Version FT/3

EGT1
ENGINEERING TRIPOS PART IB

Wednesday 6 June 2018 2 to 4.10

Paper 5

ELECTRICAL ENGINEERING

*Answer not more than **four** questions.*

*Answer not more than **two** questions from any one section and not more than **one** question from each of the other two sections.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS

Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed

Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

Solutions

1.

a) The FET input stage has a very high input impedance due to the gate capacitor of the FET. BJT collector-emitter-follower has a higher gain.

Point P is the balance point for the circuit. For differential input the voltage at P will be constant, for common mode the signal at both inputs change simultaneously and the voltage at point P will change as well.

b) The CMRR is define at

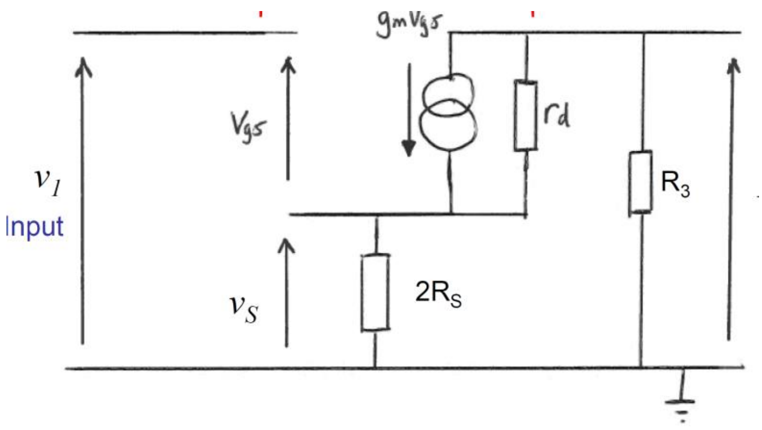
$$CMRR = \frac{Differential\ gain}{Common\ mode\ gain}$$

Using the small signal model and the symmetry of the circuit at the balance point P.

For differential mode the signal at point P is constant and P appears virtually connected to ground.

The gain in this case is A_D gain= $-g_m R_1$.

For common mode signals, we exploit the symmetry of the circuit if we replace Rs with two resistors in parallel of value 2Rs. The circuit can be split into two and studying only the left hand side with the 2Rs resistor connected to ground. The small signal model can be sketched as below:



$V1 = Vgs + V'$ where $V' = gmVgs\,2Rs$

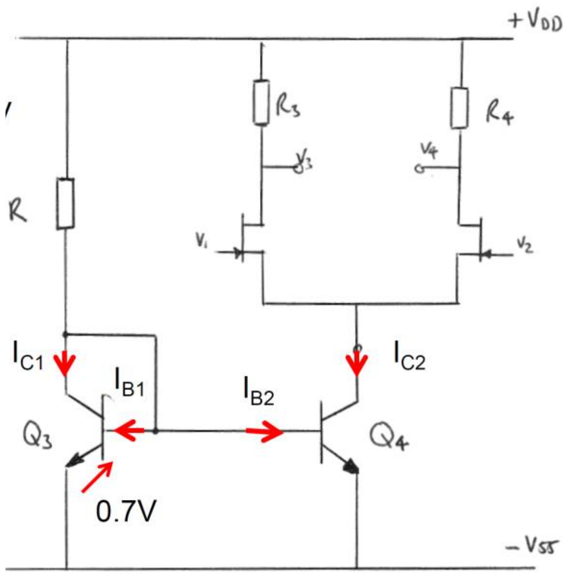
Hence $V1 = Vgs\,(1 + 2gmRs)$

Output $(0-Vo)/R1 = gmVgs$; $Vo = -gmVgsR1$;

So the common-mode gain will be $A_{cm} = -\,gmR1/(1+2gmRs)$.

The CMRR will then be: CMRR= Ad gain/Acm gain = 1+2gmRs

c) Assume that Q1 = Q2



$I_{c1} = (VDD - 0.7V)/R$; if h_{FE} is very large then $I_B << I_C$

By symmetry $I = I_{C1} = (V_{DD} - 0.7V)/R$

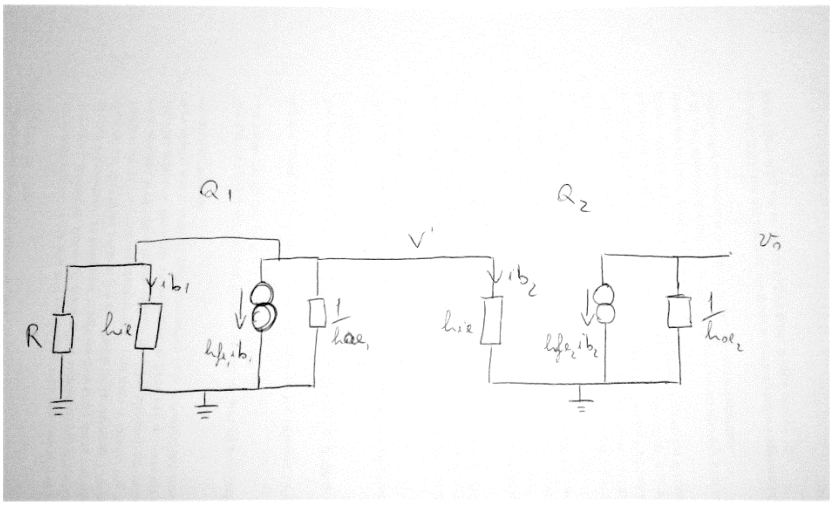
$Vbe_{1,2} = 0.7$ V

Hence $V_{ce2} = 0.7$V

d)

If $Q_1 = Q_2 \Rightarrow h_{fe1} = h_{fe2}$, $h_{ie1} = h_{ie2}$, $h_{ce1} = h_{ce2}$

The small signal model is



Both Q_1 and Q_2 have voltage v` at the base => $i_{b1} = i_{b2}$

V_{ce1}=0.7 constant => grounded. There fore v` is grounded. Hence Ro = $1/h_{oe}$.

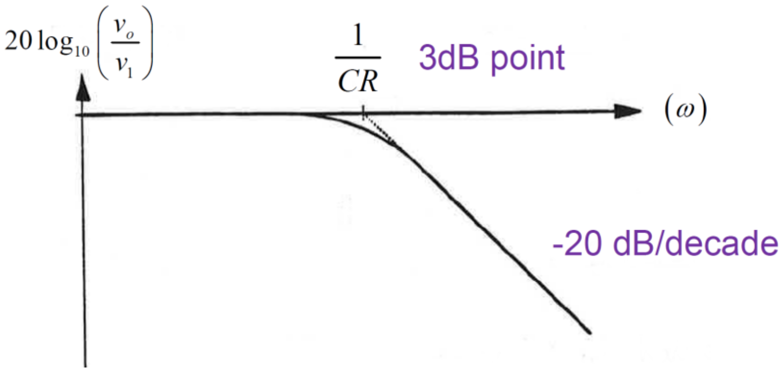
e)

The Circuit in Figure 2 is only made of BJTs and it is compatible with integrated circuits technology, unilike the Zener diode current mirror circuit.

2.

a) The gain compensation is used to stabilise the OpAmp at higher frequency.

A general gain compensation is:



Where the function described is: $A(\omega) = A_0/1+j\omega/\omega_0$

Because it is a first order system, the gain-bandwidth product is constant

Hence we get Gain(closed loop) = gain-bandwidth (open loop)/bandwidth (closed loop) = 100 A_0/100,000 = A_0/1000

b)

In an ideal OpAmp the gain is infinite, the (V_+- V_-) = 0

So, we get I_{in} = (V_{in} – V_o)/R; V_{in} = V_o Z/(R+Z) => V_o = (R+Z)/Z V_{in}

I_{in} = V_{in}/R – V_{in} (R+Z)/RZ = - V_{in}/Z => V_{in}/I_{in} = -Z (which is a NIC).

c)

Assuming that each NIC inverts the sing of the impedance preceding it, we can write:

The total impedance in Fig.4 is Z_{in} = - (Y) where Y = R+ (-R)(R+Z)/Z = - R^2/Z => Z_{in} = R^2/Z

d)

Remembering that $\omega = \frac{1}{\sqrt{LC}}$ => $L = \frac{!}{\omega^2 C} = \frac{1}{(2\pi 10^{-2})^2\ 10^{-4}}$ =2.5 x 10^4 H (This inductance is too high and impractical to manufacture).

So the inverted gyrator can be used to simulate the inductance.

The impedance of an inductor is Z=jωL.

Using the gyrator we get: Z_{in}= R2/jωL => $Z_{in} = \frac{1}{j\omega\left(\frac{L}{R^2}\right)}$

From this we get the following equivalence: C = L/R^2, where $R = \sqrt{\frac{L}{C}}$=1.59x10^4 Ω

3.

(a) For star connection

$$I_{line} = I_{ph} = \frac{V_{ph}}{Z_{ph}} = \frac{V_{line}}{\sqrt{3}Z_{ph}}$$

For delta connection

$I_{line} = \sqrt{3}I_{ph} = \sqrt{3}\, V_{line} \backslash Z_{delta}$

For equivalence $V_{line} \backslash\, I_{line}$ must be the same, so

$Z_{delta} = 3Z_{ph}$

(b)
For balanced star connection currents form a balanced 3 – phase set.
So $I_A = I_B = I_C = 0$

From Kirchhoff's laws we have $I_A + I_B + I_C + I_N = 0$ where I_N is the current through the neutral conductor. So $I_N = 0$

(c)
(i) delta connected load: $Z_{star} = Z_{delta}/3 = 1/3\ k\Omega$

So Z_{ph} of equivalent star connected load is

$(500 + 4 \times 50 \times 2\pi j)//333 = 293 + 61.4j\ \Omega$

(ii)

$$I_{line} = \frac{V_{ph}}{|Z|} = \frac{V_l/\sqrt{3}}{|Z|} = \frac{11 \times 10^3}{\sqrt{3}|293 + 61.4j|} = 21.2\ A$$

$P = 3I_{ph}^2 Re(Z_{ph}) = 3 \times (21.2)^2 \times 293 = 396\ kW$

$Q = 3I_{ph}^2 Im(Z_{ph}) = 3 \times (21.2)^2 \times 61.4 = 83.6\ kVAR$

(iii) Reactive power unchanged.

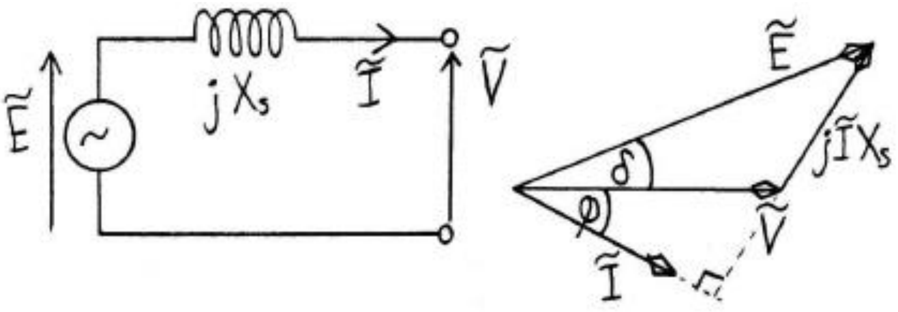
$$Real\ power = 396\ k + V_{ph}^2\left(\frac{1}{1.5k} + \frac{1}{1.0k} + \frac{1}{1.2k}\right) = 496\ kW$$

When the conductor between the star connections is not connected, the voltage at the star connection of the unbalanced load is not zero, hence the answer would change.

4.

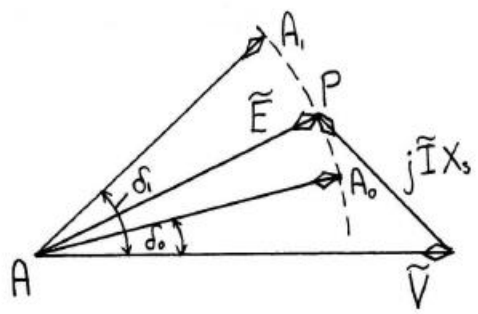
(a) Bookwork.

$\tilde{E} = \tilde{V} + j\tilde{I}X_s$

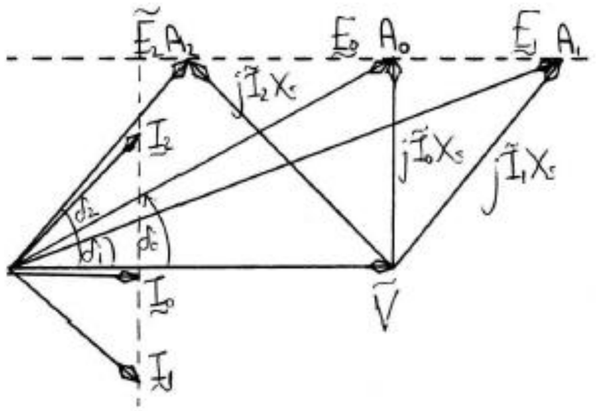


(b)
(i) Connection to an infinite bus means that the voltage and frequency are fixed and cannot be changed by any single generator.

(ii) Increasing torque with E constant increases δ.



(iii) Reactive power is controlled by changing E. Constant power means $E\sin(\delta)$ is constant.



I_1 lags V: Generator produces reactive power.

I_2 leads V: Generator consumes reactive power.

Advantages: power factor correction is continuous, and can be adjusted at fast timescales. Reactive power can also be absorbed.

(c)

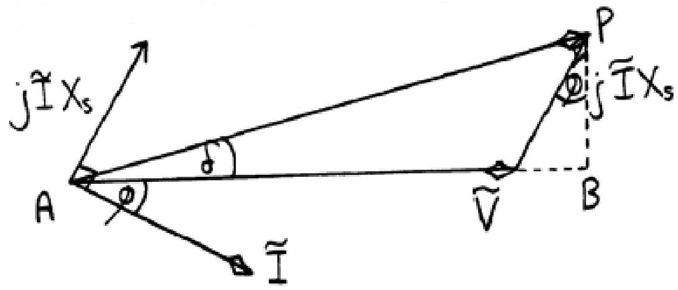
(i)

$$V_{ph} = \frac{V_l}{\sqrt{3}} = \frac{22k}{\sqrt{3}}$$

$P = 3I_{ph}V_{ph}\cos(\phi)$. So

$$I_{ph} = \frac{P}{3V_{ph}\cos(\phi)} = 11.7\ kA$$

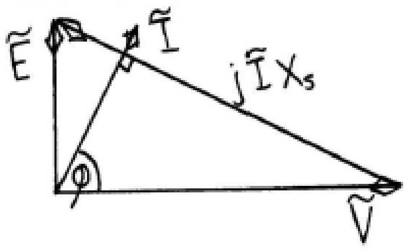
$$\phi = \cos^{-1}(0.9) = 25.8^o$$



$$E^2 = (IX_s\cos(\phi))^2 + (V + IX_s\sin(\phi))^2$$

So $E = 21.6\ kV$

(ii)



P_{max} occurs for $\delta = \pi/2$.

$$I_{ph} = \frac{\sqrt{E^2 + V^2}}{X_s} = 22.8\ kA$$

$$\phi = \frac{\pi}{2} - \tan^{-1}\frac{E}{V} = 2.21$$

$Q = 3I_{ph}V_{ph}\sin\phi = 440\ MVAR$

$P = 3I_{ph}V_{ph}\cos\phi = 749\ MW$

5.

(a) Increased connectivity provides:
- greater efficiency, can use the most efficient/clean generation for base load.
- greater reliability
- power stations can be located at best locations

Gas turbines: efficient, clean can be brought online faster
Coal: now used for peak load and is phased out.

(b) Types of faults:
- 3 phase symmetrical fault to earth
- line-line fault (two lines touch each other)
- line to ground fault

(c)
(i) Choose base values for VA and voltages

$$VA_b = 100\ MVA$$

$$V_b = 11\ kV, 132\ kV, 22\ kV$$

Perform changes of base for the reactances of the transformers:

11/132 kV transformer $X_{pu(100)} = X_{pu(150)} \times \frac{100}{150} = 0.133\ pu$

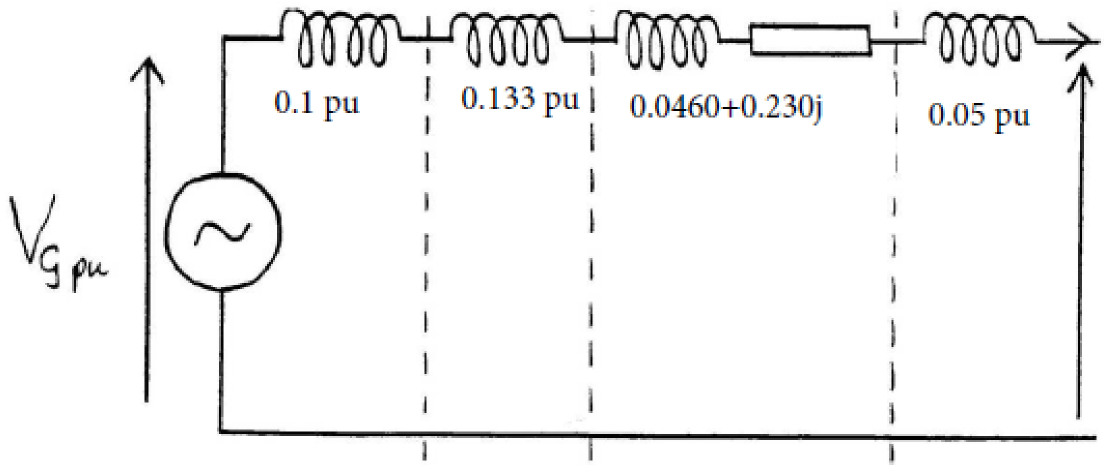
132/22 kV transformer $X_{pu(100)} = X_{pu(200)} \times \frac{100}{200} = 0.05\ pu$

Find the feeder impedance in pu

$$Z_b = \frac{V_b^2}{VA_b} = \frac{(132\ k)^2}{100\ M} = 174\ \Omega$$

$$Z_{pu} = \frac{Z}{Z_b} = \frac{8 + 40j}{174} = 0.0460 + 0.230j$$

The single phase line diagram in pu is therefore



$$S_{load} = \frac{P_{load}}{\cos(\phi)} = \frac{60M}{0.9}$$

$$S_{load,pu} = \frac{S_{load}}{VA_b} = \frac{60M}{0.9 \times 100M} = 0.667$$

$$S_{load,pu} = V_{pu} \times I_{pu} = I_{pu} = 0.667$$

$$P_{pu} = I_{pu}^2 R_{feeder,pu} = (0.667)^2 \times 0.0460 = 0.0204$$

$$P_{total} = P_{load} + P_{pu} \times VA_b = 60M + 0.0204 \times 100M = 62.0\ MW$$

$$Q_{pu} = I_{pu}^2 X_{total,pu} = (0.667)^2 \times (0.1 + 0.133 + 0.230 + 0.05) = 0.2280$$

$$Q_{load} = P_{load} \tan(\phi) = 29.1\ MVAR$$

$$Q_{total} = Q_{load} + Q_{pu} \times VA_b = 51.9\ MVAR$$

(ii)

$$I_{F,pu} = \frac{1}{|0.1j + 0.133j + 0.0460 + 0.230j + 0.05j|} = 1.94$$

$$I_b = \frac{VA_b}{\sqrt{3}\,V_b} = \frac{100M}{\sqrt{3}\,132k} = 437$$

$$I_F = I_{F,pu} \times I_b = 849\ A$$

Circuit breaker rating = $V_{pu} \times I_{F,pu} = I_{F,pu}$

Rating = $I_{F,pu} \times VA_b = 194\ MVA$

To decrease the fault current add additional reactance in series (increases also the need for reactive power).

6.

a)

From

$$\nabla \times E = -\dot{B} = -j\omega\mu_0 H$$

We have since $\nabla \times E$ has only one component in the x-direction of

$$-i.\frac{dE_y}{dz}$$

$$-j\omega\mu_0 H = -i.-j\beta E_0 e^{j(\omega t-\beta z)}$$

Hence

$H_x = -\frac{1}{\mu_0 c} E_0 e^{j(\omega t-\beta z)}$ (where c is the speed of the wave)

And

$$H_x = -\frac{E_0}{Z_0} e^{j(\omega t-\beta z)}$$

Mean power = $\frac{1}{2}\mathcal{R}e(E \times H^*) = \frac{E_0{}^2}{2Z_0}$

b) H is perpendicular to the loop hence:

V = - dphi/dt

$$V = -\frac{\pi a^2}{4} E_0 \frac{\omega\mu_0}{z_0} = -\frac{\pi a^2}{4} E_0 \omega\sqrt{\varepsilon_0\mu_0} = \frac{\pi^2 a^2}{2\lambda} E_0$$

(since c= 1/(mu*epsilon)^2 and v = f*lamda and lamda = 2*pi* omega)

c) voltage is inversely proportional to lamda. E0 is proportional to the square root of the power hence we have

$$\sqrt{\frac{P_{BBC}}{P_{C4}}} = \frac{\lambda_{BBC}}{\lambda_{C4}}$$

d) Iso Power density at a distance r

$P = \frac{P_{BBC}}{2\pi r^2}$ (since gain is 2)

Equate this to E_0

$$\frac{E_0^2}{Z_0} = \frac{E_0^2}{120\pi} = \frac{P_{BBC}}{2\pi r^2}$$

Antenna is power matched to the load hence:

$$\frac{1}{2}\frac{V_0^2}{2R} = 4.10^{-9}$$

Hence

$$V^2 = 4 * 75 * 4.10^{-9} = 1.2 * 10^{-6}$$

Using voltage from part b we get

$$V^2 = \left(\frac{\pi^2 a^2}{2\lambda} E_0\right)^2 = \left(\frac{\pi^2 a^2}{2\lambda}\right)^2 * \frac{60P}{r^2}$$

Lamda=0.54m; a=0.15m; P=1000W

Hence

R= (1/(1.2*10^-6))*3.141^2*(0.15)^2*(60000)^0.5/1.08=49.6 km

7.

$$V - L\delta x \frac{\partial I}{\partial t} = V + \frac{\partial V}{\partial x}\delta x$$

$$I - C\delta x \frac{\partial V}{\partial t} = I + \frac{\partial I}{\partial x}\delta x$$

Hence

$$-L\frac{\partial I}{\partial t} = \frac{\partial V}{\partial x}$$

$$C\frac{\partial V}{\partial t} = \frac{\partial I}{\partial x}$$

Differentiate both with respect to x

$$-L\frac{\partial}{\partial t}\left(\frac{\partial I}{\partial x}\right) = \frac{\partial^2 V}{\partial x^2}$$

$$-C\frac{\partial}{\partial t}\left(\frac{\partial V}{\partial x}\right) = \frac{\partial^2 I}{\partial x^2}$$

Hence subbing back in for dv/dx and dI/dx

$$LC\frac{\partial^2 V}{\partial t^2} = \frac{\partial^2 V}{\partial x^2}$$

$$LC\frac{\partial^2 I}{\partial t^2} = \frac{\partial^2 I}{\partial x^2}$$

b)

$$\frac{\partial^2 V}{\partial x^2} = f_+''(x - vt) + f_-''(x + vt)$$

$$\frac{\partial^2 V}{\partial t^2} = v^2 f_+''(x - vt) + v^2 f_-''(x + vt)$$

Hence:

$$f_+''(x - vt) + f_-''(x + vt) = LCv^2 f_+''(x - vt) + v^2 f_-''(x + vt)$$

Hence

$$v = {}^{1}\!/_{\sqrt{LC}}$$

Physical significance is that a transmission line supports both forward and backward travelling voltage waves which travel at a speed of $1/\sqrt{LC}$

c)

Using the two equations given and the databook expressions for Ib and If in terms of V0 and Z0 together with ohms law we get

$$Z(x) = \frac{\left(V_F e^{-j\beta x} + V_B e^{j\beta x}\right)}{\left(\frac{V_F}{Z_0} e^{-j\beta x} - \frac{V_B}{Z_0} e^{j\beta x}\right)}$$

Dividing through by VF and using the voltage reflection coefficient we get:

$$Z(x) = Z_0 \frac{\left(e^{-j\beta x} + \rho_L e^{j\beta x}\right)}{\left(e^{-j\beta x} - \rho_L e^{j\beta x}\right)}$$

Using Eulers theorem we then get the expression given

d)

Curve is a smooth curve. Example is given below. The significance is that there will be no reflections.