Part IB Paper 8 2018

Crib

		Assessor	Question numbers
A	Introductory Business Economics	Dr Antonio Rosato (ar915)	1, 2
B	Civil and Structural Engineering	Prof G Viggiani (gv278)	3, 4, 5
C	Mechanical Engineering	Dr M P.F. Sutcliffe (mpfs1)	6, 7, 8
D	Aerothermal Engineering	Prof R Miller (rjm76)	9, 10, 11
E	Electrical Engineering	Dr S Sambandan (ss698)	12, 13, 14
F	Information Engineering	Prof R Cipolla (rc10001)	15, 16, 17
G	Bioengineering	Prof M Lengyel (ml468)	18, 19, 20
H	Manufacturing, Management and Design	Prof T.H.W. Minshall (thwm100)	21, 22, 23

Part IB Paper 8 2018 SECTION A: ECONOMICS

1 (a) Suppose the market for beer is perfectly competitive. Supply is given by the following function: $Q^s=3+p$. Demand is given by the following function: $Q^d=9-p$. Compute the equilibrium price and quantity, as well as consumer surplus. [6]

(b) Define and explain the different types of price discrimination. [6]

(c) Describe and compare the Cournot and the Bertrand models of oligopolistic competition. [6]

(d) Illustrate the fundamental principles of Keynesian consumption theory. [7]

(a) *To find the equilibrium price, we need to equate Q^s with Q^d and solve for p, which yields p=3. Substituting p=3 into either the demand function or the supply one we obtain Q=6. Consumer surplus then is given by (9-3)*6/2=18.*

(b) The three types of price discrimination are:

- *First-degree (or perfect) price discrimination: the monopolist sells different units of output for different prices and these prices may differ from person to person; this enables the monopolist to extract as much surplus as possible from each consumer, for example through a bargaining process (this enables the monopolist to discover the reservation price of each consumer).*

- *Second-degree price discrimination: the monopolist sells different units of output for different prices, but each individual who buys the same amount of the good pays the same price (e.g. bulk discounts for large purchases);*

- *Third-degree price discrimination: the monopolist sells output to different consumers for different prices, but every unit of output sold to a given person sells for the same price (e.g. student discounts at the cinema, subscription prices to academic journals).*

The better students will be able to produce and discuss the diagrams for each type of price discrimination as treated in the lectures.

(c) A good answer will include the main assumptions behind the models (the number of firms producing homogenous or slightly differentiated goods, perfect

information, entry barriers, the strategic and simultaneous nature of decisions). The difference in the output- vs. pricesetting decision of Cournot vs. Betrand respectively is of particular importance. The students are expected to discuss how the Cournot equilibrium will lie between competitive and monopoly prices, while the outcome of Bertrand competition will be equivalent to a perfectlycompetitive outcome.

(d) *The students should start from components of aggregate demand in a closed economy (no trade) and therefore consider consumption (C), investment (I) and government demand (G). They should define disposable income as total income minus total taxes: Y – T and specify a simple consumption function in the form C = C (Y – T). They should then define the marginal propensity to consume (MPC) as the change in C due to changes in disposable income. The main conjectures of the theory are: 1) Income is the main determinant of consumption; 2) 0 < MPC < 1 and 3) the average propensity to consume (APC) will fall as income rises. The best students will provide accurate diagrammatic representations of the curves. The basic model predicts that the distribution of income will affect total consumption, that economies may suffer from 'underconsumption' as they grow; and that governments can remedy potential underconsumption by expanding demand through fiscal policy. The best students will comment that the Keynesian theory of consumption was found able to explain cross-sectional differences between households, but unable to explain longitudinal variations (the so-called 'consumption puzzle').*

2 (a) Consider a monopolistic supplier whose total production costs are described by the function $C(q)=4+q^2$, where q denotes the monopolist's quantity. Suppose that market demand for the monopolist's product is given by $q=12-(p/2)$, where p denotes price. Find the monopolist's profit-maximising price and quantity, and compute the *deadweight loss.* [6]

(b) Illustrate the Sweezy kinked-demand model of oligopoly. [6]

(c) Discuss the importance of the concept of Game Theory in making economic decisions by firms and explain the concept of the Prisoner's Dilemma. [6]

(d) Using either the Life Cycle Model or the Permanent Income Model, explain the potential impact of a temporary increase in income tax on the level of aggregate demand. [7]

(a) In order to maximize profts, the monopolist sets marginal revenue (MR) equal to marginal cost (MC): 24-4q=2q. Solving this equation yields $q^M = 4$ *which, plugged back into the inverse demand function, yields* $p^m = 16$*. In order to calculate the deadweight loss, we need to first compute the perfectly competitive solution. Setting 24-2q=2q yields* $q^{PC} = 6$ *and* $p^{PC} = 12$*. Finally, deadweight loss is equal to: (16-8) (6-4)/2=8.*

(b) The Sweezy oligopoly model applies to a market where:

- *A small number of firms sell similar but slightly differentiated products*
- *Firms believe that their competitors would follow them if they were to reduce their price below the prevailing market price, and that it is therefore not possible to gain much in the way of extra sales by reducing price (i.e. demand is relatively inelastic below the market price)*
- *Firms believe that their competitors would not follow them if they were to raise their price above the prevailing market price (i.e. demand is relatively elastic above the market price)*
- *Key result is price stability*

The oligopolist's belief that rivals will match price cuts but not price increases produces a kink in the demand curve.

(c) *Economists use game theory to analyse strategic situations. The key components of a game are: the set of players, the set of strategies available to each player and a complete specification of the payoffs of each player. We discussed the Prisoner's Dilemma in lecture.*

(d) *The Life Cycle and Permanent Income Models are based on the notion that consumption is determined by long-run or normal income. Thus, temporary income tax changes will have less impact on current consumption than permanent income tax changes. Thus the impact of the temporary tax increase will have little impact on the level of aggregate demand and hence output or inflation. Candidates may comment on the limitations of the models; for instance, the assumptions of perfect foresight and no liquidity constraints.*

Antonio Rosato July 2018

CIVIL

SECTION B

3(a) Need to find bending moment to resist. Position 30kN load at end of overhang, so the cantilever moment $M = 30 \times 2 + 30 \times 2 \times 1$
$= \underline{120\text{kNm}}$ [10%]

$f_{cd} = (0.85) \times \frac{30}{1.5} = 17 \text{ N/mm}^2$

$f_{yd} = 500/1.15 = 435 \text{ N/mm}^2$

Singly reinforced so $M = (10.2) \times 200 \times x \times (450 - \frac{x}{2})$

$\Rightarrow 120 \times 10^6 = 918 \times 10^3 x - 1020 x^2$

so $x = 159$ mm $< 0.5d$ ✓ [5%]

Then, $x = (1.67) \cdot \frac{435}{17} \left(\frac{A_s}{200}\right) = 159$

So $\underline{A_s = 744 \text{ mm}^2}$ [5%]

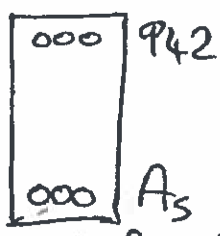
Choose three 20mm bars (942 mm^2) or anything sensible. [5%]

[Check: $x = 1.67 \left(\frac{435}{17}\right)\left(\frac{942}{200}\right) = 201$ mm

$\varepsilon_y = \left(\frac{450 - 201}{201}\right) \times 0.0035 = 0.43\%$ (yield ✓)

$M = 942 \times 435 \times (450 - \frac{201}{2}) = \underline{143\text{kNm}}$ ✓
$\underline{> 120\text{kNm}}$]

3(b) At cross section C, we now have:



[5%]

What moment needs to be resisted?

Place 30kN at midspan.

(c) $\gamma = \frac{G_s + e}{1+e}\gamma_w = \frac{2.70+1.20}{2.20} \times 10 = 17.73\ kN/m^3$

$\therefore K_{oNC} = 1 - \sin\varphi = 0.55$

$\therefore K_{oOC} = 1 - \sin\varphi \cdot OCR^{1/2}$

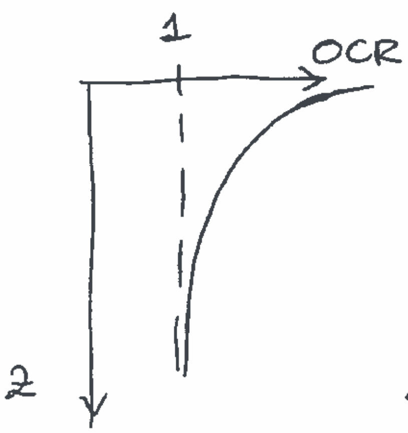
(i)

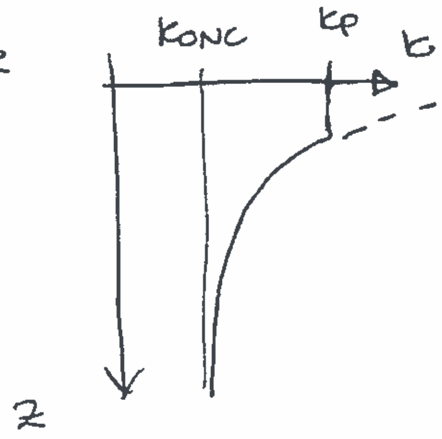
(m) z	(kPa) σ_v	(kPa) u	(kPa) σ'_v	(kPa) σ'_h	(kPa) σ_h
30	531.9	250	281.9	155.05	405.05
50	886.5	450	436.5	240.08	690.08

(ii)

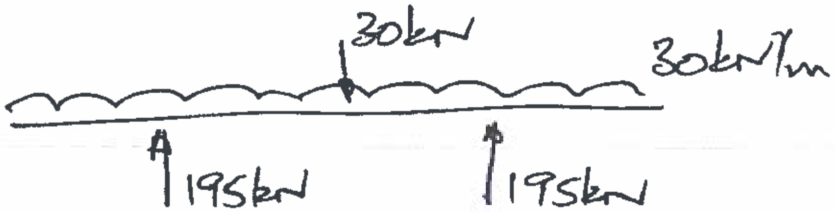
(m) z	(kPa) σ_v	(kPa) u	(kPa) σ'_v	(kPa) σ'_{vmax}	(-) OCR
5	88.65	50	38.65	281.9	7.29
25	443.25	250	193.25	436.5	2.26

(-) K_o	(kPa) σ'_h	(kPa) σ_h
1.47	56.82	106.82
0.82	158.47	408.47





ε



[10%]

$M_{max} = 195 \times 4 - 30 \times 6 \times 3 = \underline{240 kNm}$ sagging

This is double previous moment, so assume

$A_s = 6$ 20mm bars $= \underline{1884 mm^2}$

[There is a nice "spot" here, as $x \simeq 200mm$ in the first part such that if the top steel yields now, the total compression is double what it was in (a), which exactly matches new assumption above].

Choose $x = 200mm$

$C_c = 200 \times 200 \times (0.6) \times 17 = \underline{408kN}$

ε_y in top steel $= \frac{150}{200} \times (0.0035) = 0.2625\%$ $\underline{\text{yield}}$ ✓

$C_s = 435 \times 942 = \underline{410kN}$ [25%]

$T_s = 435 \times 1884 = \underline{820kN}$

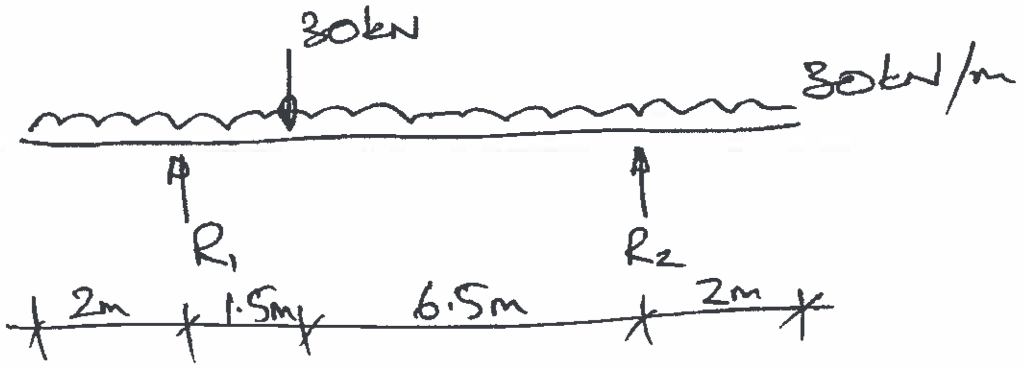
So, $T_s \simeq C_c + C_s$ as expected (or by trial and error!)

So $M = (0.6)(17)(200)(200)(450-100)$ ✓

$+ 942 \times 435 \times (450-50) = \underline{307 kNm} > 240 kNm$

3(c) To maximise shear force at B, position 30kN load there.

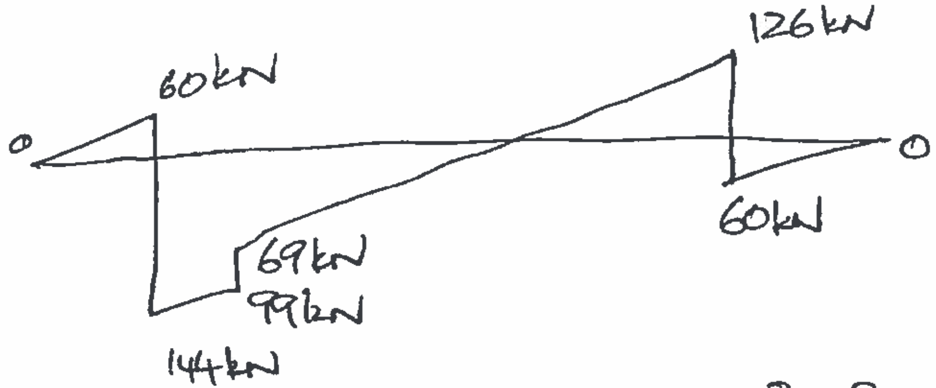
[5%]



$R_2 \times 8 = 30 \times 1.5 + 30 \times 12 \times 4$

$\Rightarrow R_2 = \underline{186\text{ kN}}$ and $R_1 = \underline{204\text{ kN}}$ [5%]

S:



[10%]

So $S = \underline{99\text{ kN}}$ at cross section B.

"k" → 2; Not 50. → 200

$$V_{RD,c} = \frac{0.18}{1.5} \times \left(2\left(\frac{100 \times 942}{200 \times 200} \times 30\right)^{\frac{1}{3}}\right) \times 200 \times 200$$

[5%]

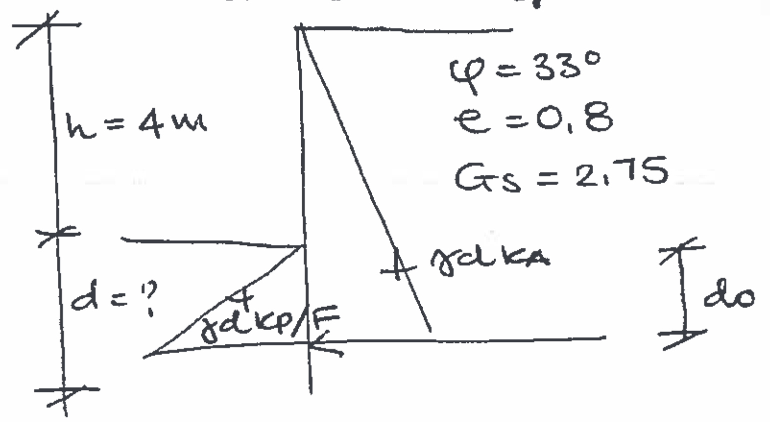
$= \underline{20\text{ kN}} << \underline{99\text{ kN}}$

Therefore shear reinforcement is certainly needed.

3(d) Shear reinforcement should be supplied as a flexible polymeric mesh, rather than in the form of individual stirrups, which would be impractical.

[10%]

QUESTION 4



(a) $\gamma_d = \frac{G_s}{1+e}\gamma_w = \frac{2{,}75 \times 10}{1.8} = 15{,}3 \ kN/m^3$

$\gamma_{sat} = \frac{G_s + e}{1+e} \cdot \gamma_w = \frac{3{,}55 \times 10}{1.8} = 19.7 \ kN/m^3$

(b) $k_A = \frac{1-\sin\varphi}{1+\sin\varphi} = \frac{1-\sin 33°}{1+\sin 33°} = 0{,}295$

$k_p = \frac{\cos\delta}{1-\sin\varphi}\left[\cos\delta + \sqrt{\sin^2\varphi - \sin^2\delta}\right] e^{2\theta \tan\varphi}$

$2\theta = \sin^{-1}\left(\frac{\sin\delta}{\sin\varphi}\right) + \delta = \sin^{-1}\left(\frac{\sin 11°}{\sin 33°}\right) + 11° = 31.51° = 0{,}55$

$k_p = \frac{\cos 11°}{1-\sin 33°}\left[\cos 11° + \sqrt{\sin^2 33° - \sin^2 11°}\right] e^{0{,}55 \tan 33°} =$

$= 2.156 \times 1.490 \times 1.429 = 4.596$

(c) $M_R = \frac{1}{6}\gamma_d k_A (h + d_o)^3$

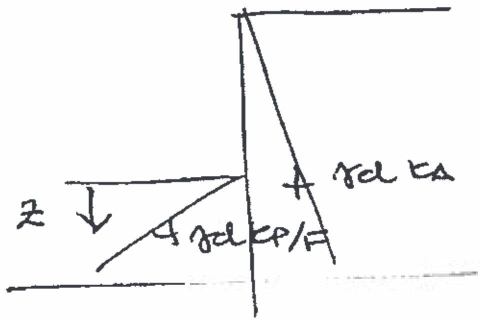
$\frac{M_S}{F} = \frac{1}{6}\gamma_d \frac{k_p}{F} d_o^3$

$\Delta M = M_R - \frac{M_S}{F} = \frac{1}{6}\gamma_d \left[k_A (h+d_o)^3 - \frac{k_p}{F}(d_o)^3\right]$

solve in d_o to find $\Delta M = 0$ (2)

d_o	ΔM
4	-56.2
3.5	21.5
3.6	8.3
3.7	-6.0
3.66	-0.1 ✓

$d = 1.2\, d_o \simeq 4.4$ m



(d) $T(z) = \frac{1}{2}\gamma_d K_A (h+z)^2 - \frac{1}{2}\gamma_d \frac{K_P}{F} z^2 =$

$= \frac{1}{2}\gamma_d \left[K_A h^2 + K_A z^2 + 2K_A h z - \frac{K_P}{F} z^2\right] =$

$= \frac{1}{2}\gamma_d \left[\left(K_A - \frac{K_P}{F}\right) z^2 + 2K_A h z + K_A h^2\right] = 0$

$\Delta = 4K_A^2 h^2 - 4\left(K_A - \frac{K_P}{F}\right) \cdot K_A h^2 =$

$4h^2 \left[K_A^2 - K_A\left(K_A - \frac{K_P}{F}\right)\right] =$

$4h^2 \left[K_A^2 - K_A^2 + K_A \frac{K_P}{F}\right] = 4h^2 \frac{K_A K_P}{F}$

$z_{12} = \dfrac{-2K_A h \mp 2h\sqrt{K_A \frac{K_P}{F}}}{2\left(K_A - \frac{K_P}{F}\right)} = \dfrac{-K_A h - h\sqrt{K_A \frac{K_P}{F}}}{K_A - \frac{K_P}{F}} =$

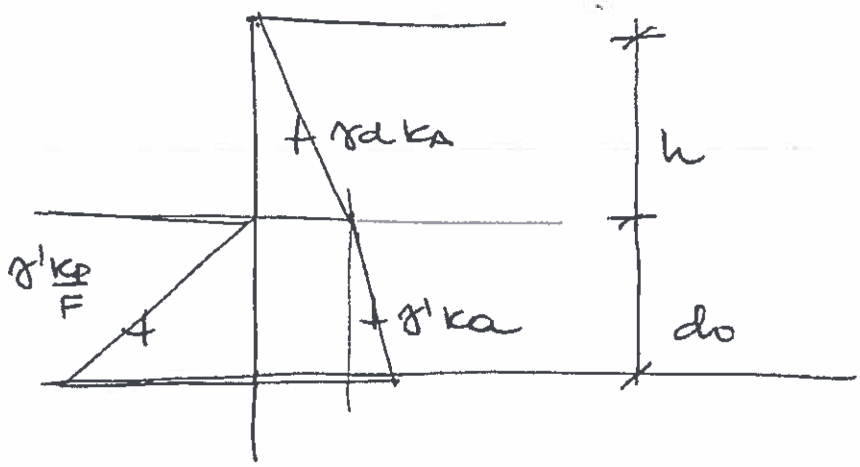
$= \dfrac{-0.295 \times 4 - 4 \times \sqrt{0.295 \cdot \frac{4.596}{1.7}}}{0.295 - \frac{4.596}{1.7}} = 1.973$ m

$$M_{max} = \frac{1}{6} \gamma d \, K_A (h+z)^3 - \frac{1}{6} \gamma d \frac{K_P}{F} z^3 =$$

$$\frac{1}{6} \gamma d \left[K_A (h+z)^3 - \frac{K_P}{F} z^3 \right] =$$

$$\frac{1}{6} \times 15.3 \left[0.295 \, (4 + 1.973)^3 - \frac{4.596}{1.7} \cdot 1.973^3 \right] =$$

$$\triangleq 107 \quad kNm/m$$



$\gamma' = \gamma_s - \gamma_w$

$$M_R = \frac{1}{2}\gamma_d k_a h^2\left(\frac{h}{3}+d_o\right) + \gamma_d k_a h \frac{d_o^2}{2} +$$

$$+ \frac{1}{2}\gamma' k_a \frac{d_o^3}{3}$$

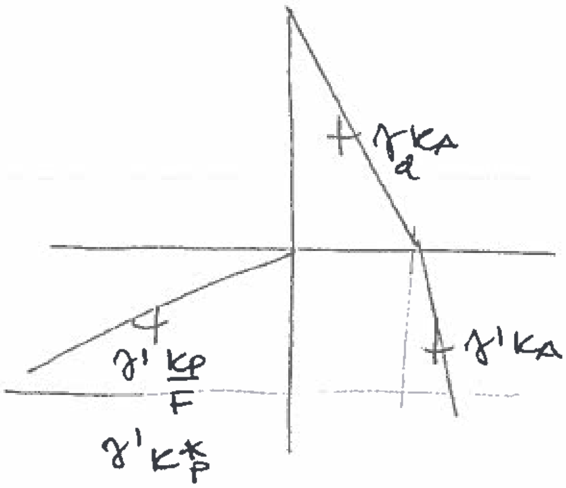
$$\frac{M_S}{F} = \frac{1}{2}\gamma' \frac{k_p}{F}\frac{d_o^3}{3}$$

$$\Delta M = M_R - \frac{M_S}{F}$$

$$\Delta M = \frac{1}{2}\gamma_d k_a\left[h^2\left(\frac{h}{3}+d_o\right) + h\, d_o^2\right] + \frac{1}{6}\gamma'\left(k_a - \frac{k_p}{F}\right)d_o^3$$

d_o	ΔM
4	86,50
5	-34,49
4,5	36,92
4,8	-3,12
4,78	-0,19 ✓

$d = 1{,}2\, d_o \cong 5{,}7$ m



$\gamma' = \gamma_{sat} - \gamma_w = 9.7 \text{ kN/m3}$

$K_p^* = \frac{K_p}{F} = \frac{4.596}{1.7} = 2{,}704$

$T(z) = \gamma K_A \left[\frac{h^2}{2} + h z \right] + \frac{1}{2} \gamma' (K_A - K_p^*) z^2$

find $\bar{z}$ | $T(\bar{z}) = 0$

$\bar{z}$	T
2	25,2
3	−15,3
2,5	7,9
2,7	−0,7
2,68	0 ✓

$M(\bar{z}) = \frac{1}{2} \gamma_d K_a \left[h^2 \left(\frac{h}{3} + \bar{z} \right) + h \bar{z}^2 \right] + \frac{1}{6} \gamma' (K_A - K_p^*) \bar{z}^3$

$= 0{,}5 \times 15{,}3 \times 0{,}295 \left[16 \left(\frac{4}{3} + 2{,}68 \right) + 4 \times 2{,}68^2 \right] +$

$+ \frac{1}{6} 9.7 \times (0{,}295 - 2{,}704) \cdot 2{,}68^3 =$

$= 2{,}257 \, [92{,}943] - 1{,}617 \times (2{,}409) \times 19.249 =$

$209{,}772 - 74{,}982 \cong 135 \text{ kNm/m}$

QUESTION 5

(a) $V_o = 500\,cc = 0.5 \times 10^{-3}\,m^3$

$W_o = 750\,g = 7.5\,kN$

$\Delta V_1 = 0.01\,V_o = 5 \times 10^{-6}\,m^3 \qquad V_1 = 0.495 \times 10^{-3}\,m^3$

$\Delta V_2 = 0.1\,V_o = 0.05 \times 10^{-3}\,m^3 \qquad V_2 = 0.450 \times 10^{-3}\,m^3$

$\gamma_d = \gamma = \frac{W}{V} \qquad e = \frac{G_s \gamma_w}{\gamma_d} - 1$

(i) $\gamma_{do} = \frac{7.5}{0.5 \times 10^{-3}} \frac{N}{m^3} = 15\,kN/m^3$

$e_o = 2.65 \times \frac{10}{15} - 1 = 0.77$

(ii) $\gamma_{d1} = \frac{7.5}{0.495 \times 10^{-3}} \frac{N}{m^3} = 15.15\,kN/m^3$

$e_1 = 2.65 \times \frac{10}{15.15} - 1 = 0.75$

$\gamma_{d2} = \frac{7.5}{0.45 \times 10^{-3}} \frac{N}{m^3} = 16.67\,kN/m^3$

$e_2 = 2.65 \times \frac{10}{16.67} - 1 = 0.59$

[10%]

(b) $S_r = w \frac{G_s}{e} = 0.25 \times \frac{2.75}{0.86} = 0.8$

$\gamma = \gamma_w \frac{(G_s + S_r e)}{1+e} = 10 \times \frac{2.75 + 0.8 \times 0.86}{1.86} =$

$= 18.48\,kN/m^3$

[10%]

(c) $\gamma = \frac{G_s + e}{1+e}\gamma_w = \frac{2.70 + 1.20}{2.20} \times 10 = 17.73\ kN/m^3$

$\therefore K_{0NC} = 1 - \sin\varphi = 0.55$

$\therefore K_{0OC} = 1 - \sin\varphi \cdot OCR^{1/2}$

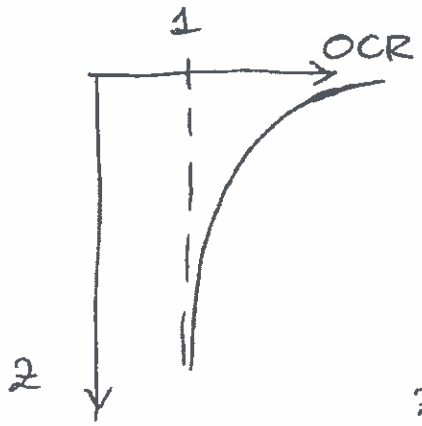
(i)

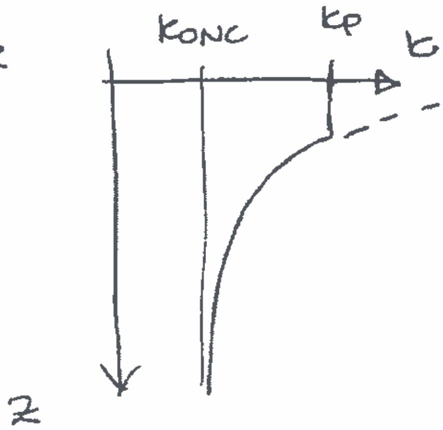
z (m)	σ_v (kPa)	u (kPa)	σ'_v (kPa)	σ'_h (kPa)	σ_h (kPa)
30	531.9	250	281.9	155.05	405.05
50	886.5	450	436.5	240.08	690.08

(ii)

z (m)	σ_v (kPa)	u (kPa)	σ'_v (kPa)	σ'_{vmax} (kPa)	OCR (–)
5	88.65	50	38.65	281.9	7.29
25	443.25	250	193.25	436.5	2.26

K_0 (–)	σ'_h (kPa)	σ_h (kPa)
1.47	56.82	106.82
0.82	158.47	408.47





(a) (i) the stability ratio is defined as:

$$N = \frac{\sigma_v - \sigma_T}{s_u}$$

where:

- σ_v is the total vertical stress at the tunnel axis

$$\sigma_v = \gamma z$$

- σ_T is the support pressure applied to the tunnel face
- s_u is the undrained shear strength of the clay

For open face tunnelling, $\sigma_T = 0$
If $N > 5$ the tunnel is unlikely to be stable and will require face support

(ii) For a tunnel at 15 m depth, the stability ratio would be:

$$N = \frac{20 \times 15}{80} = 3.75$$

This is sufficiently low to ensure that open face tunnelling (no support to the tunnel face) would be safe.
The tunnels might be excavated by open shield

For a tunnel at 25 m depth

$$N = \frac{20 \times 25}{80} = 6.25$$

This is quite high - if $N > 5$ it is

impossible to excavate the tunnel by open shield, because the tunnel face would not be stable. Mechanised tunnelling by closed face shields (either slurry shield or earth pressure balance shields) would be necessary to provide continuous support to the face during excavation

(iii) Tunnelling induced ground movements transmit to existing buildings as settlement rotation and distortions of their foundations. These can induce damage affecting visual apperance and aesthetics, serviceability or function, and, in extreme cases, stability. Masonry buildings are particularly affected by differential settlements that can cause tensile strain in the masonry leading to cracking. Compensation grouting is a mitigation measure where horizontal steel tubes are inserted in the ground at an intermediate level between the tunnel crown and the foundations of the building, and cement grout is injected from the tubes to compensate for settlements caused by tunnelling.

instrumentation and monitoring of ground and building response are crucial to inform decisions about where and when to grout to prevent / limit damaging differential building settlements

ENGINEERING TRIPOS PART IB

Paper 8 – Section B: Civil and Structural Engineering

Examiners Report

Question 3 was answered **poorly**, on the whole. There were two key reasons for this. Firstly, too many students could not determine bending moments and shear forces at key locations after assuming key positions for the concentrated load in a statically determinate structure. Secondly, too many students tried to solve equations in trying to find areas of steel, and got into a horrible mess with wrong units, when minor iteration based on an initial sensible estimate would have served much better.

Specifically:

3(a). Answered fairly well overall, although many students thought it necessary to draw the entire BM diagram, thereby wasting precious time.

3(b). Answered poorly on the whole, when in fact all that was required was to assume the top steel from 3(a), and to design the bottom steel by initially assuming twice this amount (the moment in sagging is double), as covered in lectures, as a really good first approximation, followed by minor iteration.

3(c). Answered really poorly, very surprisingly. Most students did not place the 30kN load in the appropriate position, and most students did not plug the right values into the formula, which was most surprising.

3(d). Answered really poorly. Most students chugged out the 'put in stirrups' comment, when this is a fabric-formed beam."

Question 4 was answered **really poorly** on the whole. The main reason for this is that the majority of students failed to recognise this was a cantilevered wall, not a propped wall and hence moments should have been taken around pivot point close to the bottom of the wall, and not about its top. In many cases, force equilibrium rather than moment equilibrium was considered, with the side effect that the wall was not in equilibrium and hence it was not possible to work out shear forces nor bending moments. Many students tried to solve equations and got lost in calculations, whereas a few iterations based on an initial sensible estimate would have served much better.

Specifically:

4(a) was generally answered well

4(b) a significant proportion of the students failed to understand that the value of θ in the equation for K_P should be in radians rather than degrees, thus getting impossibly large values of K_P (very few of those who made this mistake commented that there must be something wrong with their calculations)

4(c) and 4(d) the earth pressures were generally computed OK, but then moment equilibrium about the top of the wall was considered or force equilibrium. Many students lost in calculations (see above). Nobody managed to compute maximum bending moment in wall, only very few sketched sensible diagrams for bending moment.

Question 5 was answered **well** on average, particularly where the more discursive question on tunnel stability, whereas the question on the in situ state of stress was answered quite poorly.

Specifically:

5(a) not a popular question: many students did not even attempt to solve it. Some confusion seem to have arisen from the units of volume being litres.

5(b) answered correctly by most students

5(c) very poorly answered. None of the students realised the horizontal effective stress should have been computed using the coefficient of earth pressure at rest. Some of them used either the active or the passive coefficient of earth pressure. Also none of the students realised that after erosion OCR and K0 change with depth.

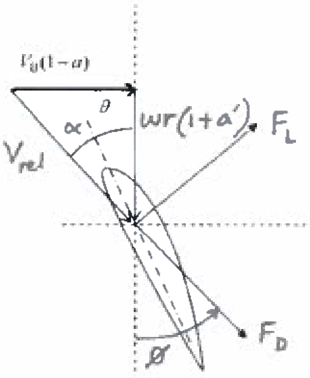
5(d) typically answered very well

18/06/2018 Giulia VIGGIANI G. Viggiani

Paper 2P8, Section C, Crib 2018 (J Cullen, T Flack, H Hunt, M Sutliffe)

Question 6

The diagram below is from the notes, and refers to a) i) and ii).



a) i) The sketch should show clearly:

- **the relative wind speed V_{rel} with a direction of $\phi = \alpha + \theta$, where α is the angle of attach and θ is the local twist angle.**
- **the lift force F_L and the drag force F_D aligned with the relative wind speed V_{rel}.**

ii) The axial and tangential velocities should be shown as vectors, corrected for the slow down of air through the turbine using the the axial induction factor a and the angular induction factor a' . From the diagram:

$$V_{rel} \sin\phi = V_0(1-a) \quad \textbf{and} \quad V_{rel}\cos\phi = \omega r(1-a')$$

Therefore:

$$\tan\phi = \frac{(1-a)V_0}{(1+a')\omega r}$$

iii) Resolving the lift and drag forces, normal and tangential to the rotor plane:

$$F_N = F_L\cos\phi + F_D\sin\phi \quad \textbf{and} \quad F_T = F_L\sin\phi + F_D\cos\phi$$

Normalising these forces gives:

$$C_N = \frac{F_N}{\frac{1}{2}\rho V_{rel}^2 c} \quad \textbf{and} \quad C_T = \frac{F_T}{\frac{1}{2}\rho V_{rel}^2 c}$$

Hence:

$$C_N = C_L\cos\phi + C_D\sin\phi \quad \textbf{and} \quad C_T = C_L\sin\phi - C_D\cos\phi$$

ii) The flapwise bending moments is calculated by taking moments in the flapwise direction for each section of the blade, remembering that the moment is displaced because of the blade root.

$$M_N = \int F_N(r - r_{min})dr$$

where F_N is the normal force, r is the section midpoint, r_{min} is the blade root.

$$M_N \text{ [5 to 30 m]} = 465 \times (17.5 - 5) \times 25 = 145 \text{ kNm}$$
$$M_N \text{ [30 to 55 m]} = 1946 \times (42.5 - 5) \times 25 = 1,825 \text{ kNm}$$
$$M_N \text{ [total]} = 1,970 \text{ kNm}$$

The axial thrust on the turbine is found using $B \sum F_N l$, where B is the number of blades, F_N is the normal force and l is the length of the section.

$$F_A = 3 \times \left[(465 \times 25) + (1946 \times 25)\right] = 181 \text{ kN}$$

iii) The tangential (edgewise) bending moment is calculated by taking moments in the tangential direction for each section of the blade, remembering that the moment is displaced because of the blade root.

$$M_T = \int F_T(r - r_{min})dr$$

where F_T is the tangential force, r the section midpoint, r_{min} the blade root,

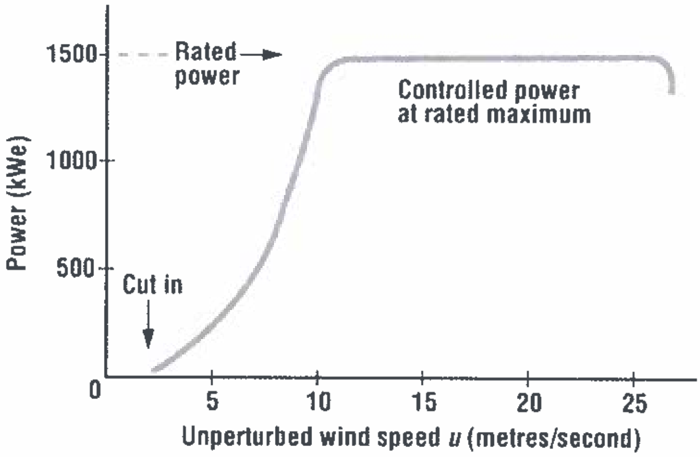
$$M_T \text{ [5 to 30 m]} = 161 \times (17.5 - 5) \times 25 = 50 \text{ kNm}$$
$$M_T \text{ [30 to 55 m]} = 124 \times (42.5 - 5) \times 25 = 116 \text{ kNm}$$
$$M_T \text{ [total]} = 167 \text{ kNm}$$

The torque at the turbine hub (i.e. at $r = 0$m) is found by calculating the the torque for each section (length 25 m and at the midpoint) acting on the hub, summing these, and multiplying by the number of blades.

$$T = 3 \times \left[(161 \times 17.5 \times 25) + (124 \times 42.5 \times 25)\right] = 607 \text{ kNm}$$

Examiner's comment:
This question was answered well by many students. Most students had little trouble with part a) and were rewarded for a clearly labelled diagram. Several students failed to spot in part b)i) that starting with the given values for a and a' meant only one iteration cycle was required. Students found calculating the bending moments, thrust and torque in part b) ii) and iii) more challenging.

7 (a) The 'cut-in' speed of a wind turbine is the wind speed above which it becomes worthwhile producing power. 'Rated' wind speed is the wind speed at which the turbine-generator produces its rated output power. 'Stall' wind speed is the wind speed above which it becomes unsafe to continue to operate the wind turbine, and so it is stalled. A typical power vs wind speed characteristic is sketched below.



(b) From $P = \frac{1}{2}C_p \rho A v^3$ and substituting in $C_p = 0.35$, $\rho = 1.23$ kgm^{-3}, $A = \pi R^2$, v = 12 ms^{-1} and P = 1.8 MW gives R = 39.2 m so d = 78.5 m.

(c) (i) For rated wind speed of 12 ms^{-1}

Wind speed, v (ms^{-1})	Number of days per year	Hours	Power (MW)	Energy (MWhr)
<2	10	240	0	0
7	180	4320	0.357	1544
12	105	2520	1.8	4536
16	60	1440	1.8	2592
>21	10	240	0	0

Total energy is 8672 MWhr, income is 30×8672 = £260,160. Rated power would be 1.8 MW and so capacity factor is 8672/(24×365×1.8) = 0.550

For rated wind speed of 16 ms^{-1}

Wind speed, v (ms^{-1})	Number of days per year	Hours	Power (MW)	Energy (MWhr)
< 2	10	240	0	0
7	180	4320	0.357	1544
12	105	2520	1.8	4536
16	60	1440	4.3	6144
>21	10	240	0	0

Total energy is 12224 MWhr, income is 30×12224 = £366,720. Rated power would be 4.267 MW and so capacity factor is 12224/(24×365×4.267) = 0.327

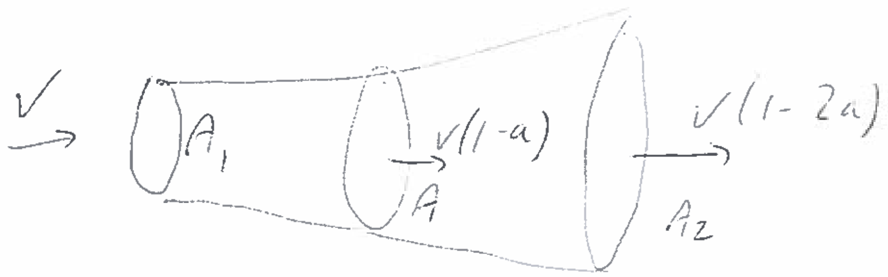
(ii) Capital cost for 1.8 MW turbine is £1.8M, annual income of £260,160 means 6.9 year payback period.

Capital cost for 4.267 MW turbine is £4.267M, annual income of £366,720 means 11.6 year payback period.

(iii) Better to choose £1.8M turbine, it will operate at a higher capacity factor giving a shorter payback period.

Examiner's comments: (a) Most students sketched the power vs wind speed characteristic correctly, but many overlooked explaining the terms cut-in, rated and stall wind speeds. Part (b) was straightforward bookwork, a few students mixed up radius and diameter but otherwise well answered. Part (c) caused more problems, but many completely correct answers were received. The main issue was failing to attempt this part methodically, causing errors in the calculations.

7(d)



(i) Continuity $VA_1 = VA(1-a) = VA_2(1-2a)$

$$\Rightarrow A_1 = A(1-a)$$

$$A_2 = \frac{A(1-a)}{1-2a}$$

(ii) $\Delta KE = \frac{1}{2}\rho V^3\left((1-2a)^3 A_2 - A_1\right) =$ Power out

$$\therefore P = \frac{1}{2}\rho V^3 A\left((1-2a)^3 \frac{1-a}{1-2a} - (1-a)\right)$$

$$= \frac{1}{2}\rho V^3 A(1-a)\left((1-2a)^2 - 1\right)$$

$$= \frac{1}{2}\rho V^3 A(1-a)(4a^2-4a)$$

$$= 2\rho V^3 A(1-a)^2 a$$

With $a = 0.4$ $P = 0.288\rho V^3 A$

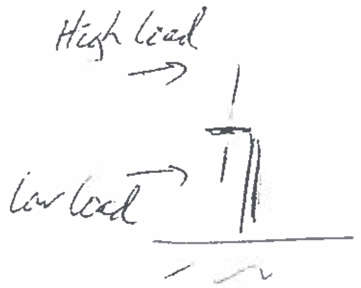
(iii) For η evaluate $\dfrac{P}{\frac{1}{2}\rho V^3 A} = 57.6\%$

Betz limit with $a = \frac{1}{3}$ is the maximum possible value of η.

Examiner's comment: Generally well done

8(a) - Can measure wind speed or loading and use rainflow analyses to derive a spectrum.
- standard spectra used in design calcs
- adjust for different sites
- spectra eg on towers could exhibit resonant behaviour

(b) -> blade rotation -> 3P for tower interaction
1P for self weight
1P for shear flow loading



-> random wind loading due to fluctuations day to day and turbulence
-> load and pitch out of balance

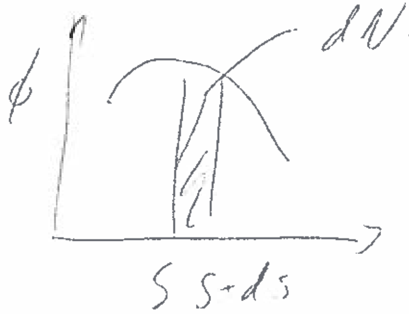
(c) Fatigue - of bolts, joints, blades. Use fatigue analyses and testing

Impact - eg bird strike, lightening
- testing

Delamination - good manufacturing QA

Excessive tip deflection - calculations knowing extreme loads

8(d)



$dN = \phi\, dS \times n$

n = total number of cycles

Assume Miner's rule $\sum \frac{N_i}{N_{fi}} = 1$ with $N_f = \left(\frac{S}{S_0}\right)^{-m}$

Neglect effect of mean stress on failure

$$\frac{1}{n} = \int_0^\infty \frac{\phi(s)\, dS}{N_f(s)} = \frac{1}{\bar{s}} \int_0^\infty \frac{\exp\left(-\frac{S}{\bar{S}}\right)}{S^{-m} S_0^{\,m}} dS$$

Put $t = S/\bar{S} \Rightarrow dt = dS/\bar{S}$

$$\frac{1}{n} = \int_0^\infty \frac{e^{-t}}{S_0^{\,m} \bar{S}} (S)^m \bar{S}\, dt = \int_0^\infty e^{-t} \left(\frac{S}{\bar{S}}\right)^m \left(\frac{\bar{S}}{S_0}\right)^m dt$$

$$\frac{1}{n} = \int_0^\infty e^{-t} t^m dt \left(\frac{\bar{S}}{S_0}\right)^m = m! \left(\frac{\bar{S}}{S_0}\right)^m$$

$$\bar{S} = \left(\frac{1}{n\, m!}\right)^{1/m} \times S_0 = 4.9 \text{ MPa}$$

(e) Uncertainty in high stress spectrum will be critical, Also errors in calculating any stress concentrations.

- Material properties (eg m) ~~may~~ may depend on construction details.

Examiner's comment: (d) was done well. Marks were lost in the descriptive parts for only identifying one or two factors.

SECTION D

Qu 9

(a) $\psi = \frac{\Delta h_o}{U^2}$ Δh_o IS THE CHANGE IN STAGNATION ENTHALPY

U IS THE BLADE SPEED

FOR A COMPRESSOR ~ 0·4
FOR A TURBINE ~ 2·0

THE REASON IS THE ADVERSE PRESSURE GRADIENT FROM INLET TO EXIT ~~...~~ IN A COMPRESSOR. THE BOUNDARY WILL SEPARATE.

(b) $\frac{T_{02s}}{T_{01}} = \left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}$ $T_{02} = 288 \times 15^{\frac{0.4}{1.4}}$
$= 624.3\,K$

$\eta_c = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}} = 0.88$

$T_{02} = T_{01} + \frac{(T_{02s} - T_{01})}{0.88} = 670.2\,K$

$\psi = \frac{C_p \Delta T_0}{U^2} = 0.4$

$\Delta T_{0_{STAGE}} = \frac{670.2 - 288}{12} = 31.9\,K$

$U^2 = \frac{31.9 \times 1005}{0.4}$ $U = 283.1\ ms^{-1}$

$U = \Omega r$ $\Omega = \frac{283.1}{0.55} = \underline{\underline{514.7}}\ RAD\,s^{-1}$

$\Omega = 514.7 \times \frac{60}{\pi} = \underline{\underline{4915\ RPM}}$

Qu 9

(c) $C_p(T_{01} - T_{02}) = C_p(T_{03} - T_{04})$

COMPRESSOR TURBINE

$$T_{04} = 1600 - (670.2 - 288)$$

$$= \underline{\underline{1218 K}}$$

$$\left(\frac{P_{03}}{P_{04}}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_{03}}{T_{04s}}$$

$$\frac{T_{03} - T_{04}}{T_{03} - T_{04s}} = 0.88$$

$$T_{04s} = T_{03} - \frac{(T_{03} - T_{04})}{0.88}$$

$$T_{04s} = 1600 - \frac{(1600 - 1218)}{0.88}$$

$$T_{04s} = \underline{\underline{1166 K}}$$

$$P_{04} = P_{03}\left(\frac{T_{04s}}{T_{03}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$P_{04} = 15\left(\frac{1166}{1600}\right)^{\frac{1.4}{0.4}}$$

$$P_{04} = \underline{\underline{4.96 bar}}$$

$$\frac{V^2}{2} = C_p(T_{04} - T_9) = C_p(T_{04})\left(1 - \left(\frac{1}{4.96}\right)^{\frac{\gamma-1}{\gamma}}\right)$$

$$= \underline{\underline{4.49 \times 10^5}}$$

$$V = \underline{\underline{948 ms^{-1}}}$$

(d) $\psi_{\text{STAGE TURBINE}} = 2.0$

$$\psi = \frac{C_p \Delta T_{0\,\text{STAGE}}}{U^2}$$

$$\Delta T_{0\,\text{STAGE}} = \frac{2.0 \times 283.1^2}{1005}$$

$$= \underline{\underline{159.5\text{K}}}$$

$$\Delta T_{0\,\text{TURB}} = T_{03} - T_{04} = 1600 - 1218$$

$$= \underline{\underline{382\text{K}}}$$

$$N_{\text{STAGE}} = \frac{382}{159.5} = \underline{\underline{2.39}}$$

THE DESIGNER WOULD PROBABLY CHOOSE 3 STAGES. IF THE STAGE WAS WELL DESIGNED THEY MIGHT GET AWAY WITH 2 STAGES.

THE DESIGNER SHOULD CONSIDER WEIGHT, MANUFACTURE ACCURACY, ACCURACY OF BOUNDARY LAYER PREDICTION, COOLING IN THEIR CHOICE.

Qu 10

(a) $\eta_p = \dfrac{\text{POWER OF AIRCRAFT}}{\text{POWER OF JET}}$

$$P_t = V \times F_N = V\left((\dot{m}_A + \dot{m}_f)V_j - \dot{m}_a V\right)$$

$$P_j = \frac{1}{2}\left((\dot{m}_A + \dot{m}_F)V_j^2 - \dot{m}_A V^2\right)$$

$$\eta_p = \frac{V\left[(\dot{m}_a + \dot{m}_f)V_j - \dot{m}_a V\right]}{\frac{1}{2}\left[(\dot{m}_a + \dot{m}_f)V_j^2 - \dot{m}_a V^2\right]}$$

ASSUME $\dot{m}_f << \dot{m}_A$

$$\eta_p = \frac{\dot{m}_A}{\dot{m}_A} \frac{V(V_j - V)}{\frac{1}{2}(V_j - V)} = \frac{2V}{V + V_j}$$

CIVIL AIRCRAFT HAVE CONTINUOUSLY RAISED η_p

BY $V \Rightarrow V_j$ $\eta_p \Rightarrow 1$

THIS MEANS HIGH BYPASS RATIOS AND LOW FAN PRESSURE RATIOS

TYPICAL BPR $\approx 6-9$

AS BYPASS RATIO RISES WEIGHT + DRAG OF NACELLE RISES

Q.10

(b) $P_{o4} = 5\,bar$

$T_{o4} = 1500K$

$$T_{o5} = T_{o4} - \frac{250}{1.005} = \underline{\underline{1251.2K}}$$

$$R_T = \frac{T_{o4} - T_{o5}}{T_{o4} - T_{o5s}} = 0.9$$

$$T_{o5s} = \underline{\underline{1223.6K}}$$

$$\left(\frac{P_{o4}}{P_{o5}}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_{o4}}{T_{o5}}$$

$$\frac{P_{o4}}{P_{o5}} = \left(\frac{1500}{1223.6}\right)^{\frac{\gamma}{\gamma-1}} = \underline{\underline{2.04}}$$

$$P_{o5} = \frac{5}{2.04} = \underline{\underline{2.45\,bar}}$$

$$\frac{T_{o5}}{T_j} = \left(\frac{P_{o5}}{P_A}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{2.45}{0.26}\right)^{\frac{\gamma-1}{\gamma}} = 1.90$$

$$V_j^2 = C_p\left(T_{o5} - T_j\right) = 2 \times 1.005 \times 1251.2 \times \left(1 - \frac{1}{1.9}\right)$$

$$V_j = \underline{\underline{1091.4\,ms^{-1}}}$$

Qu 10

(c) $V = M \times \sqrt{\gamma R T} = 0.89 \times \sqrt{1.4 \times 287 \times 222}$

$= \underline{265.8\ ms^{-1}}$

$\frac{F_N}{\dot{m}} = V_j - V = 1091.4 - 265.8$

$= \underline{825.6}$

$R_P = \frac{2 \times 265.8}{265.8 + 1091.4} = \underline{0.392}$

(d) MACH NUMBER SAME SO

$$\left(\frac{\dot{m}\sqrt{C_p T_{02}}}{A_2 P_{02}}\right)_{NEW} = \left(\frac{\dot{m}\sqrt{C_p T_{02}}}{A_2 P_{02}}\right)_{OLD}$$

T_{02} P_{02} UNCHANGED

$\therefore \dot{m}_{NEW} = \dot{m}_{OLD} \frac{A_{NEW}}{A_{2OLD}} = \underline{3\dot{m}_{OLD}}$

$\frac{F_{NEW}}{\dot{m}_{NEW}} = \frac{825.6}{3} = (V_j - V)$

$V_j = 265.8 + \frac{825.6}{3}$

$= \underline{541}$

$R_P = \frac{2 \times 265.8}{265.8 \times 541} = \underline{0.659}$

Qu 11

(a)

$$S = \frac{V}{g}\frac{L/D}{SFC}\ln\frac{W_{START}}{W_{END}}$$

$$W_{START} = W_e + W_p + W_f$$

(W_e ↑ EMPTY, W_p ↑ PAYLOAD, W_f ↑ FUEL)

$$\frac{W_e + W_p + W_f}{W_e + W_p} = e^{\left(\frac{S}{V}\frac{g}{L/D}SFC\right)}$$

$$\frac{W_f}{W_e + W_p} + 1 = e^{\left(\frac{S}{V}\frac{g}{L/D}SFC\right)}$$

$$W_f = (W_e + W_p)\left(e^{\left(S/V \frac{g}{L/D} SFC\right)} - 1\right)$$

$$\frac{W_f}{SW_p} = \frac{1}{S}\left(1 + \frac{W_e}{W_p}\right)\left(e^{\left(S/V \frac{g}{L/D} SFC\right)} - 1\right)$$

↑ MEASURE OF TRANSPORT EFFECTIVENESS

TO MINIMISE →
- MINIMISE $\frac{W_e}{W_p}$
- MINIMISE $\frac{L}{D} \times V$
- MINIMISE SFC

Qu 11

(b) $W_e = 360$ TONS

$W_p = 70$ TONS

$S = 15000 \times 10^3$ m

$\frac{L}{D} = 20 \qquad V = 260 ms^{-1}$

$SFC = 0.016 \times 10^{-3}$

$$\frac{W_f}{SW_p} = \frac{1}{15000}\left(1 + \frac{360}{70}\right)\left[e^{\left(\frac{15000 \times 10^3 \times 9.81 \times 0.016 \times 10^{-3}}{260 \times 20}\right)}\right]$$

$$= 2.35 \times 10^{-4}$$

(c) $$EI_{CO_2} = \frac{\text{MASS } CO_2}{\text{MASS FUEL BURNT}}$$

$$C_8H_{18} + 8.5O_2 \longrightarrow 8CO_2 + 9H_2O$$

THUS $EI_{CO_2} = \frac{8 \times 44}{8 \times 12 + 18} = 3.1$ kg CO_2/kg FUEL

(d) $$\frac{m_{CO_2}}{SN_{PASS}} = \frac{m_f}{Sm_p} \times EI(CO_2) \times m_{PASS}$$

$$= 2.35 \times 10^{-4} \times 3.1 \times 100$$

$= 0.0728$ kg PER PASSENGER per km

$= 73$g OF CO_2 PER PASSENGER per km

THIS IS SIMILAR TO A CAR WITH TWO PASSENGERS. BUSES CAN BE AS LOW AS 50g PER PASSENGER km. TRAINS EVEN LOWER.

Qu 11

(e)

$$\frac{W_f}{\Sigma W_P} = \frac{1}{6000} \times \left(1 + \frac{360}{70}\right)\left[e^{\left(\frac{6000\times10^3 \times 9.81 \times 0.016\times10^{-3}}{260\times20}\right)}\right]$$

$$= 2.033\times10^{-4}$$

NB EMPTY WEIGHT AND PAYLOAD WEIGHT ARE THE SAME

$$\frac{M_{CO2}}{\Sigma N_{PASS}} = \frac{M_f}{\Sigma M_P} * EI(CO_2) \times M_{PASS}$$

$$= 2.033\times10^{-4} \times 3.1 \times 100$$

$$= 0.063$$

$= 63g$ CO_2 PER PASS PER KM

REASON FOR REDUCTION IS THAT AIRCRAFT IS NOT BEING USED AS A TANKER FOR LATER PART OF JOURNEY.

MULTIPLE SHORT HOPS BETTER FOR CO_2 EMISSIONS.

Examiners report – Paper 8: Section D - Aerothermal Engineering

The average mark for the section D was 62.7%. Attempts were evenly split between the first two questions. The third question had around a third less attempts. The questions seem to accurately examine the students understanding.

Question 9 (64 attempts)

The question involved designing a compressor and turbine for an aeroengine. The average mark for the question was 62%. Part (a) was answered reasonably well by most candidates. However, many lost marks for not defining the terms in the work coefficient. Part (b) and (c) were answered correctly by most candidates. Part (d) was only answered correctly by around 30% of candidates. Many candidates did not manage to calculate the correct turbine stage number or manage to explain all the practicalities which a designer must consider. Overall this was a well-balanced question with the correct average mark.

Question 10 (64 attempts)

The question involved replacing the engine on an aircraft to improve its propulsive efficiency. The average mark for the question was 64%. The proof of propulsive efficiency in part (a) of the question was answered correctly by most candidates. The discussion part of the question was only well answered, with around 50% of the students not giving a full answer. Part (b) and (c) were well answered by most students. Part (d) was not well answered. Very few students understood the reason for the change in mass flow. Overall this was a well-balanced question with the correct average mark.

Question 11 (43 attempts)

The question involved calculating how the CO_2 produced per passenger per kilometer varies for different lengths of flight and different modes of transport. The average mark for the question was 63%. The question was slight unusual in that students had to calculate the emission index EI. This is on the syllabus and in the lectures but had not been examined in recent years. For those who understood the subject the question could be answered relatively quickly. The answers were split into two groups. Around 30% had trouble with the question and around 70% seemed to find it straight forward. Most students produced good answers for part (a), (b) and (c). Part (d) was answered correctly by most students but the discuss was less well answered. Part (e) was only answered in full by around 30% of the students.

Rob Miller 15th June 2018

IB Paper 8 - Section E

Crib

12.)

a.) (i) Looking for the following keywords or comments:

- Drift velocity is proportional to electric field at low electric fields. Proportionality constant is mobility
- No acceleration forever in a lattice due to scattering with lattice, carriers, defects etc.
- Therefore the microscopic picture: Relation between mobility, mean collision time, effective mass (and a comment on effective mass)

Carries 4 marks

Most students: Have only defined with the first of these points listed .I have given 2 marks in those cases.

(ii) Direct application of formula. Key point – use mobility in m^2/Vs. Ans: 1.8e-13 s

Carries 1 mark

Most students got this right.

b.) Straightforward use of equations. Answers:

(i) 416 kOhm-cm

(ii) 1e5 /cc

(iii) 6.25 Ohm-cm

Most students got this right. Some got the dimensions wrong. Some got the order wrong.

c.) Straightforward use of equations. Answer: 2.37e-20 /cc.

If they have the formula right and answer (order of the answer) correct: full marks

If they have the formula right but wrong answer (mostly it is due to the wrong usage of formulas): 3 marks (there is nothing but careful calculations needed here)

If they got the formula wrong (and therefore answer): 0 marks

d.) (i) Straightforward writing of Schr. Equation (1 Mark)

(ii) Here I am looking for the following keywords or comments. (4 marks)

- Finite potential energy of barrier with Total energy < potential energy
- Non zero probability of finding the wave-particle in the barrier or the other side of the barrier.
- Exponentially decaying component to the wave function. (If they have shown why by writing Schr. Equation –excellent, but no penalty if not) or A diagram
- Observed in Quantum mechanics. Classical mechanics cannot explain it.

13.)

a.) (i) We are looking for:

- pixel circuit diagram correctly and clearly indicating the two thin film transistors (TFTs) and the OLED.

- The comments that the pixel select transistor operates as a switch and is in linear operation

- The LED driver transistor operates as a current source and is in saturation

- Finally, the detailed operation of how data is written on the pixel capacitor and used to drive the LED, i.e. the programming mode and running mode. A waveform of all signals.

Most students have completely skipped the last point which was the essence of the question.

Total 7 marks

(ii) – Identifying that the TFT is in saturation.

- Straightforward use of formula. And 0.18 micro-Amps. (2 marks).

b.) (i) We are looking for comments on:

- MOSFET: ~Presence of gate insulator ~Gate insulator provides high gate impedance~Some details on operation and speed.

-MESFET: ~Presence of a Schottky metal-semiconductor contact ~The depletion region provides high gate impedance~Some details on operation and speed.

(ii) We are looking for comments on:

- Depletion mode: Diagram/comments on the architecture having a doped channel with same doping as contacts ~comments on the operation resulting in the doped channel being depleted by the application of a gate voltage.
- Enhancement mode: Diagram/comments on the architecture with discussion on doping of body and contacts~ comments on the operation resulting in semiconductor insulator interface reaching flatband to depletion to inversion with current due to minority carriers.

All students have fallen short – but the marking has been generous.

c.) Straightforward solution of Poissons' equation.

- Written the equation right

- solved correctly

- applied right boundary condition of field=0, when x=W

-Arrived at correct answer E=(qN/epsilon)*(x-W)

d.) (i) As the lengths (L,W, tox) are scaled down by k, so are the voltages scaled down by k to keep the field constant.

Most students have simply states that field is kept constant (which is obvious from the question) and have not made the simple comment that this is done by scaling voltage down – which was the essence of this question.

(ii) Area scales down by k^2

(iii) If field effect and field has to be constant, reducing thickness alone has a bottleneck of reducing gate impedance due to SCLC and tunnelling. Therefore field effect is maintained by increasing permittivity rather than reducing thickness. Therefore high permittivity dielectrics are useful.

(2marks each)

14.)

(a) (i) looking for a good diagram and a proper derivation with the following comments.

- how to pick wavelength
- why the diffraction happens
- what is the path difference
- what is the Bragg condition.

Most students have been very sketchy, but the marking has been generous.(5 marks)

(ii) Direct application of the formula. Ans: 2.29e-10 m

(b) (i) Looking for a good explanation supported with diagrams explain the entire process of patterning with photoresist. Key points

- how to deposit the resist
- masking
- exposure - comments on +ve and –ve resist
- etching and resist removal

Specifics of chemicals used was not required but has been rewarded if mentioned.

(ii) Direct use of formula. Ans: 1.45e-10 (1 mark)

(iii) Direct use of formula – Ans 7e-12 being the under estimate. Estimate is actually larger due to scattering. (1 mark)

(c) Clear explanation supported by diagram. Looking for two key words: During writing – the importance of overcoming coercitvity ; During reading – a mention of the relation between the field, velocity of tape and current/emf sensed. Generously marked.

(d) (i) Any answer that mentions the wide band-gap and its application in high voltage, high power, high speed or optoelectronic (UV/blue LED) devices is marked positively.

(ii) Looking for a diagram with 3 comments: ~ low field, the linear relation between drift velocity and field, ~ velocity saturation , ~ gunn effect i.e. velocity drop due to interband movement of electron increasing their effective mass.

(3 points to each part)

SECTION F: Information Engineering

15 (a) (i) The main problem with the *RGB* space is that each colour affects the apparent brightness (luminance) by different amounts, and the human eye is much more sensitive to changes in luminance than it is to changes in colour (chrominance). Usually most of the information about a scene is contained in its luminance rather than its colour (chrominance).

To allow us to take advantage of the different ways that the human visual system responds to luminance and chrominance components of an image, we can move to the YUV colour space. Here Y is the *luminance* (brightness) component, and U and V are two colour-difference or *chrominance* components, indicating how far away from grey the colour is, in the blue and red directions respectively.

HSV stands for *Hue, Saturation, Value*. It is a non-linear version of some of the YUV ideas, except that it codes the chrominance information using polar coordinates (H,S) instead of cartesians (U,V).

The Value component, V, of HSV indicates the amplitude of the largest component out of R, G and B. It represents the *approximate* intensity (luminance) of the pixel. The Saturation component, S, is the difference between the value V and the smallest of R, G and B. It represents how far the colour of the pixel is from some shade of grey. The Hue component, H, is a function of the colour of the largest component, adjusted by the other components.

The advantage of an image being in *HSV* form, is that the apparent brightness can be controlled by V, the strength of colour by S, and the colour tint by H. This independent control of brightness, saturation and tint is not possible with either RGB or YUV, although YUV does allow control of the brightness with just Y. [4]

(ii) If $U=V=0$, then $B=Y$ and $R=Y$. Hence $Y=0.3Y+0.6G+0.1Y$, so that $0.6Y=0.6G$, so $G=Y$ and therefore $R=G=B$, which is some form of grey. [2]

(iii) If we substitute for Y in the expressions for U and V we have

$$U=\frac{\alpha}{8}(-0.3R-0.6G+0.9B)$$

$$V=\frac{\beta}{8}(0.7R-0.6G-0.1B)$$

Therefore, the minimum value of U occurs when $[RGB]=[110]$, giving $U_{min}=-\frac{\alpha}{8}0.9$. The maximum value of U occurs when $[RGB]=[001]$, giving $U_{max}=\frac{\alpha}{8}0.9$. If $U_{min}\geq -0.5$, we need $\alpha\leq\frac{0.5\times 8}{0.9}=4.4$, so α is 4. U_{max} will then also be ≤ 0.5.

Q16 Q~~4~~(a) (i) Remove high frequency noise which is amplified by differentiation.

$$g_\sigma(x) \rightleftharpoons G_{\sigma'}(f) \qquad \sigma' \propto \frac{1}{\sigma}$$

Use Gaussian kernel.

(ii). $$S(x,y) = \sum_{-n}^{n} \sum_{-n}^{n} I(x-u, y-v)\, g_\sigma(u)\, g_\sigma(v)$$

(iii) $$\nabla S(x,y) = \nabla (G_\sigma * I)$$

$$S(x+\Delta x, y) = S(x,y) + \Delta x \frac{\partial S}{\partial x} + \frac{\partial^2 S}{\partial x^2}\frac{\Delta x^2}{2} \ldots$$

$$S(x-\Delta x, y) = S(x,y) - \Delta x \frac{\partial S}{\partial x} + \frac{\partial^2 S}{\partial x^2}\frac{\Delta x^2}{2}$$

$$\therefore \frac{\partial S}{\partial x} = \frac{S(x+\Delta x) - S(x-\Delta x)}{2\Delta x}$$

or.

$\longrightarrow x$

$\frac{1}{2}$	0	$-\frac{1}{2}$

$\downarrow y$

$\frac{1}{2}$
0
$-\frac{1}{2}$

2(b)

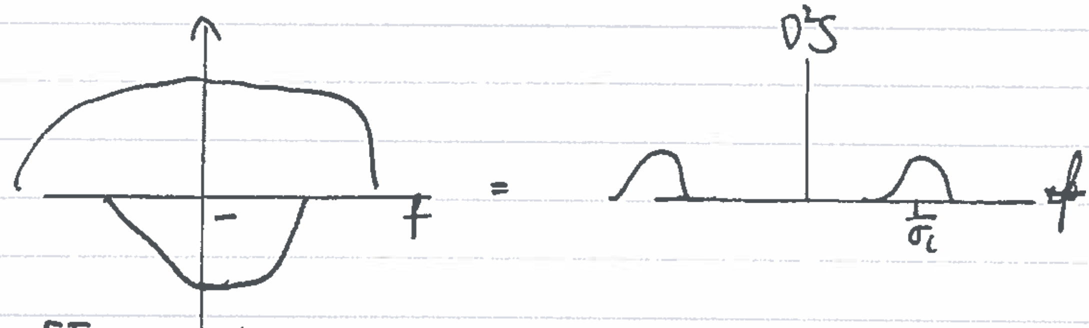
(i). Band-pass filtering —— Laplacian of a gaussian.

$$\nabla^2 (G_\sigma * I) = \nabla^2 S_\sigma(x,y)$$
$$\simeq S(\sigma_{i+1}) - S(\sigma_i)$$

Implement $S(x, y, \sigma_i)$ as an image pyramid

where $\sigma_{i+1} = 1.2 \rightarrow 1.5\,\sigma_i$

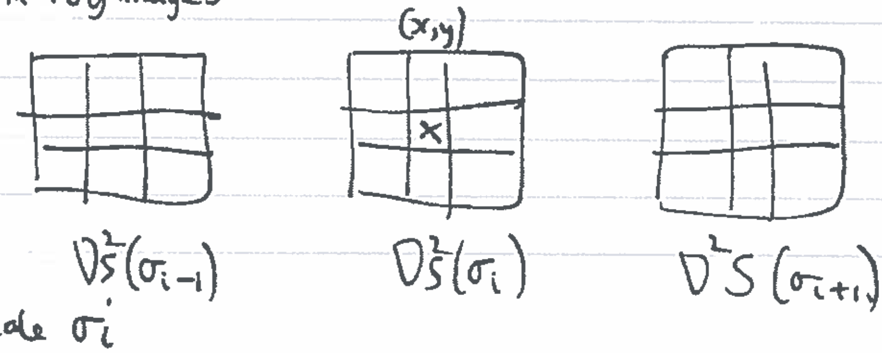
This is a gaussian pyramid. Band-pass filter since difference of 2 low-pass filters in frequency domain.



Also: $\frac{d^2 G}{dx^2} \xrightarrow{FT} -\omega^2 G(\omega)$ — low pass filter.

(ii) Look for max/min of $\nabla^2 S_{\sigma_i}$ by searching over scale-space

26 neighbours in DOG images



Position (x,y) and scale σ_i

B

2(c)

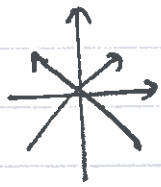
(i). Sample 16x16 pixels from $S(x, y, \sigma_i)$ at (x_i, y_i)

Compute $\nabla S(x, y, \sigma_i) \rightarrow$ gradient magnitude and orientation

[Produce histogram in bins of 10° + look for maximum ≡ dominant orientation
Smooth histogram by blurring $* g_\sigma(1.5\sigma_i)$

Sample 16x16 at dominant orientation aligned to 0.

(ii) SIFT
- Produce 16x16 patch
- Compute ∇S at each pixel
- Weigh by $G_\sigma(x,y)$
- Bin into 4x4 ~~bins~~ cells = 16 cells
- Each cell look at orientation histogram (8 directions)



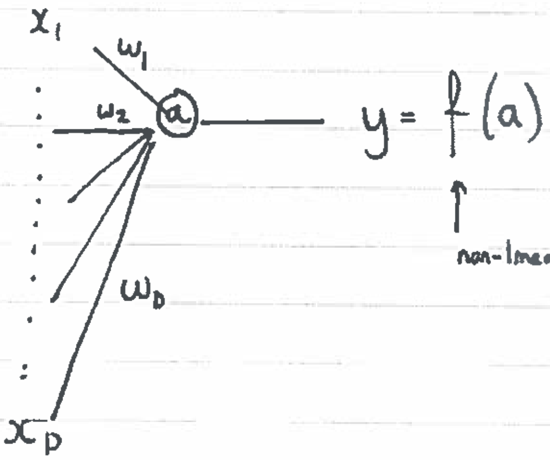
- Produce 128 D vector by concatenation. Normalize
- threshold any element > 0.2
- renormalize to unit vector (128D).

Invariant to lighting by using gradients and exact alignment (histogram) and scale (blob centre + scale) and orientation (dominant orientation)

(iii). Look for NN in 128 descriptor by using euclidean distance
Accept if $\frac{x \cdot x_2}{x \cdot x_1} < 0.7$ where x_1 is NN and x_2 is 2nd match.

Q17 Q8.

(a) (i).

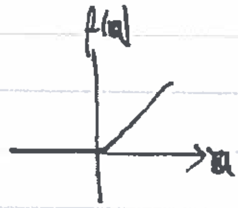


$$\therefore \; y(\underline{x}) = f\left(\sum_{i=1}^{D} w_i x_i \right)$$

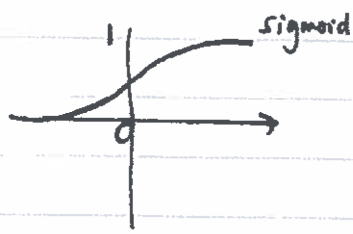
(ii) Convolution ⟶ filter response to create a filter/feature map
⟶ repeated weights for each position/neuron (shared)

(iii). $y = f(a)$ where $a = \sum w_i x_i$

$f(a) = \text{ReLU}(a)$



or $f(a) = \dfrac{1}{1 + e^{-a}}$



or $f(a) = \tanh(a)$

Non-linear in a.

17 Q 3 a (iv) Pooling function

Sub-sample to reduce size (dimension) of vector $\underline{x}$

eg max-pooling uses maximum from each cluster to represent o/p's into a single input to next layer neuron.

Downsize from 2x2 to 1x1. by taking maximum.

(b)(i) Supervised learning

Train with i/p data $\underline{x}_i$ and labelled o/p y_i pairs

ii) Training data —
Validation data — use to set hyper-parameters: learning rate, # parameters, batchsize
Test data — never seen and used to evaluate performance
Epoch – run once through each dataset.

(iii) Loss function = fn(data, weights) — eg. L2 norm or cross-entropy

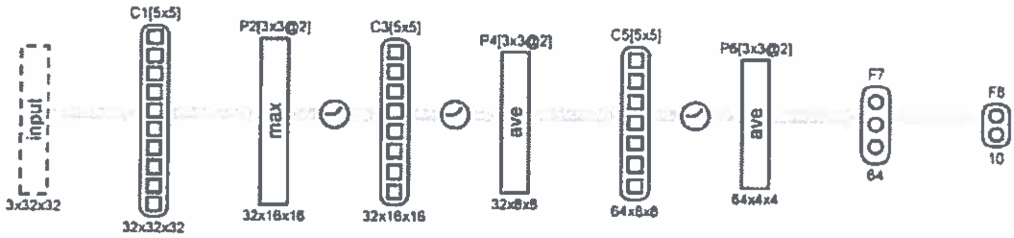
$$L2 = \sum_D (y_i - f(w, \underline{x}_i))^2$$

Minimize loss wrt weights by gradient descent and backpropagation

(iv) Stochastic gradient descent. $\underline{w}^{t+1} = \underline{w}^t + \eta x_n t_n$
Hyper-parameters:

Anatomy of a Deep Net

Using these new building blocks, we can now fully examine the anatomy of a modern deep net for CIFAR-10.



We can compute the number of parameters per trainable layer: Channels $\times$ Size^2 $\times$ Nodes.

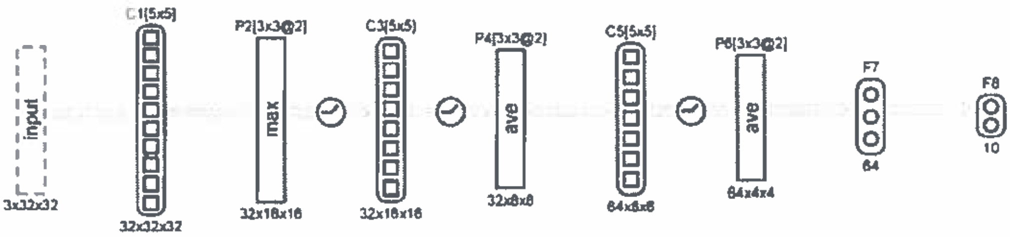
Name	Channels	Size	Nodes	# Parameters
C1	3	5	32	2400
C3	32	5	32	25,600
C5	32	5	64	51,200
F7	1024		64	65,536
F8	64		10	640
Total				145,376

We can also calculate the input and output dimensions. For convolutional and pooling layers, output is a function of stride, size and padding: $dim_o = \lfloor \frac{dim_i + 2 \times \text{padding} - \text{size}}{\text{stride}} \rfloor + 1$.

Name	Input	Size	Stride	Pad	Nodes	Output
C1	3x32x32	5	1	2	32	32x32x32
P2	32x32x32	3	2	1		32x16x16
C3	32x16x16	5	1	2	32	32x16x16
P4	32x16x16	3	2	1		32x8x8
C5	32x8x8	5	1	2	64	64x8x8
P6	64x8x8	3	2	1		64x4x4
F7	1024				64	64
F8	64				10	10

Anatomy, cont.

Now that we understand the parameters and the dimensionality, let us look at this in more detail.



Each layer in the network plays an important role.

C1 Extracts low-level features in the image, like edges, corners and blobs.

P2 Provides some flexibility of location to the low-level features

C3 Looks for parts that are combinations of lower-level features

P4 Smoothes the part responses before subsampling

C5 Finds structures that are built from parts

P6 Smoothes and subsamples the structural responses. The output of this final layer acts as a CIFAR-10 specific feature vector of length 1024.

F7 Sub-category classifiers

F8 Final classifier

Engineering Tripos Part IB Paper 8 2018

Section F: Information Engineering Elective Assessor's Report

Section F was taken by a total of 115 candidates. The candidates were required to answer 2 from 3 questions. The examination was without incident. The average raw mark obtained for Part IB students was 33/50 (66%).

1. **Q15 Image processing: Colour spaces and high-pass filtering.**

   A popular and straightforward questions on the material of Part A and attempted by 106/115 candidates with raw average mark 17.9/25 (72%). Marks were lost on describing the advantages of the alternative colour spaces.

2. **Q16 Image matching: Gaussian convolution, band-pass filtering and feature description.**

   Attempted by 95/115 candidates, average mark 15.9/25(64%). A popular question which covered the material of Part B. Most candidates failed to show how the orientation of the features could be determined from the 16×16 patch of pixels around the feature centre and giving details of the methods to find correspondences.

3. **Q17 Single-layer perceptron and CNNs.**

   Attempted by 29/115 candidates, average mark 14.7/25 (59%). An unpopular question on the material of Part C. Role of convolution in neural networks (to generate feature maps and reduce parameters) was poorly explained.

Roberto Cipolla 18/6/2018

Version ML/2 (based on CYB/5) – CRIB

SECTION G

Bioengineering

18 (a) Describe the main limitation of *pit eyes* and the three main improvements that have evolved to improve on it. [4]

Answer: Pit eyes indicate the overall level of illumination in the environment and provide a vague indication about the direction of light, but their main limitation is that they do not form images. The three main improvements are: (1) the pin hole camera, in which an image forms thanks to a small aperture, (2) the lens, and (3) the mirror, which form an image by focusing light, thus using refraction (lens) or reflection (mirror) to break the contrast-resolution trade-off of pin hole cameras.

(b) A neuron's response, $r = a\,s + \epsilon$, is a linear function of a normally distributed stimulus, $s \sim \mathcal{N}(\mu_s, \sigma_s^2)$ with mean μ_s and variance σ_s^2, where the neuron's sensitivity is $a > 0$ and it has normally distributed response variability $\epsilon \sim \mathcal{N}(0, \sigma_r^2)$ with variance σ_r^2. (Note that, for simplicity, we assume the neural response can take on negative as well as positive values.)

(i) Using the parameters defined above, provide the formula for the information about the stimulus encoded by the response of the neuron. [3]

Answer: For efficient coding, we need to maximise the mutual information, I, between s and r. Based on the lecture notes, $I = \frac{1}{2}\log\left(\frac{a^2\,\sigma_s^2+\sigma_r^2}{\sigma_r^2}\right)$.(Note that the lecture notes show the result for the special case $\mu_s = 0$, but it is nevertheless directly applicable as μ_s only changes the mean of the response distribution which in turn does not affect its entropy and thus information.)

(ii) The energy consumed by the cell when emitting response r is $\beta\,r^2$, where $\beta > 0$ is a positive constant. Using the parameters defined above, provide the formula for the average energy consumed by the cell. [3]

Answer: The marginal distribution of the responses is $\mathrm{P}(r) = \mathcal{N}(r; a\,\mu_s, a^2\,\sigma_s^2 + \sigma_r^2)$, and so the average energy consumed by the cell is $\int_{-\infty}^{+\infty} \mathcal{N}(r; a\,\mu_s, a^2\,\sigma_s^2 + \sigma_r^2)\;\beta\,r^2\,\mathrm{d}r = \beta\,\left[a^2\,\left(\mu_s^2 + \sigma_s^2\right) + \sigma_r^2\right]$.

(iii) Express the optimal value of a as a function of the other parameters for efficient coding under energy constraints when the average energy consumed by the cell cannot be more than E_{max}. [4]

Answer: As both information and energy increase with a (when $a > 0$), information must be maximised when energy is maximised. This is achieved when $a = \sqrt{\frac{E_{max}/\beta - \sigma_I^2}{\mu_s^2 + \sigma_s^2}}$.

(c) With regard to efficient coding in colour vision, explain the following.

(i) What mathematical technique can be used, and how can it be used, to apply the theory of efficient coding to colour vision? [5]

Answer: Efficient coding of the spectra of natural images can be formalised by principal components analysis (PCA). In practice, this means that each image can be represented by a vector describing the intensity of each wavelength in the visible spectrum within that image. Distinguishing between N different wavelengths, this means that each image can be characterised as a point in an N-dimensional space, and so our data (the ensemble of natural images) is a cloud of points in this space. PCA finds a smaller number, M, of (orthogonal) dimensions in this space that capture most of the variance of the data. (Optional: the kth principal component is the kth largest eigenvalue of the data covariance matrix.)

(ii) What aspects of colour vision does efficient coding account for? [4]

Answer: The first three principal components of natural image spectra are (1) a constant power spectrum (overall luminance), (2) one that is the difference between (the intensity of) long (red) and medium wavelength colour (green), and (3) one that is the difference between (the intensity of) short (blue) and the sum of (the intensity of) long and medium wavelength colour (red+green=yellow). These correspond to the three types of retinal ganglion (or LGN) cells: M-cells which are not colour sensitive, and two types of P-cells which have receptive fields composed of a red centre and a green surround (or vice versa) or a blue centre and yellow surround (or vice versa). The P-cells give rise to the phenomenon of colour opponency, whereby we are unable to perceive a colour that would be a mixture of two colours that correspond to the two opponent parts (so called complementer colours) of a receptive field (greenish red or yellowish blue).

(iii) What experiments can be used to demonstrate these aspects? [2]

Answer: The phenomenon of colour opponency can be demonstrated by complementary-colour afterimages, when we saturate (fatigue) the responses in one part of such a channel (e.g. yellow) after which the threshold for the opponent (complementary) colour is lowered, such that even an otherwise white object is seen in this opponent colour.

19 (a) For both the *fundus camera* and the *scanning laser ophthalmoscope*, describe the way in which the imaging data are constrained to a particular depth in the eye, also noting what properties affect the depth resolution. [5]

Answer: For the Fundus Camera, the only constraint to the depth is the axial location of the focal point. Light will be received over a smaller region at the focus of this lens than far from the focus, with lenses of higher NA giving better axial resolution.

The Scanning Laser Ophthalmoscope uses confocal optics: the reflected light is focused through a small aperture, and it is the size and position of this aperture (as well as the focal point of the imaging lens) which determines at which depth the SLO is most sensitive. Making the aperture smaller improves the depth resolution, but also allows less light to pass through, decreasing the sensitivity. Moving the aperture changes the depth location: a fairly low NA lens is needed to allow for variation in depths.

(b) Figure 7 shows a lens, with diameter D and focal length f, which focusses a light beam through an aperture of diameter d in a plate of thickness t. The aperture is located at the focal point of the lens, and the radial extent $r(x)$ of the light is given by:

$$r^2(x) = r_0^2 \left[1 + \left(\frac{\lambda x}{\pi r_0^2} \right)^2 \right]$$

where x is the horizontal distance measured from the focal point, r_0 the radius at the focal point, and λ is the wavelength of light. All components are in air.

(i) Define the Numerical Aperture (NA) of the lens, and show that it can be related to r_0 by:

$$r_0 \approx \frac{\lambda}{\pi \, \text{NA}}$$

[6]

Answer: NA is given by $n \sin\theta$, where θ is the maximum subtended angle of light entering the lens,

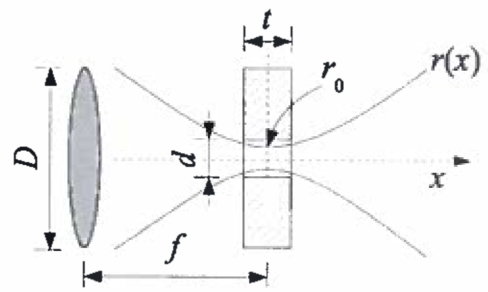


Fig. 7

(cont.

and n (the refractive index) is equal to 1 for air. Hence:

$$\begin{aligned} \text{NA} &= \sin\theta \\ &= \sin\tan^{-1}\frac{D}{2f} \\ &\approx \frac{D}{2f} \end{aligned}$$

The lens is located at $x = -f$ and at this point for the light to pass through the lens, $r(x) = \frac{D}{2}$. Substituting these into the given equation, and noting that $D \gg r_0$:

$$\begin{aligned} \left(\frac{D}{2}\right)^2 &= r_0^2\left(1+\left(\frac{-\lambda f}{\pi r_0^2}\right)^2\right) \\ &\approx r_0^2\left(\frac{-\lambda f}{\pi r_0^2}\right)^2 \\ &= \left(\frac{\lambda f}{\pi r_0}\right)^2 \end{aligned}$$

Hence:

$$\begin{aligned} r_0 &= \frac{2\lambda f}{\pi D} \\ &\approx \frac{\lambda}{\pi \text{NA}} \end{aligned}$$

(ii) What is the maximum value of λ, in terms of NA, t and d, for which all the light within the radius $r(x)$ will pass through the aperture? [6]

Answer: At each edge of the aperture, we need $r(x) = \frac{d}{2}$ at $x = \frac{t}{2}$. Substituting these into the given equation:

$$\begin{aligned} \left(\frac{d}{2}\right)^2 &\geq r_0^2 + \left(\frac{\lambda t}{2\pi r_0}\right)^2 \\ &\geq \left(\frac{\lambda}{\pi \text{NA}}\right)^2 + \left(\frac{t\text{NA}}{2}\right)^2 \end{aligned}$$

Hence

$$\begin{aligned} \lambda &\leq \pi\text{NA}\left(\left(\frac{d}{2}\right)^2 - \left(\frac{t\text{NA}}{2}\right)^2\right)^{\frac{1}{2}} \\ &\leq \frac{\pi\text{NA}}{2}\left(d^2 - (t\text{NA})^2\right)^{\frac{1}{2}} \end{aligned}$$

(iii) What is the minimum thickness of the plate t, in terms of NA and d, for which there is no value of λ at which all light within the radius $r(x)$ would pass through the aperture? [3]

 BAD PAGEBREAK

Answer: We need $\lambda \leq 0$ in the previous equation, in which case:

$$d^2 \leq (t\text{NA})^2$$
$$t \geq \frac{d}{\text{NA}}$$

(iv) Suggest reasonable numerical values for d and t when using this aperture in an optical imaging device with a lens of NA $= 0.5$. [3]

Answer: The previous answer suggests that the thickness of the plate must be no more than $\frac{d}{\text{NA}}$, in this case about twice as thick as the aperture diameter. For wavelengths up to 1 μm, then (from (b)(i)) $r_0 \approx 0.64$, hence $d \approx 1.3\,\mu$m, and $t \approx 2.6\,\mu$m. In reality we may want to use a slightly larger aperture to let more light through for a greater range of scanning depths.

(v) What are the consequences of your answer to (b)(ii) for the spectrum of light passing through such an aperture? [2]

Answer: The implication is that longer wavelengths will have more trouble passing through the aperture, in which case we would expect them to be attenuated with respect to light at shorter wavelengths.

20 (a) Compare the composition and structure of the *cornea* and *crystalline lens* tissues in the eye, detailing how their structure relates to their function. [8]

Answer: The lens is about a cm across and half a cm thick. The transparent, biconvex lens structure changes shape to change focus. Changes in stiffness with age affect the ability of the lens to be deformed. There is an exterior capsule that contains the lens, which is in two parts, the nucleus and the cortex. The nucleus is older lens fibers and the cortex is the newer lens fibers; the capsule is the source of new lens fiber cells. The lens is made up of "lens fibers" which are specialised elongated epithelial cells surrounded by unusual proteins called crystallines (30% by mass). The overall structure of the lens is complex and "onion-like" in terms of being in layers. There are no blood vessels in the lens.

The cornea is effectively a fibre-reinforced hydrogel. The fibres are collagen, the most ubiquitous protein in the body and the basis of strength for most structural tissues. The "matrix" is a highly hydrated sugar and protein gel. Collagen in the cornea is crystalline: it is organized into very regular perpendicular lamellae with uniform spacing between individual collagen fibrils. The lamellar structure is important both mechanically and optically. Mechanically, the lamellar organization provides resistance to intraocular pressure and allows the cornea to serve its critical function of providing two thirds of the optical power of the eye overall. There is also preferential orientation towards the edges to support muscle attachment. Optically, the regular crystalline structure allows for corneal transparency, again required for unconstructed vision. There are no blood vessels in the cornea either.

The cornea is a very "typical" soft biological tissue in terms of its composition and structure whereas the lens is quite atypical.

(b) Explain the individual contributions to visual focussing of the cornea and crystalline lens. [4]

Answer: The crystalline lens sits behind the iris and contributes 1/3 of the total focussing power of the eye. Lens accommodation is the process of lens shape change that allows the eye to adjust for focus on objects nearer or further away. Lens curvature is controlled by ciliary muscles, and by changing curvature, one can focus the eye on objects at different distances. Amplitude of accommodation is the max amount that the lens can accommodate in diopters (D), equal to the reciprocal of the focal length measured in metres. The lens continually grows throughout life, laying new cells over the old cells, which results in stiffening of the lens as well as growth of the lens size with age, which both contribute to the decreased ability of the lens to accommodate with age.

The cornea contributes 2/3 of the total focussing power of the eye but is fixed focus, there is no accommodation. LASIK surgery is sometimes used to improve vision by altering the local thickness of the cornea to adjust its fixed focus.

(c) Describe how problems with the cornea and crystalline lens can interfere with vision and how these issues can be treated. [5]

Answer: Glasses or contact lenses can be used to correct long or short-sightedness associated with defects in focussing power associated with shape errors.

LASIK (laser surgery) can be used to adjust the shape of the cornea by vaporising parts of the cornea, after first removing a flap to expose the stroma. The flap is closed and heals naturally.

A cataract is a cloudy or opaque lens, and is the leading cause of blindness in the developing world, and a major problem with older people in the developed world. In treating cataracts, the lens is removed completely and replaced with an artificial intraocular lens (IOL). The artificial lenses were originally very stiff, glassy polymers with elastic moduli in the range of GPa. However, more modern lenses are flexible and rubber-like, with moduli in the MPa-range. This enhances the minimally invasive nature of the surgery, as the flexible rubbery lenses can be folded up for deployment, and thus the incisions needed for surgery are smaller. IOLs are fixed focus, so the 1/3 of total eye focussing power that was once associated with lens accommodation is gone. Multifocal and accommodating IOLs are under development.

The cornea can become torn or opaque due to injury, illness, or congenital factors. It must be replaced with a donor cornea because there is as of yet no alternative polymer or tissue engineering solution. Keratoconus is when the cornea becomes thinner and misshapen, bowing out and causing vision problems. This can occur after LASIK surgery since it purposefully thins the cornea.

(d) Figure 8 illustrates a standard linear material model, with parameters E_1, E_2 and η.

(i) How do the different elements of the model affect the response of the material to mechanical loading? Illustrate your answer with sketches of the response to different loading cases. [4]

(ii) Describe how the material properties E_1, E_2 and η for an eye lens could be measured. [4]

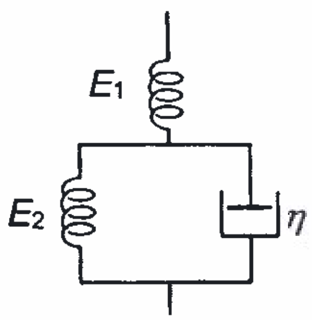


Fig. 8

Answer:

(i) E_1 defines the long-term response to steady loading. E_1 and η together give the time dependant response with an appropriate time constant. Sketches as per the notes from 1B materials.

(ii) An indentation test would be a good way of measuring the overall properties, though this would be an averaged value and would need to have the properties extracted by analysis. Cutting up small parts of the lens might be effective, again using compression loading or indentation. Sinuisoidal loading at an appropriate frequency, or step or hold loading, will be needed to extract the time-dependent properties, perhaps by using a DMA.

END OF PAPER

Part IB Paper 8 2018 Section H CRIB:

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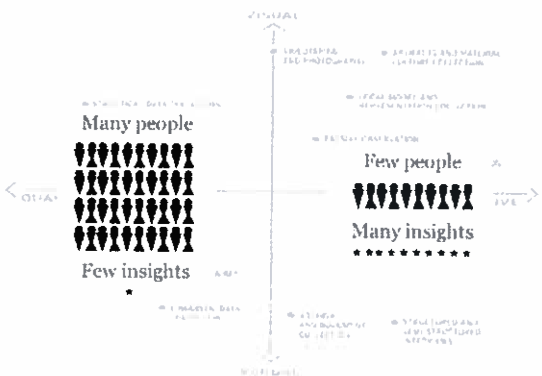
(a) Many inventions arise from the realisation that 'we can do it' = ***technology push.*** *Possible examples could include:*

1. *The Post-It note arose by accident*
2. *The DVD player arose by analogy and the Dyson was a transfer of an existing industrial technology to a new domestic applications*
3. *The domestic breadmaker arose from a structured search for new kitchen appliances*
4. *The 'inerter' arose due to a gap in an existing map of possibilities*
5. *New materials allowed the hair dryer to move from an expensive metal body to a cheaper plastic body.*

The difficulty of inventions of this type is that they may reflect the inventors belief that "this ought to be useful" rather than a group of customers' statement that "we want that." So an alternative source of inventions is driven by customers = ***market pull.*** *Possible examples could include:*

1. *The 'aural' thermometer for babies arose from the difficult experience of using conventional mercury thermometers measuring babies temperatures with*
2. *The chopper bicycle arose from modifications to existing bikes by enthusiastic users*
3. *Fridges and washing machines are now sold as fashion items as the kitchen has become the main entertaining room*
4. *The ink-jet printing industry around Cambridge has grown due to legislation on sell-by dates for food*
5. *The model-T Ford was successful because Ford found ways that by making cars cheaper he could turn a luxury product into a common one*

(b) User observation is one way of getting information about customers' needs that customers themselves might be unable to articulate if asked. It sits in the top right-hand quadrant of the figure below:



Basic answers should draw upon the material below to describe where and how they would observe potential users:

There are many areas, where people either cannot answer direct questions, are too embarrassed to answer, or simply do not know the answer.

For example "what would you like me to cover in a lecture on understanding user needs?"

Observation enables insights on issues that the consumer might not have considered for themselves.

For example, "what makes you cross when cooking?" might elicit some interesting answers, but seeing someone struggling to open a tin of tomatoes might provide some real inspiration.

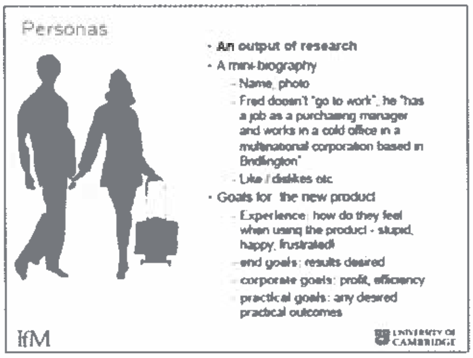
But, be careful, as people modify their behaviour when they know they are being studied, and are often a lot more careful about what they are doing.

Observation can also be time consuming to do and analyse, and therefore costly.

Observation will not answer 'why' any particular action has been done. For that reason, it is often good to combine observation with interviews. For example "I noticed that you frowned when you opened the case, why was that?" or "the pen obviously makes you cross, why did you buy that particular one?"

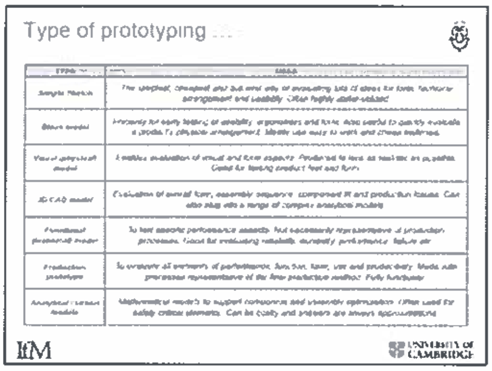
For the example of the bicycle helmet, given the Cambridge is such a cycling city, you could go and stand by some bike racks and observe how cyclists deal with the putting-on, taking-off, and storing of their helmets to see where there areas that could be improved. You could see how student come in and leave the department or railway station on bicycles to get some additional input. This could then allow you to develop some hypotheses that could then be tested through interviews.

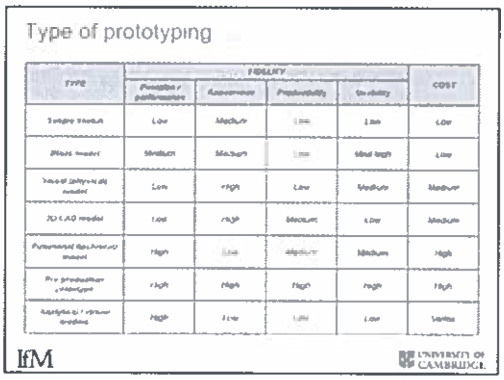
<u>Personas</u> are a representation of a key stakeholder, a hypothetical 'archetype'. They remove the subjectivity of design choices, and help bring the voice of the customer into the design team.



For the specific case of the development of a new bicycle helmet, you could create personas such as the young Cambridge student, the London commuter, and family weekend cyclists, etc and see that this might reveal about cycle helmet needs and designs.

(c) The material below provides that generic structure that can be used to describe the development of a bicycle helmet. It should also consider context-specific issues (i.e. a bicycle helmet and its materials will need to pass standard tests for safety, etc).





22

(a)

(i) Licensing of IP

Pros = Get the cash quite fast, little responsibility for building the business/achieving success, little further funding /capital investment needed. Cons = Get relatively small % of value, need to find and manage licensees.

Examples = ARM (IP for semiconductor)s; Microsoft and Windows; Pilkington and the float glass process.

(ii) Manufacturing and selling a product

Pros = Increase revenue with higher volumes, focus on single, repeatable activity, costs required can make barriers to prevent others entering the market; Cons = Need to develop, make, distribute, support; high capital costs (unless outsource manufacturing – but then have problems of managing contract manufacturers), rigidity of operations.

Examples: Ford, Toyota, Boeing, Airbus, Apple, etc.

(iii) Manufacturing and selling a product with consumables

Pros = As (ii) but with additional benefit of on-going revenue stream from customers, plus on-going connection with customers enabled by sale of consumables to ensure direct line of feedback. Cons = have to develop infrastructure for selling consumables; may not be able to 'lock-in' customers to your consumables, as other companies may be able to provide consumables.

(iv) selling a service

Pros = No manufacturing costs – hence lower capital requirements to get started, easy to launch, flexible – easy to pivot (in some cases); Cons = Can be hard to scale up the business (in many cases can only grow through recruiting and training lots of people), how to cope with peaks and troughs in demand, how to deal with low barriers to entry.

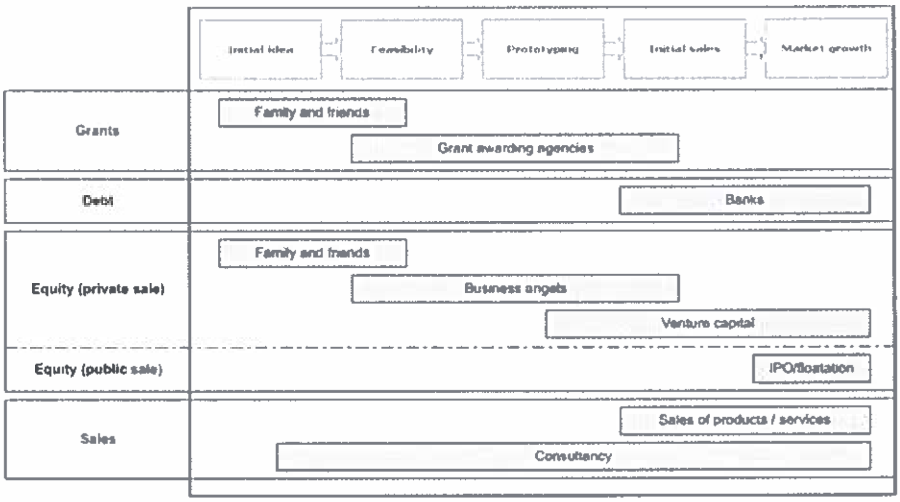
Examples: McKinsey, Uber, AirBnB, Expedia.

(b) *Grant = a 'gift' from an organisation such as InnovateUK or Gates Foundation. The type of projects that are eligible for funding through this route may be very restricted. It does not require repayment, but make have conditions attached that limit how the money can be spent.*

Debt = borrowing money from a bank or specialist finance organisation. You can only borrow money if you can convince the bank that you can repay the money, plus interest, exactly when they want it. This is usually very hard for a new company to do, as banks want to see a track record of successful commercial performance to reduce the bank's risk.

Equity = selling part of the ownership (shares) of your business in return for cash. The assumption is that whoever buys part of your business will wish to sell this part of the business to someone else in the future at a higher price. The only reason for doing this is if the investor can be convinced that: (a) the business is really going to grow (and quite fast) and that (b) there will be someone else willing and able to buy their share in the company at a later date at a higher value. Sources of private equity are: business angels and venture capital funds. Public equity is the selling the company shares on a public market, but this is only possible for firms with a reasonably long commercial track record (though this depends on which market the shares are floated).

(c)



(d) *The market - Who has the problem that you attempting to fix? What is the scale of the problem?*

Product or service - What solutions are going to be used to address the problem? Can you prove that it works? Have you tested it with real customers?

Management team - Who is going to do it? What is their track record?

Business operations - How are you going to do it? What will you do yourself, and what will you get others to do? How will you ensure the product gets manufactured, and how will you ensure the availability of consumables?

Financial projections - How and when will money be made? On what evidence are these assumptions and projections made? Are they convincing?

Marketing strategy - How will get people to buy your product / service? Do you understand the buying patterns of people/companies in your target market? How will you ensure that the buyers come back to you for consumables?

*Resources required - What do you need to start your business? What are you going to spend the money on? – there is the manufacturing of the product **and** the distribution of the consumables. Where will you get the resources from? What if you can't get all the resources you need?*

Exit opportunities – If you are going for a equity funding, how will your investors get their returns? Is there a clear exit route via a possible trade sale (if so, who might be the buyer) or is the intent to go for an Initial Public Offering? If so, where and when?

23

(a) *(i) it must be **novel**;*

*(ii) involve an **inventive step**: i.e. not be obvious to someone in the light of what has been done before (the 'prior art');*

*(iii) have a **practical application**: be capable of being made or used in some kind of industry; and*

*(there are also issues of **exclusion** that need to be considered (e.g. scientific theory or mathematical method, method of doing business, perpetual motion machine)*

(b)
- *A patent provides ensures that the nature of your invention is clear, and that if anyone attempts to copy your idea, you have a clear reference point around which to build a legal defence.*
- *There are costs associated with patenting (though the filing is free, the cost of legal advice and processing will be £5-10k. For international coverage, these costs can grow to >£100k. There is then the annual fee to be paid, and this will increase each year.*
- *Having the patent as the basis for legal action against someone suspected of infringing is just the starting point. Fighting a legal action can be extremely costly and time consuming. The Dyson and Kearns examples described in class provide examples that illustrate the time, costs and potential payouts possible. Recent cases between Samsung and Apple also provide interesting examples.*
- *A patent is basically a public disclosure of an invention, showing the world how to do something. Even if the idea is not directly copied, it can provide stimulation for innovation in similar areas. An alternative is to keep the idea confidential. The example in class of the Bessemer paint process is a good example of this. By simply ensuring that nobody new all the stages of the process, Bessemer was able to retain a monopoly in the 'gold' paint process.*
- *The use of confidentiality agreements for employees (and suppliers) can be very effective as a means of ensuring that an idea is not communicated to others, and the value creating potential of the idea is maintained within the organisation. However, confidentiality agreements only apply to those who have signed them.*
- *You can choose just to keep things secret, but this gives you no legal protection should someone start using your idea.*

- *If you chose to keep something secret, then someone then patents your idea, you may then find that you are liable to pay them a license fee if you wish to continue using that idea.*
- *For firms seeking to raise money from investors, not having any legal protection over your core invention may make them very nervous. For some sectors (e.g. advanced materials, drug development, etc) patents are critical, and any potential investor would want to know that you have clear registered ownership before investing any money. Analysis of the patent position of a start-up is often a key part of the due diligence process undertaken by would-be investors.*

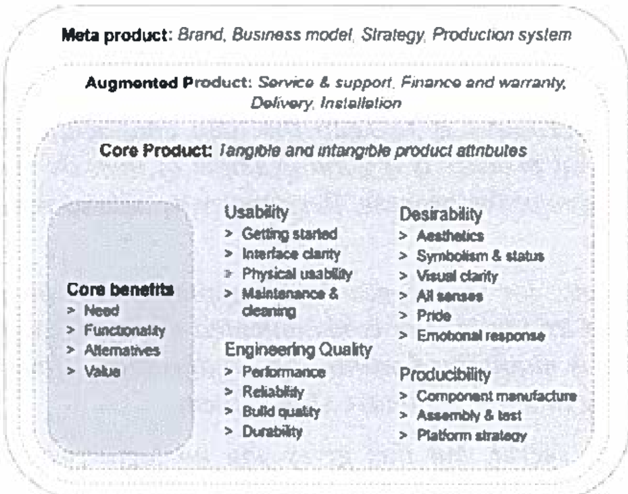
A good answer should show understanding of issues such as these. The results could be presented as a comparative table to aid the discussion.

(c) Good answers should demonstrate both clarity in explaining both the product spec and the design of the manufacturing process and the relationship between them.

Product specification:

- *Defines what needs to be designed*
- *Expresses customer needs in the 'language of the customer'*
- *Does not limit ways in which the requirements are met*
- *Provides design targets and sets design constraints*
- *Provides precise, unambiguous, measureable detail about what the product must do*
- *Enables the evaluation of solutions*
- *Evolves as new information becomes available.*

The product specification encompasses all the aspects shown in this figure, not just on the 'core product':

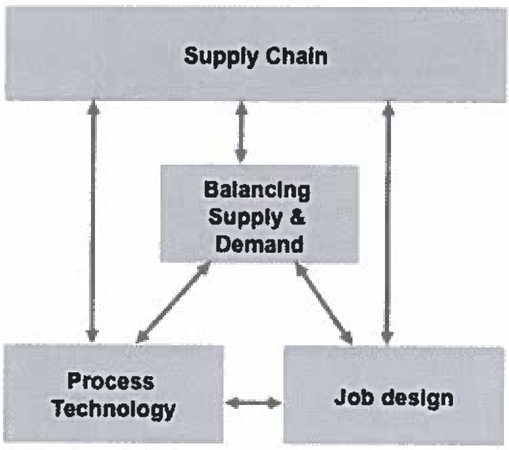


Manufacturing process design

'Manufacturing process' could refer to just the way in which the core product is made in a factory, but actually needs to encompass consideration of four core areas:

1. *How can the business balance supply and demand? Supply is the rate at which the business can make products, and demand is the rate at which customers buy them. For a new product, the demand will be uncertain, and we hope it will grow – so the business must be designed to have a flexible capacity for supply*
2. *Should production be manual or automated – and what are the effects of different choices of production technology?*
3. *What types of jobs will be created by the production system? Are these appropriate for the types of people available to work in it?*
4. *To which other companies must the business connect in order to complete its products and deliver them to customers, and how should the relationships with these companies be nurtured?*

These issues can be summed up in the figure below:

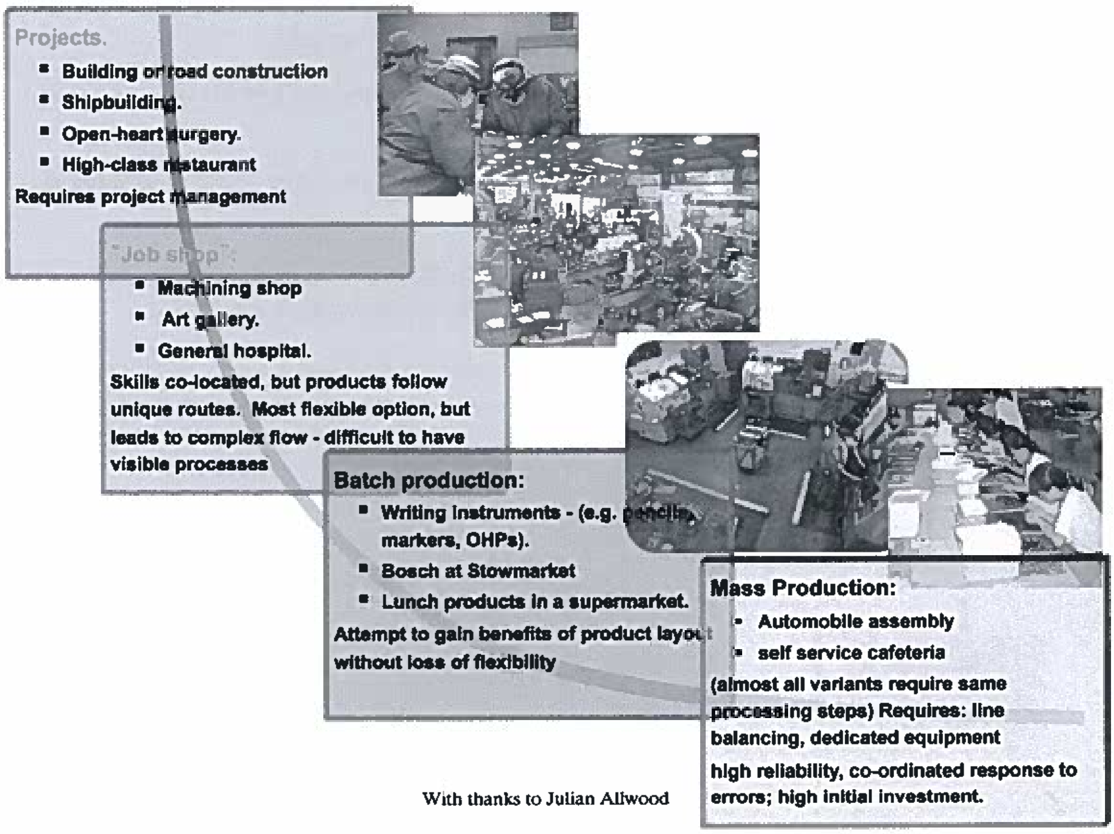


The relationship between the product specification and manufacturing process should address issues such as:

For the given product specification: How many components will be necessary? How will the components be joined? What shape of component was easy/difficult to manufacture and by what process? Will the design lend itself to the use of automated or manual processes for each component, and for assembly; how will the jobs implied by these decisions relate to availability of skills and expectations of pay wherever the factory is based; which companies will be able to supply components or raw materials and which other companies will be required to get the completed products to the customers? These issues are all related to the design of the product, and also depend on it. The choice of materials, component shapes, components and the processes required to join them are coupled.

Focusing beyond the specification of the core product, consideration could also be given to the way in which the business model suggested by the augmented and meta product could *impact the design of the manufacturing process. For example, if the product is to require regular updates and maintenance – and this could be a major part of the business model - then the choice of production processes might be constrained to, e.g., openable, replaceable parts, rather than snap-fit, integrated parts.*

(d)



With thanks to Julian Allwood

Producing at high volume requires stability and consistency to allow repeatability at low cost. Changing the process invariably messes with this and causes things to slow down, and costs to rise. One example technology that is, however, changing this is additive manufacturing / 3D printing.

Assessor's report on Part IB, Paper 8, Section H, 2018

All questions in Section H were answered reasonably well as shown in the average mark of 64.78%. No significant problems were noted.

Q21: 85 attempts, average = 64%

(a) Almost every student could explain the difference between market pull and technology push, and provide four examples of each.

(b) Most students could explain what user observation and personas are, but some gave very short or incomplete answers. The better responses were those that gave fuller responses that also put these in the context of the bike lock design and development process.

(c) All students who attempted this section could describe basic forms of prototyping, but the weaker responses were those that just listed rather than discussed strengths and weaknesses and/or did not discuss prototyping in the context of the bike lock design process.

Q22: 101 attempts, average = 65%

(a) The presentation of the relative merits of these different business models was generally well done. Stronger responses were those that gave greater levels of detail, and supported their answers with relevant and well explained examples.

(b) Most students could describe each type financing, but stronger answers were those that explained the differences between them.

(c) Almost all students could sketch a version of the required diagram.

(d) This section resulted more mixed responses. The strong answers were those that not only highlighted the particular sections of a generic business plan that would be draw most attention for a VC, but did so in the context of this particular type of business, i.e. a manufacturing (or 'hard' start-up) that was using a product + consumables business model.

Q23: 30 attempts, average = 63%

(a) Almost all students could list the three main tests, with some students also noting the additional factor (not strictly a test) relating to exclusions.

(b) This resulted in some quite vague answers, with the better ones focusing on specific issues to do with impact on the business model and future options of a firm considering these options.

(c) This section was generally not well attempted. Many answers just talked in general terms about how the design of the product might be linked to choice of manufacturing process. Some really good answers did present the required level of balance and depth as given in the crib.

(d) Almost all students could sketch a version of this diagram, but not all could provide a required explanation.

Professor Tim Minshall, 17/6/18.