Version AMB/3.4

EGT0
ENGINEERING TRIPOS PART IA

Wednesday 6 June 2018 9 to 12.10

Paper 1

MECHANICAL ENGINEERING

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

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*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

SECTION A

1 **(short)** A two-dimensional water jet with constant density ρ, and negligible friction is deflected in a right angle by a stationary circular obstacle as shown in Fig. 1. While in contact with the solid obstacle, the jet has circular streamlines between radii $r = R_1$ and $r = R_2$, with a velocity distribution $v(r) = \frac{C}{r}$.

(a) Find the pressure difference Δp between the streamlines at $r = R_1$ and $r = R_2$ in terms of the constants ρ, C, R_1, and R_2. [6]

(b) Find the components, parallel and perpendicular to the impinging jet, of the force by the jet on the obstacle, indicating their direction. [4]

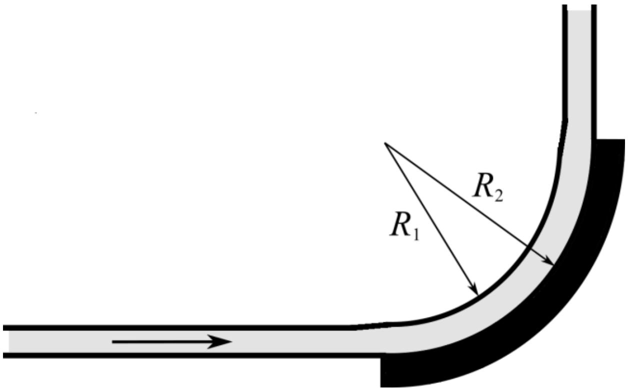

Fig. 1

2 **(short)** A connection between two pipes of cross sectional areas A_1 and A_2 can be made (i) with a smooth, long transition, where mixing is negligible, or (ii) with a sudden expansion, as indicated in Figs. 2(a) and 2(b), respectively. An incompressible fluid of density ρ flows at velocity V_1 through A_1 in each case. By mass conservation, the velocity through A_2, far enough from the connection, is $V_2 = (A_1/A_2)\, V_1$. In case (ii), the pressure on the back-face of the expansion can be approximated as being equal to the upstream pressure p_1.

(a) Find the change in pressure between sections 1 and 2 for case (i), and show that it is of the form $\Delta p_{\mathrm{a}} = \frac{1}{2}\rho V_1^2 K_{\mathrm{a}}$, where K_{a} is only a function of A_1 and A_2. [5]

(b) Find the change in pressure between sections 1 and 2 for case (ii), and show that it is of the form $\Delta p_{\mathrm{b}} = \frac{1}{2}\rho V_1^2 K_{\mathrm{b}}$, where K_{b} is only a function of A_1 and A_2. [5]

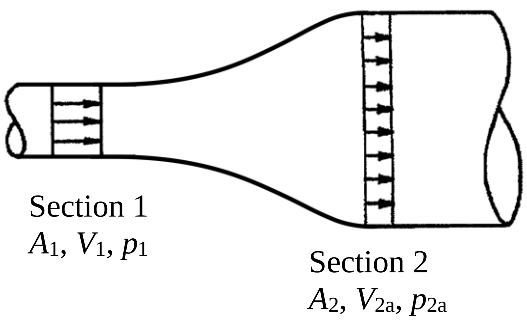

Fig. 2(a)

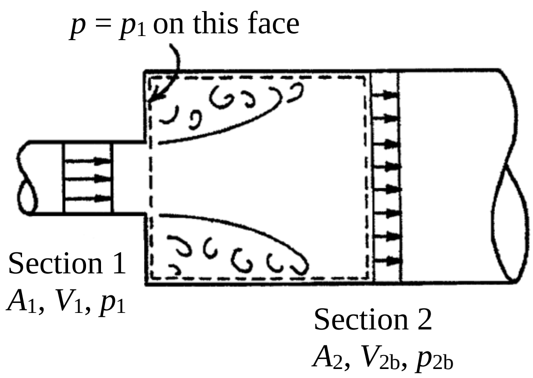

Fig. 2(b)

3 **(long)** A cylindrical, open tank discharges an incompressible fluid of density ρ without friction to the atmosphere as shown in Fig. 3(a). The tank has diameter D and time-varying instantaneous water level height $h(t)$, which is initially $h(0) = H$. The discharge pipe has diameter d and negligible length, and the streamlines on exit can be assumed to be parallel. The acceleration of gravity is g.

(a) Find the expression that relates the discharge velocity v with the rate of change of the water level, $\mathrm{d}h/\mathrm{d}t$. Under what conditions can the discharge be considered quasi-steady? [4]

(b) Assuming that the flow is quasi-steady, find an expression for v in terms of D, d, g, and the instantaneous water level, h. [8]

(c) Derive a differential equation for $h(t)$ and find the total discharge time T_D. To simplify your solution you may define a constant $G = \sqrt{\dfrac{gd^4}{2\left(D^4 - d^4\right)}}$. [10]

(d) A length of pipe L with the same diameter d is added to the discharge outlet, as shown in Fig. 3(b). Find the new total discharge time, T_D'. Show that, for $L = H$, the new discharge time is $T_D' \approx 0.41 T_D$. [8]

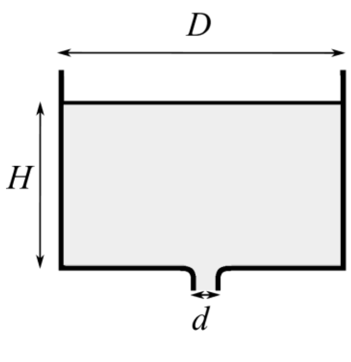

Fig. 3(a)

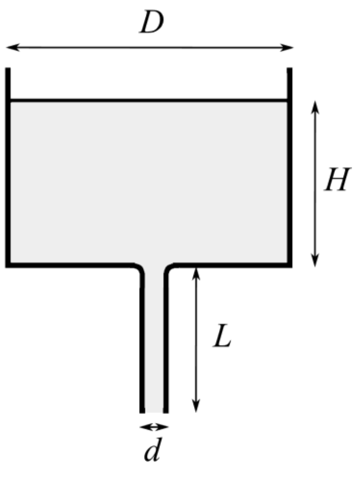

Fig. 3(b)

4 (**short**) A rigid vessel contains two closed systems of two ideal gases separated by a piston held in position by a pin, as shown in Fig. 4. Initially gas A and gas B have equal volumes of $V_1 = 1$ m^3, and gas A has twice the pressure of gas B, $p_1 = 2\,p_B$. The pin holding the piston is released. The pressures of both gases are equal at the final state and gas A has a final volume V_2. The piston is thermally conductive and both systems have a final temperature that is unchanged, $T_2 = T_1$.

(a) Determine whether the heat transfer and work from the initial to the final state are positive or negative for systems A and B. [4]

(b) Calculate V_2. [6]

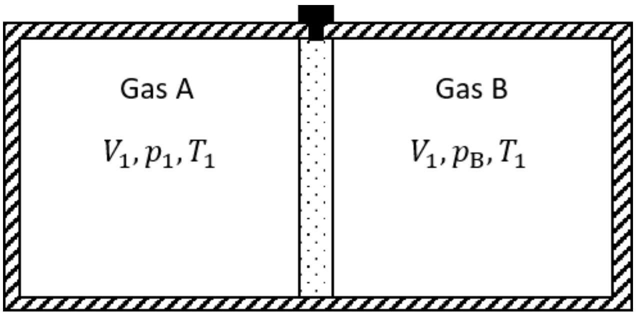

Fig. 4

5 **(short)** A process involving a perfect gas is shown in Fig. 5, whereby two gas streams are mixed, and heat is extracted, such that $\dot{Q}_{3-4}$ = –1.55 MW. At inlet 1, the mass flow rate is $\dot{m}_1$ = 1 kg s^{-1} and the temperature is T_1 = 1000 K. At inlet 2, the mass flow rate is $\dot{m}_2$ = 3 kg s^{-1} and the temperature is T_2 = 2000 K. The pressure changes are negligible, and the gas has a constant specific heat capacity at constant pressure of c_p = 350 J kg^{-1} K^{-1}.

(a) Neglecting changes in potential and kinetic energy of the gases, calculate the temperatures after mixing T_3, and after heat extraction T_4. [5]

(b) Calculate the total rate of entropy generation for the mixing process, $\dot{S}_{1\&2-3}$. [5]

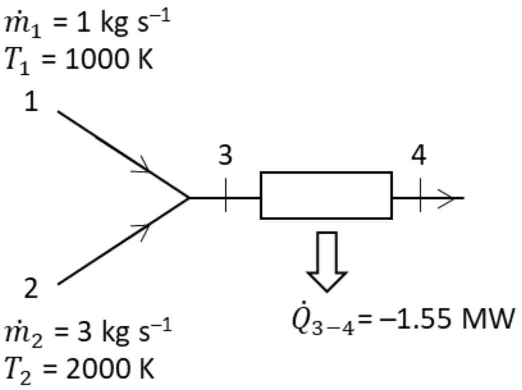

Fig. 5

6 **(long)** A supercharger compressor with an intercooler is fitted to a reciprocating engine as shown in Fig. 6. The air flow conditions at the inlet to the compressor are T_1 = 300 K and p_1 = 1 bar. The air flow at the compressor exit is fed into an intercooler which reduces the air temperature at constant pressure p_2 = 2 bar before entering the engine. The engine can be modelled as a steady flow device with heat input per unit mass of air flow of q_{3-4} = 1 MJ kg^{-1}, and an exhaust temperature of T_4 = 900 K at p_4 = 1 bar. The volumetric flow rate of air into the engine is limited to $\dot{V}$ = 0.1 m^3 s^{-1}. Assume that the cycle fluid is air which can be treated as a perfect gas with heat capacity ratio γ = 1.4, gas constant R = 287 J kg^{-1} K^{-1} and specific heat capacity at constant pressure c_p = 1.005 kJ kg^{-1} K^{-1}.

(a) Sketch a representative $T-s$ diagram for the total process cycle, indicating states 1-4. [5]

(b) Calculate the temperature at the compressor exit T_2 and specific work w_{1-2} required by the compressor from the engine output assuming isentropic compression. [5]

(c) Given that the intercooler has a heat transfer per unit mass of air flow of q_{2-3} = –60 kJ kg^{-1}, calculate the density of air at the exit of the compressor ρ_2 and at the exit of the intercooler ρ_3. [5]

(d) Calculate the output power $\dot{W}_{3-4}$ of the engine with and without the intercooler. [5]

(e) Calculate the total system efficiency of the engine including the compressor and intercooler, and the engine efficiency <u>without</u> the compressor and intercooler, i.e. the engine only. [5]

(f) Discuss the impact of the compressor and intercooler on the system power and efficiency. [5]

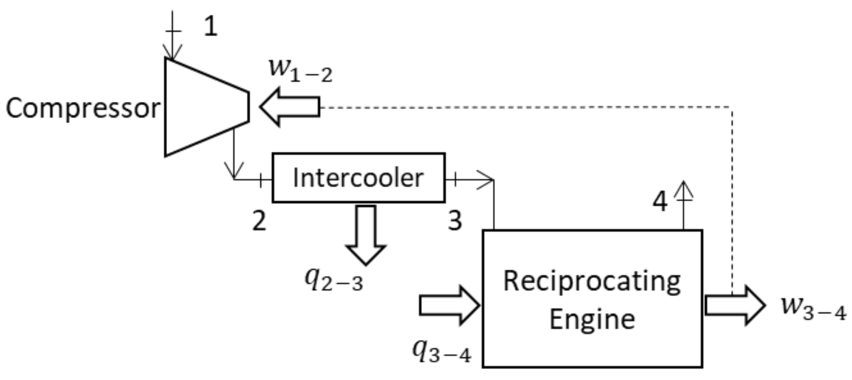

Fig. 6

SECTION B

7 **(short)** A uniform rod of length l and mass m is released from the vertical position shown in Fig. 7 and commences to fall under gravity. The rod rotates clockwise about the pivot point O where θ is the angular position of the rod with respect to the vertical. For the case when $\theta = 45^{o}$ determine:

(a) the angular acceleration of the rod; [5]

(b) the angular velocity of the rod. [5]

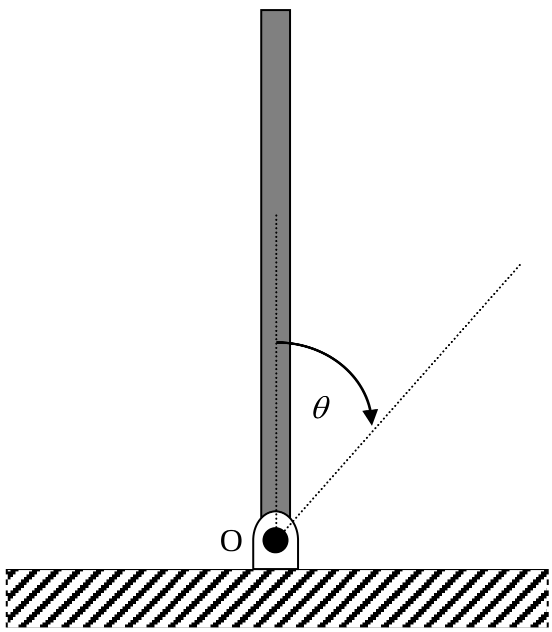

Fig. 7

8 **(short)** A mechanism consisting of two uniform bars is held in place by a linear spring of spring constant k. Points A and C are fixed, while B is constrained to move horizontally as shown in Fig. 8. Each bar has mass m and length L. The spring is unstretched when the angle α is 0°, and the system is in equilibrium when α is 45°.

(a) Obtain an expression for the potential energy of the system as a function of the spring extension x. [5]

(b) Determine a value for the spring constant k. [5]

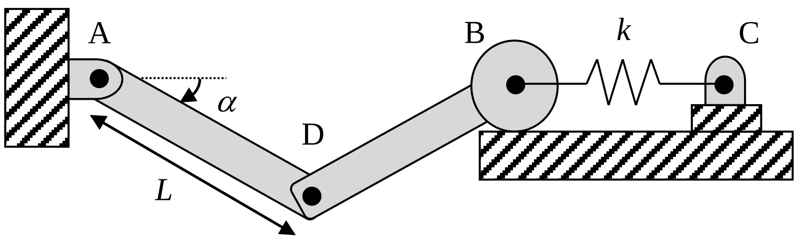

Fig. 8

9 **(short)** A fidget spinner consists of four identical annular weights held in a triangular configuration by a light plastic casing. Each weight has mass m, outer radius a and inner radius b. The weights are separated by a distance $2b$ such that the centre of a weight in the spinner's arm is a distance $2(a+b)$ from the spinner's centre as shown in Fig. 9.

(a) Find the moment of inertia of the fidget spinner about an axis through its centre and perpendicular to the plane of the masses. [6]

(b) The angular velocity of the spinner about the central axis is increased linearly as a function of time t such that $\dot{\theta} = ct$, where c is a constant. Find the total force $F(t)$ on a single weight in an arm of the spinner, expressing your answer as a vector using polar coordinates. [4]

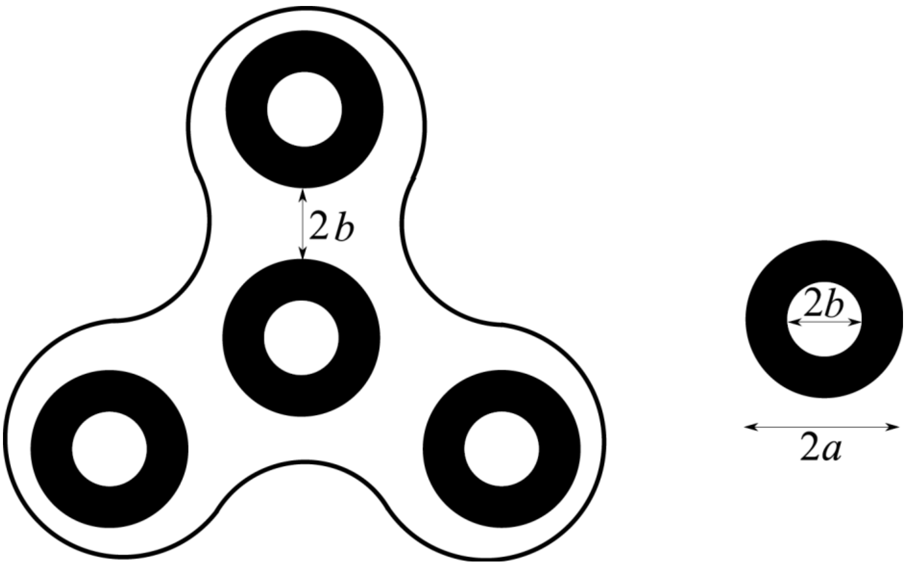

Fig. 9

10 **(long)** A satellite of mass *m* is orbiting the earth which has radius *R*. The position of the satellite at any point is specified by polar coordinates (r, θ), as shown in Fig. 10. At the perigee, the velocity of the satellite is given by $v = v_0$ and the position of the satellite is given by (r_0, 0). The acceleration due to gravity at the surface of the earth is g.

(a) Obtain expressions and values for the radial and polar components of the acceleration of the satellite in the elliptical orbit. Explain why the angular momentum of the satellite is conserved about an axis passing through the centre of the earth. [6]

(b) The motion of the satellite can be expressed by the following differential equation

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right)+\left(\frac{1}{r}\right)=\frac{gR^2}{r_0^2 v_0^2}$$

Solve the above equation by subsituting $u = 1/r$ to obtain an expression for the satellite orbit, and the eccentricity of the orbit. [8]

(c) At a given instant when the satellite is at the apogee, an impulse of magnitude I is imparted to the satellite, transferring it to a circular orbit of radius equal to the distance away from the centre of the earth at that instant. Determine the new velocity of the satellite v_1 and the magnitude of impulse I as a function of the other parameters. [8]

(d) At a later stage, following transfer to the circular orbit, the satellite commences to break up in orbit, and splits into two equal masses travelling in opposite directions. The velocity of the object travelling in the same direction as the original satellite is $3v_1$. Estimate the mechanical energy released instantaneously during satellite breakup. [8]

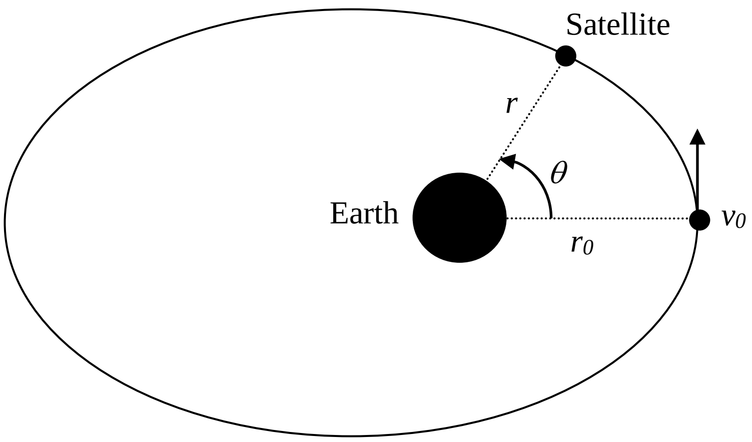

Fig. 10

11 **(short)** A tow truck of mass M is connected to a small car of mass m by a stretchable elastic rope, which is modelled with a viscous dashpot λ and a spring k, as shown in Fig. 11.

(a) The truck and car are both stationary, and the rope is at its natural length. At time $t = 0$, the truck moves away from the car at constant velocity v. If the position of the car is x, show that the equation of motion for x is

$$m\ddot{x} = k(vt - x) + \lambda(v - \dot{x}) .$$ [4]

(b) The viscous dashpot is tuned so the system is critically damped. Assuming $x = 0$ at time $t = 0$, find the motion of the car $x(t)$. [6]

Hint: You may assume that the solution to the critically damped equation of motion $\ddot{y}/\omega_n^2 + 2\dot{y}/\omega_n + y = 0$ is of the form $y = (At + B)e^{-\omega_n t}$.

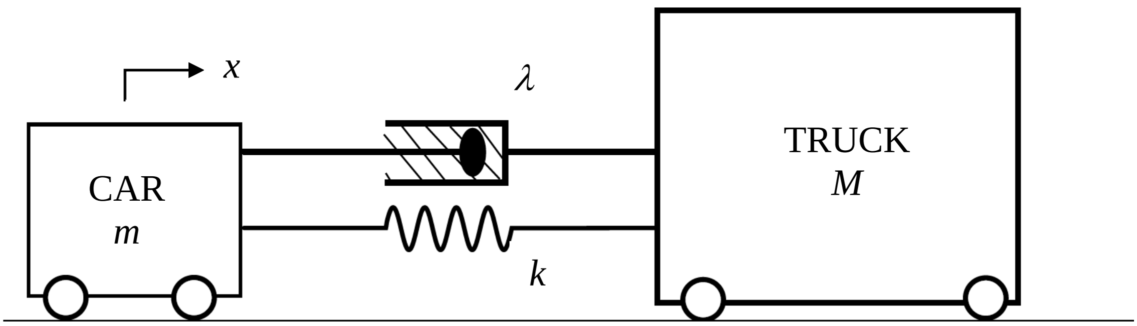

Fig. 11

12 **(long)** A mass m sits at the origin and is connected to fixed plates at $z = \pm\frac{4}{3}l$ by a pair of stretched springs, each with spring constant k and natural length l, as shown in Fig. 12.

(a) How many normal modes do we expect for the mass? Sketch the expected displacement pattern for each mode. [5]

(b) Find the angular frequency, ω_1, of the normal mode in which the mass moves up and down in the z direction. [5]

(c) If the mass moves in the horizontal (z = 0) plane, the change in length of the springs is negligible for small displacements. By using a small angle approximation ($\sin\theta \approx \tan\theta \approx \theta$) or otherwise, show the angular frequency of the normal mode in which the mass moves side-to-side in the x direction is $\omega_2 = \omega_1/2$. [10]

(d) The mass is subject to a driving vertical force $\mathbf{f} = f_0 \cos\left(\frac{3}{4}\omega_1 t\right)\hat{\mathbf{z}}$. Find the amplitude and phase of the steady state displacement of the mass relative to the driving force, assuming that damping is negligible once the steady state is achieved. [5]

(e) A viscous fluid is added between the plates that exerts a damping force $\mathbf{f} = -\lambda\mathbf{v}$, where $\mathbf{v}$ is the mass's velocity and λ is the damping constant. If the mass is subject to the same driving force, use the databook to estimate what value of λ would reduce the displacement of the mass by half of its undamped value. [5]

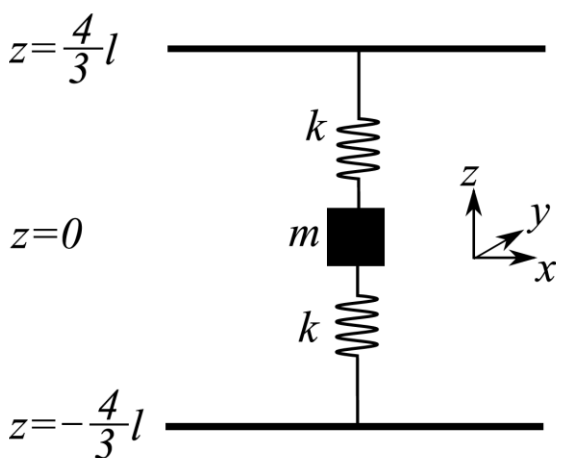

Fig. 12

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