$$

$$\therefore \quad \overline{c}(p+\lambda) = \frac{\beta}{\Lambda}\overline{n} + c_0 = \frac{\beta}{\Lambda}\overline{n} + \frac{\beta}{\Lambda\lambda}n_0 \qquad \text{substituting for } c_0$$

$$\therefore \quad \lambda\overline{c} = \frac{\beta}{\Lambda}\frac{(\lambda\overline{n} + n_0)}{(p+\lambda)}$$

$$(1) \qquad p\overline{n} - n_0 = \frac{\rho - \beta}{\Lambda}\overline{n} + \frac{\beta}{\Lambda}\frac{(\lambda\overline{n} + n_0)}{(p+\lambda)}$$

$$\therefore \quad \overline{n} \left[p - \frac{\rho - \beta}{\Lambda} - \frac{\beta\lambda}{\Lambda(p+\lambda)} \right] = n_0 \left[1 + \frac{\beta}{\Lambda(p+\lambda)} \right]$$

$$\therefore \quad \overline{n} = \frac{n_0 \left[1 + \frac{\beta}{\Lambda(p+\lambda)} \right]}{\left[p - \frac{\rho - \beta}{\Lambda} - \frac{\beta\lambda}{\Lambda(p+\lambda)} \right]}$$

To take inverse transforms we need to reduce this expression to partial fraction form. This entails finding the roots of the equation formed by setting the denominator of the RHS to zero, i.e.

$$p - \frac{\rho - \beta}{\Lambda} - \frac{\beta \lambda}{\Lambda(p + \lambda)} = 0$$

$$\therefore \quad \rho = \Lambda p + \beta - \frac{\beta \lambda}{p + \lambda}$$
[40%]

[15%]

4M16 2018

Final Crib

(c) From (b)
$$c_0 = \frac{\beta}{\Lambda\lambda} n_0 \implies \frac{c_0}{n_0} = \frac{\beta}{\Lambda\lambda} = \frac{0.0075}{10^{-4} \times 0.1} = 750$$

To find the time constants for the excursion solve

$$\rho = \Lambda p + \frac{\beta p}{p + \lambda}$$

$$\therefore \quad \frac{\rho}{\Lambda} (p + \lambda) = p(p + \lambda) + \frac{\beta p}{\Lambda}$$

$$\therefore \quad p^2 + p \left[\lambda + \frac{\beta - \rho}{\Lambda} \right] - \frac{\rho \lambda}{\Lambda} = 0$$

$$\therefore \quad p^2 + p \left[0.1 + \frac{0.0075 - 0.005}{10^{-4}} \right] - \frac{0.005 \times 0.1}{10^{-4}} = 0$$

$$\therefore \quad p^2 + 25.1p - 5 = 0 \implies p = 0.19765 \text{ or } -25.29765 \text{ s}^{-1}$$

Therefore the dominant time constant (the positive one) is

$$T_{+} = \frac{1}{p_{+}} = \frac{1}{0.19765} = 5.0595 \,\mathrm{s}$$
 [20%]

(d) Without precursors, the neutron kinetics equation is $\frac{dn}{dt} = \frac{\rho}{\Lambda}n$, so, by inspection, the dominant time constant is

$$T_{+} = \frac{\Lambda}{\rho} = \frac{10^{-4}}{0.005} = 0.02 \text{ s (a much more rapid response)}$$
 [10%]

(e) In a Fast Breeder Reactor (FBR), the prompt neutron reproduction time Λ is about 1000 times shorter than that in a PWR, i.e. $\sim 10^{-7}$ s. Assuming β and λ are unchanged, for the same reactivity change, the quadratic equation for *p* becomes

$$p^2 + 25000.1p - 5000 = 0 \implies p_+ = 0.2 \text{ s}^{-1}$$

So, T_+ would be very slightly reduced due to this effect.

In practice, β is likely to be rather smaller due to the use and breeding of plutonium in the FBR. For $^{239}_{94}$ Pu β is ~0.003. Assuming an 'average' value of $\beta = 0.0065$ instead, the quadratic equation for *p* becomes

$$p^2 + 15000.1p - 5000 = 0 \implies p_+ = 0.33332 \text{ s}^{-1}$$

So, T_+ would be ~3 seconds rather than ~5 seconds – a significant reduction. [15%]

Assessor's Comments:

All candidates: 70 attempts, Average mark 13.9/20, Maximum 19, Minimum 3.

A very popular question attempted by all but 3 candidates, many of whom made good attempts. Part (a) was answered surprisingly poorly with much sloppy terminology failing to distinguish carefully enough between prompt neutrons, delayed neutrons and (delayed neutron) precursors.

4M16 2018

Attempts to derive the in-hour equation in (b) that went astray usually did so due to poor algebra rather than lack of knowledge about what to try to do. Several answers were undermined by the candidate's inability to write λ and Λ distinguishably.

Quite a few candidates failed in recognise that they were simply being asked to solve the in-hour equation (a quadratic equation) for the case specified in (c) and instead tried to use the prompt jump approximation or another approximate method.

Several candidates found the dominant inverse periods in (c) and (d) rather than the time constant as required.

A number of candidates failed to recognise that both β and *c* would be zero if delayed neutrons were not present in (d).

In (e) many candidates correctly recognised that Λ would be much shorter in a FBR but failed to appreciate the small impact this would have on T_+ .

[20%]

Q3

(a) Using the partial reactivity model and taking the reactivity to vary as $\rho_0 \left(1 - \frac{\tau}{T_1}\right)$, the end-ofcycle (EOC) condition for the first cycle is

$$\rho_0 \left(1 - \frac{\tau_1}{T_1} \right) = 0$$

as all batches are the same. Therefore the length of the first cycle is given by

$$\tau_1 = T_1$$

The EOC condition for the second cycle is

$$\frac{1}{M}\rho_0\left(1 - \frac{\tau_2}{T_1}\right) + \frac{M-1}{M}\rho_0\left(1 - \frac{\tau_1 + \tau_2}{T_1}\right) = 0$$

where the first term is the partial reactivity of the fresh batch and the second term the partial reactivity of the retained batches.

$$\therefore \quad \left(1 - \frac{\tau_2}{T_1}\right) + \left(M - 1\right) \left(1 - \frac{\tau_1}{T_1} - \frac{\tau_2}{T_1}\right) = 0$$
$$\therefore \quad 1 - \frac{\tau_2}{T_1} - \left(M - 1\right) \frac{\tau_2}{T_1} = 0$$
$$\therefore \quad M \frac{\tau_2}{T_1} = 1 \quad \Rightarrow \quad \tau_2 = \frac{T_1}{M} = \frac{\tau_1}{M}$$

(b) The EOC condition for the general case (taking M = 3) is

Noting that $\tau_1 = T_1$

$$\frac{1}{3}\rho_0 \left(1 - \frac{\tau_n}{T_1}\right) + \frac{1}{3}\rho_0 \left(1 - \frac{\tau_n + \tau_{n-1}}{T_1}\right) + \frac{1}{3}\rho_0 \left(1 - \frac{\tau_n + \tau_{n-1} + \tau_{n-2}}{T_1}\right) = 0$$

where, from left to right, the terms represent the partial reactivities of the batches from freshest to oldest.

$$\therefore (T_1 - \tau_n) + (T_1 - \tau_n - \tau_{n-1}) + (T_1 - \tau_n - \tau_{n-1} - \tau_{n-2}) = 0$$

$$\therefore 3T_1 - 3\tau_n - 2\tau_{n-1} - \tau_{n-2} = 0$$

$$\therefore \tau_n = T_1 - \frac{2}{3}\tau_{n-1} - \frac{1}{3}\tau_{n-2}$$

As the reactor power is constant, burn-up is proportional to time. $\tau_1 = T_1 = 24$ months (given)

Using the result from (a)
$$\tau_2 = \frac{\tau_1}{M} = \frac{24}{3} = 8$$
 months
 $\therefore \quad \tau_3 = T_1 - \frac{2}{3}\tau_2 - \frac{1}{3}\tau_1 = 24 - \frac{2}{3} \times 8 - \frac{1}{3} \times 24 = 10\frac{2}{3}$ months $\left(\frac{4}{9}T_1\right)$
 $\therefore \quad \tau_4 = T_1 - \frac{2}{3}\tau_3 - \frac{1}{3}\tau_2 = 24 - \frac{2}{3} \times 10\frac{2}{3} - \frac{1}{3} \times 8 = 14\frac{2}{9}$ months $\left(\frac{16}{27}T_1\right)$
 $\therefore \quad \tau_5 = T_1 - \frac{2}{3}\tau_4 - \frac{1}{3}\tau_3 = 24 - \frac{2}{3} \times 14\frac{2}{9} - \frac{1}{3} \times 10\frac{2}{3} = 10\frac{26}{27}$ months $\left(\frac{37}{81}T_1\right)$

The availability

Final Crib

From page 7 of the 4M16 Data Sheet, the equilibrium cycle length is given by

$$T_M = \frac{2T_1}{M+1} \implies T_3 = \frac{2 \times 24}{3+1} = 12 \text{ months}$$
 [30%]

(c) Including the refueling outage the total cycle length is $L = T_M + \Delta$.

$$A = \frac{T_M}{L} = \frac{T_M}{T_M + \Delta} = \frac{1}{1 + \Delta/T_M}$$

Hence, A is maximized when Δ/T_M is minimized.

(d)

$$L = T_M + \Delta = \frac{2T_1}{M+1} + \alpha + \frac{\beta}{M}$$

Taking 24 months to be 104 weeks, for M = 4

$$L = \frac{2 \times 104}{4+1} + 1 + \frac{16}{4} = 46.6 \text{ weeks}$$

From an operational (and economic) point of view, it would be better to have an annual cycle, i.e. 52 weeks, so that the outage can be scheduled in a period when electricity demand and prices are lowest (in the summer in the UK).

The analysis above shows that, perhaps surprisingly, the optimal value of M does not depend on T_1 , so this could be adjusted to give $T_M = 47$ weeks (as $\Delta = 5$ weeks for M = 4), i.e.

$$\frac{2T_1}{M+1} = 47 \implies T_1 = 117.5 \text{ weeks for } M = 4$$

 T_1 can be increased by operating the reactor at lower power (which is certainly feasible, but not economically attractive as this would reduce revenue) or by increasing the enrichment of the fuel slightly (which would increase the cost of fresh fuel somewhat). [20%]

Assessor's Comments:

All candidates: 58 attempts, Average mark 14.5/20, Maximum 19, Minimum 1. A popular question attempted by 79% of candidates, many of whom made excellent attempts. Part (a) was done pleasingly well.

[5%]

[25%]

A number of candidates found the correct cycle lengths in (b) without finding the recurrence relationship required. They only received partial credit.

Others made things more difficult than they needed to be by not knowing/using the fact that the result for the steady-state cycle length in *M*-batch refueling is given on the 4M16 data sheet.

Several candidates made (d) more complicated than it needed to be by differentiating an expression for availability with respect to *M* rather than exploiting the strong hint in (c).

A surprisingly large number of candidates calculated an incorrect cycle length in (e) by mixing up weeks and months.

Although most candidates correctly recognised that a 12 month cycle length would be best, there were fewer well thought-out suggestions as to how to achieve this.

Q4

- (a) The UK has three categories of nuclear waste:
 - Low level waste (with activity < 4 GBq/tonne of alpha activity or < 12 GBq/tonne of beta/ gamma activity)
 - Intermediate level waste (with activity > LLW but not heat-generating)
 - High level waste (with activity > LLW and heat generating)
- (b) The UK will allow wastes giving a public dose of less than 0.3 mSv/yr to go to specially designated non-nuclear landfill sites, although the average dose in such cases is usually below 0.02 mSv/yr.

Low level wastes are generally solid materials. They are first sorted to separate out very low level and non-active components, and are then compacted for disposal in half-height ISO freight-type containers under controlled conditions in a specially designated surface landfill site in Cumbria.

Intermediate level wastes can take many forms, including spent ion exchange resins, fuel cladding, sludges and filter cakes etc., and are usually encapsulated in cement in either shielded or unshielded drums and stored in a secure environment to await ultimate disposal in the proposed deep geological repository.

High level wastes comprise the fission products or Highly Active Liquor (HAL) produced in
spent fuel reprocessing. They are first evaporated to reduce volume, then vitrified to prevent
long-term leaching. The glass is poured into stainless steel containers, again to be stored
securely, to await ultimate disposal in the proposed deep geological repository.[30%]

(c) Specific activity at entry to ion exchange plant = 20.7 Bg g^{-1}

Decontamination factor of ion exchange plant = 10

 \therefore Specific activity at exit from ion exchange plant = 2.07 Bq g⁻¹

Hold-up/decay

Collection time $T = \frac{\text{volume}}{\text{flow rate}} = \frac{336}{0.8208} = 409.36 \text{ hours}$

The number of atoms of ${}^{60}_{27}$ Co per hour arising in the hold-up/decay tank is given by

$$P = \frac{\rho \times Q \times A \times 1000 \times 3600}{\lambda}$$

where ρ is the density, Q is the volumetric flow rate, A is specific activity of ${}^{60}_{27}$ Co in Bq g⁻¹, 1000 is the conversion factor from kg to g, 3600 is the conversion factor from hours to seconds, and λ is the decay constant for ${}^{60}_{27}$ Co in hr⁻¹.

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{5.26 \times 365 \times 24} = 1.504 \times 10^{-5} \,\mathrm{hr}^{-1}$$

$$\therefore P = \frac{1000 \times 0.8208 \times 2.07 \times 1000 \times 3600}{1.504 \times 10^{-5}} = 4.067 \times 10^{14} \,\mathrm{atoms \, hr}^{-1}$$

[15%]

The number of ${}^{60}_{27}$ Co is given by

$$N = \frac{P}{\lambda} (1 - \exp(-\lambda T)) \exp(-\lambda t)$$

where *T* is the collection time and *t* the hold-up time

$$N = \frac{4.067 \times 10^{14}}{1.504 \times 10^{-5}} (1 - \exp(-1.504 \times 10^{-5} \times 409.36)) \exp(-1.504 \times 10^{-5} \times 312) = 1.652 \times 10^{17}$$

The specific activity of the effluent is given by

$$A' = \frac{\lambda N}{\rho \times V \times 1000 \times 3600}$$

where V is the hold-up tank volume

$$\therefore A' = \frac{1.504 \times 10^{-5} \times 1.652 \times 10^{17}}{1000 \times 336 \times 1000 \times 3600} = 2.054 \text{ Bg g}^{-1}$$

The ion exchange plant has a significant (and obvious) effectiveness. The calculation shows that the hold-up/decay process has very little effect due to the long half-life of $^{60}_{27}$ Co, but this is just one nuclide amongst many in this particular stream. [55%]

Assessor's Comments:

All candidates: 29 attempts, Average mark 10.8/20, Maximum 18, Minimum 1. Comfortably the least popular question, attempted by only 40% of candidates, and the least well

done.

Few candidates knew/used the official high/intermediate/low level descriptors in (a). Only a handful had any idea what the threshold activities denoting low-level waste were.

Answers to (b) were often vague on specifics and interchangeably discussed past, present and future methods for handling and disposal of waste – the question specifically asks about methods currently used.

Part (c) was done very well by some candidates, but others were evidently unclear on the details of the calculation required and, in particular, the role and use of Bateman's equation.

Several candidates demonstrated a lack of appreciation of the inevitable ineffectiveness of hold-up and decay in this case, given the long half-life of Co-60, and in consequence were unable to spot that their calculations must have gone wrong.